GROVN[illegible]

ARTS TEACHING THE

perfect Worke and Practise of Arithme-
ticke, both in whole numbers and Fracti-
ons after a more easie and exact forme
then hitherto hath beene
set foorth.

Made by M^r. ROBERT RECORD
Doctour in Physicke.

And now lately diligently corrected and beautified with sundry new Rules and necessary Additions: And further endowed with a third part of Rules of Practise, abridged into a briefer method then hitherto hath beene published: with diuers such necessarie Rules as are incident to the trade of Merchandise.

Whereunto are also added diuers Tables and instructions that will bring great profit and delight vnto Merchants, Gentlemen, and others, as by the Contents of this Treatise shall appeare.

By IOHN MELLIS.

And now lastly corrected by *Iohn* [illegible]

LONDON,
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noster *Row*, at the signe of the
Greyhound. 1610.

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TO THE MOST mightie Prince, Edward the sixth, by the grace of God King of England, France, and Ireland &c.

THe excellēcy of mans nature being such, as it is by Gods fauour (most mighty prince) not ōly created in highnesse of degree far aboue all other corporall things, but by perfection of reason, & search of wit much approching toward the Image of GOD, as not onely the holy scriptures do testifie, but also those naturall Philosophers, which exactly did consider the nature of man, and namely the farre reach and infinite compasse of the workes of the minde, were inforced to cōfesse, that man scarcely was able to know himself. And if he would duly ponder the nature of himself, hee would finde it so strange, that it might seeme vnto him a very miracle. And thereof sprang that saying: *Magnū miraculum est homo, maximum miraculum sapiens homo.* For vndoubtedly, as man is one of the greatest miracles that euer God wrought, so a wise man is plainly the greatest.

And therefore was it that some did account the head of a man the greatest miracle in the world, because not onely of the strange workmanship that is in it, but much more of the efficacie of reason, wit, memorie, imagination, and such other powers and workes of the minde, which can more easily conceiue any thing in a manner, then vnderstand it selfe. And amongst all the creatures of God, it findeth none more difficult to be perceiued then the same powers of it selfe, whereby it doth conceiue and iudge, as it may be well coniectured by the diuersitie of opinions, that the wisest Philosophers did vtter touching the spirit of man and the substance of it: whereof at this present I intend to make no rehearsall, but whoso listeth to read thereof, may finde it largely set forth, not only in Aristotle his booke DE ANIMA, but also in Galen his booke called HISTORIA PHILOSOPHICA, and againe in Plutarch his work DE PHILOSOPHORVM PLACITIS, whose words are also repeated again of Eusebius in the 15 booke THE EVANGELICES PROPARASKEVES, vnto whom I remit them that haue desired to vnderstand the intricate difficulty of knowing our selues, as touching our best part, and that part whereby we deserue to beare the name of men.

This matter seemeth so obscure and difficult in knowledge, that Galene, who for his excellent wisedome and iudgement in naturall workes, is called of many men a Miracle in nature, yet in searching the nature and substance of the spirit of man, hee not onely confesseth himselfe ignorant, but counteth it plaine temeritie to attempt to finde it: so farre aboue the hope of mans knowledge is that part, whereby man doth know and iudge of things. And although the ignorant sort (which hate all things that they know not) doe little esteeme the profoundnesse of mans spirit, and of reason, the chiefe power & faculty of it, yet as here is a kinde of feare and obedience of vnreasonable beasts vnto man by the working power of GOD,

ſo is there in all thoſe ſmall reaſoned perſons, a certain kinde of reuerence toward wiſdome and reaſon, which they do ſhew often times, and by power of perſwaſion are inforced to obey reaſon, will they, nill they.

And heereby came it to paſſe, that the rudeneſſe of the firſt age of man was brought vnto ſome more ciuill trade, as it is well declared by Cicero in the beginning of his firſt booke, DE INVENTIONE RHETORICA, where he ſaith thus:

Nam fuit quoddam tempus, quo in agris homines paſſim beſtiarum more vagabantur, & ſibi victu ferino vitam propagabant; nec ratione animi quicquam, ſed plæraque viribus corporis adminiſtrabant. Nondum diuinæ religionis, non humani officii ratio colebatur. Nemo legitimas viderat nuptias, non certos quiſquam inſpexerat liberos, non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atque inſcitiam cæca ac temeraria dominatrix animi cupiditas, ad ſe explendam, viribus corporis abutebatur, pernicioſiſſimis ſatellitibus.

Quo tempore quidam magnus videlicet vir & ſapiens cognouit quæ materia eſſet, & quanta ad maximas res opportunitas in animis ineſſet hominum, ſi quis eam poſſet elicere, & præcipiendo meliorem reddere: Qui diſperſos homines in agris, in tectis ſylueſtribus abditos, ratione quadam compulit in vnum locum, & congregauit eos, in vnãquamq; rem inducens vtilem atq; honeſtam, primò propter inſolentiam reclamantes, deinde propter rationem atq; orationem ſtudioſius audientes ex feris & immanibus mites reddidit & mãſuetos.

This long repetition of Tullies words will ſeeme tedious to them which loue but little, & care much leſſe for the knowledge of reaſon: but vnto your Maieſty (I dare ſay) it is a delectable remembrance, and vnto me it ſeemeth ſo pleaſant, that I could ſcarce ſtay my pen from writing all that mine eies did ſo greedily read.

This ſentence of Cicero I am loth to tranſlate into Engliſh, partly for that vnto your Maieſtie it need-

deth no translation But specially knowing how far that grace of Tullies eloquence doth excell any Englishmãs tongue, and much more exceedeth the basenesse of my barbarous stile, yet for the fruit of the sentence I had rather vnto my meere English Countri-men vtter the rudenesse of my Translation, then to defraud them of the benefite of so good a lesson; trusting they will also gladly and greedily embrace all good Sciences, that may helpe to the iust furniture of the same, when they consider that informed reason was the onely instrumẽt or at least the chiefe mean to bring men to Ciuill gouernment, from barbarous maners and beastly cõditions.

For the time was (saith Tullie) that men wandred abroad in the fields vp and downe like beasts, and vsed no better order in feeding thẽ they, so that by reasons rule they wrought nothing, but most of their doings did they atchieue by force of strength. At this time there was no iust regard of religion toward God, nor duty toward man. No man had seene right vse of Mariage: neither did any man know their owne children from other, nor no man had felt the commoditie of iust Lawes: so that through error and ignorance, willful Lust, like a blind and heady ruler, abused bodily strength, as a most mortall minister for the satisfying of his desire. At that time was there one, which not onely in power, but also in wisdome was great, and he considered, how that in the minds of men was both apt instruments, and great occasion to the due accomplishment of most waightie affaires, if a man could apply them to vse, and by teaching of Rules frame them to better trade. This man with perswasion of reason gathered into one place the people that were wandring about the fields, and lay lurking in wild cottages and woods: And bringing them vnto one common society, did trade them to all such things as either were profitable or honest, although not without repining at the first, by reason that they had

not

ot beene so accustomed before, yet at length through
ason and perswasion of words, they obeied him more
iligently, & so of a wilde and cruell people he made
hem courteous and gentle.

Thus did Tullie set forth the efficacie of reason and
erswasion, how it was able to conuert wild people
o a mildnesse, and to change their furious cruelnesse
nto gentle courtesie. Were it not now a great reproch
n this our time (when knowledge raigneth so large)
hat men should shew themselues lesse obsequious to
eason, vnlesse it may be thought that now euery man
hauing sufficient knowledge of himselfe, needeth not
o harken to the perswasion of others.

Indeede he that thinketh himselfe wise, will not e-
steeme the reason of any other, be he neuer so wise: so
that of such a one it may be well said, he that thinketh
himself wiser then he is, may iustly be accoūted a dou-
ble foole: wherefore such men are not to be permitted
in open audience to talke, but must be put to silence
& made to giue eare to reason, which reason consisteth
not in a multitude of words, heaped rashly together, &
applied for one purpose, but reason is the expressing of
a iust matter with wittie perswasiō, furnished with lear
ned knowledge. Such knowledge had Moses, being ex-
pert in all learning of the Egyptians, as the scriptures
declare, & therfore was able to perswade the stubborn
people of the Iewes, although not without great pain.
Such knowledge and such reason did Druys shew,
which was the first Lawmaker of all the west partes
of Europe: like reason and wisdome did Xamolxus vse
amōgst the Goths, Lycurgus vnto the Lacedemoniās,
Zeleucus to the Locrians, Solon to the Athenians, and
Donuallo Molnutius two thousand years past amōgst
the old Britaines of this realme. And thereby it came
to passe, that their Lawes continued long, til more
perfect reason altered many of them, and willfull pow-
er oppressed most of them.

At the beginning when these wise men perceiued how hard it was to bring the rude people to vnderstand reason, they iudged, the best meanes to attaine this honest purpose, to depend of learning in euery kinde: for by learning, as Ouid saith, *Pectora mollescunt, asperitasq; fugit*: Stout stomacks doe wax mild, and sharpe fiercenesse is exil'd. Therfore, as Berosus doth testifie, Sarron that was the third King ouer all this West part of Europe, for to bring the people from beastly rage to manly reason, did erect Schooles of liberall Arts, which tooke so good successe, that his name continued in that sort famous aboue two thousand yeeres after: for Diodorus Siculus, which was in the time of Iulius Cæsar, maketh mention of the learned men of the Gothes, and named them Sarronides, that is to say, Sarron his scholars and followers.

Among these Arts that then were taught, some did informe the tongue, and make men able both to vtter aptly their mind, and also to perswade, as Grammar, Logicke, and Rhetoricke, although not so curiously as in this time: some other did appertaine to the iust order of partition of Land, the true vsing of weights, measures, and reckonings in all sorts of bargaines, and for order of building, and sundry other vses, those were Arithmeticke and Geometrie. Againe, to encourage men to the honour of God, they taught Astronomie, whereby the wonderfull workes of God were so manifestly set foorth, that no mans tongue nor pen can in like sort expresse his infinite power, his vnspeakeable wisedome, and his exceeding goodnesse towards man, whereby he doth bountifully prouide for men all necessaries, not onely to liue, but also to liue pleasantly. And so was their confidence in Gods prouidence strongly staied, knowing his goodnesse to be such, that he would helpe man as he could; and his power to bee so great, that he could doe what he would: and thirdly, his wisdome to be so pure, that he would doe nothing but that was best. Beside these

Sciences

Sciences, they taught alſo Muſicke, which moſt commonly they did apply partly to religious Sciences, to draw men to delight therein, and partly to ſongs made of the manners of men in praiſe of vertue, and diſcommendation of vice: whereby it came to paſſe, that no man would diſpleaſe them, nor do any thing euill that might come to their hearing: for their Songs did make euill men more abhorred in that time, then any excommunication doth in this time. The poſteritie of theſe Muſicians continue yet both in Wales and Ireland, called Bardes vnto this day, by the ancient name of Bardus, their firſt founder.

And as theſe Sciences did increaſe, ſo did vertue increaſe thereby. Againe, as theſe Sciences did decay, ſo vertue loſt her eſtimation, and conſequently was little in vſe: whereof to make a full declaration, were a thing meet for a Prince to heare, but it would require a peculiar treatiſe. Wherefore at this preſent I count it ſufficient, lightly to haue touched this matter in generall words, and to ſay no more of the particularitie thereof, but only touching one of thoſe Sciences, that is, Arithmeticke, by which not only iuſt partition of lands was made, but alſo touching buying and ſelling, all aſſiſes, weights, and meaſures were deuiſed, and all reckonings and accounts driuen: yea, by proportion of it were the true orders of iuſtice limited (as Ariſtotle in his Ethicks doth declare) and the degrees of eſtates in the common wealth eſtabliſhed. Although that proportion be called Geometricall, and not Arithmeticall, yet doth that proportion pertaine to the Art of Arithmeticke: and in Arithmeticke is taught the progreſſion of ſuch proportions, and all things thereto belonging: wherfore I may well ſay, that ſeeing Arithmeticke is ſo many waies needfull vnto the firſt plãting of a cõmon wealth, it muſt needs be as much required to the preſeruation of it alſo: for by the ſame means is any common wealth continued, by which it was erected and eſtabliſhed. And if I ſhall in ſmall matters in

appea-

appearance, but indeed very weightie, put one e-
ample or two, what ſhall we ſay for the Statutes of th
Realme, which bee the onely ſtay of good order
manner now? As touching the meaſuring of grou
by length and breadth, there is a good and ancie
Statute made by the Art of Arithmeticke, and now
ſhall be to little vſe, if by the ſame Art it be not pract
ſed and tried.

For the Aſſiſe of bread and drink, the two moſt co
mon and moſt neceſſarie things for the ſuſtentation o
man, there was a goodly ordinance in the Law mad
which by ignorance hath ſo growen out of knowledg
and vſe, that few men doe vnderſtand it, and therefor
the Statute-books are wonderfully corrupted, and th
Commons are cruelly oppreſſed: notwithſtandin
ſome men haue written, that it is too doubtfull a ma
ter to execute thoſe Aſſiſes by thoſe ſtatutes by reaſo
they depend of the ſtanderd of the Coine, which
much changed frõ the ſtate of that time, when thoſ
Statutes were made. This ſhall euery man read th
liſteth, in the Abridgement of Statutes, in the title o
Weights and meaſures, in the ſeuenth number of th
Engliſh booke, where he ſhould haue tranſlated a goo
ordinance, which is ſet foorth in the French booke
but no maruell if the Abridgement doth omit it, ſeein
the great booke of Statutes doth omit the ſame Sta-
tute, as it hath done diuers other very good Lawes.
And this is the fruit of ignorance, to reiect all that it
vnderſtandeth not, although they vſe ſome clokes for
it: but ſuch clokes, as being allowed, might ſerue to
repell all good Lawes: which God forbid.

Againe, there is an ancient order for Aſſiſe of fire-
wood and coales, which was renued not many yeeres
paſt, and now how auarice and ignorance doth canuas
the Statute, it is too pitifull to talke of, and more mi-
ſerable to feele.

Furthermore, for the Statute of Coynage, and
the Standerd thereof, if the people vnderſtood right-
ly

ly the Statute, they should not, nor would not (as they often doe) gather an excuse for their follie thereby: but, as I said, these Statutes by wisedome and good knowledge of Arithmetick, were made, and by the same must they be continued. And let ignorance no more meddle with the vse of them, then it did with the making of them. O in how miserable case is that Realme, where the Ministers and Interpreters of the Lawe are destitute of all good Sciences, which be the keyes of the Lawes. How can they either make good Lawes, or mantaine them, that lack that true knowledge wherby to iudge them? And happie may that Realme be accounted, where the prince himselfe is studious of learning, and desireth to vnderstand equitie in all Lawes. Therefore most happy are wee the louing subiects of your Maiestie, which may see in your highnesse, not onely such towardnesse, but also such knowledge of diuerse Artes, as seldome hath beene seene in any Prince of such years; whereby wee are inforced to conceiue this hope: Certainly, that he which in those yeares seeketh knowledge, when knowledge is least esteemed, and of such an age can discerne them to be enemies both to his Royall person, and to his Realm, which labour to withdraw him from knowledge to excessiue pastime, and from reasonable study to idle or noysome pleasures, he must needs when he commeth to more mature yeares be a most prudent Prince and iust Gouernour, and a right Iudge not onely of his subiects commonly, but also of the ministers of his Lawes, yea and of the Lawes themselues: and to be able to conceiue the true equitie and exact vnderstanding of all his Lawes and Statutes, to the comfort of his good subiects, and the confusion and reproch of them, which do labor to obscure or peruert the equitie of the same lawes and Statutes. How some of those Statutes may be applied to vse, as well in our time, as in any other time, I haue particularly declared

clared in this booke, and some other I haue omitted for iust considerations, till I may offer them first vnto your Maiestie, to weigh them, as to your Highnesse shal seeme good : for many things in them are not to bee published without your Highnesse knowledge and approbation, namely because in them is declared all the rates of all oiles for all Standers from one ounce vpward, with other mysteries of mint-matters, and also most part of the varieties of coines that haue beene currant in this your Maiesties Realme by the space of six hundred yeeres last past, and many of them that were currant in the time that the Romans ruled here.

All which, with the ancient description of England and Ireland, and my simple censure of the same, I haue almost compleated to be exhibited to your Highnesse.

In the meane season, most humbly beseeching your Maiestie to accept this simple Treatise, not worthy to be presented to so high a Prince, but that my lowly request to your Maiestie is, that this amongst other of my bookes may passe vnder the protection of your Highnesse, whom I beseech God most earnestly and daily, according to my dutie, to aduance in all honour and princely Regalitie, and to increase in all knowledge, iustice, and godlie policie, Amen.

Your Maiesties most obedient

Subiect and Seruant,

ROBERT RECORD.

TO THE LOVING READER.

The Preface of Master R. Record.

SOre oftentimes haue I lamented with my ſelf the infortunate condition of England, ſeeing ſo manie great Clerkes to ariſe in ſundry other parts of the world, and ſo few to appeare in this our nation, whereas for pregnancy of naturall wit (I thinke) few nations do exceede Engliſhmen; but I cannot impute the cauſe to any other thing, then to the contempt and miſregard of learning. For as Engliſhmen are inferiour to no men in mother wit, ſo they paſſe all in vaine pleaſures, to which they may attaine without great paine of labour: and are as ſlacke to any neuer ſo great commodity, if there hang vpon it any painfull ſtudie or trauelſome labour.

Howbeit, yet all men are not of that ſort, though the moſt part be, more is the pitie it is: but to them that are ſo glad, not only with painfull ſtudie, & ſtudious paine to attain learning, but alſo with as great ſtudy & paine

to communicate their learning to others, and make all England, if it might be, partakers of the same, the most part are such, that vnneth they can support their owne necessary charges, so that they are not able to beare any charge in doing of that good, that else they desire to doe.

But a greater cause of lamentation is this: that when learned men haue taken paines to do things, for the aid of the vnlearned, scarce they shall be allowed for their well doing, but derided and scorned, and so vtterly discouraged to take in hand any like enterprise againe. So that if any be found (as there are some) that do fauour learning and learned wits, and can be content to further knowledge, yea only with their word, such persons, though they be rare, yet shall they encourage learned men to enterprise some things at the least, that Englãd may reioice of. And I haue good hope that England wil (after she hath taken sure tast of learing) not onely bring forth more fauourers of it, but also such learned men, that she shall be able to compare with any Realm in the world. But in the meane season, where so few regarders of learning are, how greatly they are to be esteemed that do fauour and further it, my pen will not suffice at full to declare.

Therefore Gentle Reader, wheras I do vpon iust occasion iudge, yea & know assuredly, that there be some men in this realme, which both loue and also much desire to further good learning, and yet am not well able to write their condigne praise for the same, I thinke it better with silence to ouerpasse it, then either to say too little of it, or to prouoke against them the malice of such other, which do nothing themselues that is praisworthy, and therefore cannot abide to heare the praise of any other mans good deed.

And considering their great fauour vnto learning, although I my selfe be not worthy to be reckoned in the number of great learned men, yet am I bold to put my selfe in prease with such abilitie as God hath lent mee,

though

though not with so great cunning as many mē, yet with as great affection as any man, to helpe my countrimen, and wil not cease daily (as much as my small ability wil suffer me) to indite some thing, that shal bee to the instructiō, though not of the learned men, yet at the least to the vulgar sort, whose argument alwaies shal be such that it shall delight all learned wits, though they doe not learne any great things out of it.

But to speake of this present booke of Arithmeticke, I dare not, nor will not set it forth with any words, but remit it to the iudgement of all gentle Readers, and namely such as loue good learning, beseeching them so to esteeme it, as it doth seeme worthy. And so either to accept the thing for it selfe, or at the least to allow my good endeuor. But I perceiue I need not to vse any perswasions vnto them, whose gentle nature and fauourable minde is ready to receiue thankfully, and interpret to the best, of all such enterprises attempted for so good an end, though the thing doe not alwaies satisfie mens expectation.

This considered, did bolden me to publish abroad this little booke of the Art of numbring; which if you shall receiue fauourably, you shall encourage me to gratifie you hereafter with some greater thing.

And as I iudge some men of so louing a mind to their natiue Country, that they would much reioice to see it to prosper in good learning & witty arts, so I hope well of al the rest of Englishmen, that they will not be vnmindful of his due praise, by whose means they are helped and furthered in any thing. Neither ought they to esteem this thing of so little value (as many mē of little discretion oftentimes do,) For who so setteth smal price by the witty deuice & knowledge of numbring, hee little considereth it to be the chiefe point (in manner) whereby men differ from all bruit beasts: for as in all other things, (almost) beasts are partakers with vs, so in numbring we differ cleane from thē, and in maner particularly, sith that in many things they excel vs againe.

The Foxe in craftie wit exceedeth most men,
A dogge in smelling hath no man his Peere,
To foresight of weather if you looke then,
Many beasts excell man, this is cleare.
The wittinesse of Elephants doth letters attain,
But what cũning doth there in the Bee remain?
The Emmet foreseeing the hardnes of winter,
Prouideth victuals in the time of Sommer.
The nightingale, the linet, the thrush, the lark,
In musicall harmony passe many a Clark.
The Hedghog of Astronomy seemeth to know,
And stoppeth the caue wher the wind doth blow.
The Spider in weauing such Art doth show,
No man can him mend or follow I trow.
When a house doth fall, the Mice full quicke
Flee thence before: Can man doe the like?

Many things else of the wittinesse of beasts and birds might I here say, saue that another time I intend to write wherein they excel in maner all men, as it is daily seene: but in nũber was there neuer beast found so cunning, that could know or discerne one thing from many, as by daily experience you may wel consider. When a Bitch hath many whelpes, or a Hen many chickens, and likewise of other whatsoeuer they be, take from thẽ all their yong sauing onely one, and ye shall perceiue plainly that they misse none, though they will resist you in taking them away, & wil seeke them againe if they may know where they be; but else they wil neuer misse them truely, but take away that one that is left, & then wil they cry and complaine: and restore to them that one, then they are pleased againe: so that of nũber this may I iustly say, It is the onely thing (almost) that separateth man from beasts. He therfore that shal contemne

number

number he declareth himselfe as brutish as a beast, and vnworthy to bee counted in fellowship of men. But I trust there is no man so fouly ouerseene, though many right small regard it.

Therefore wil I now stay to write against such, and returne againe to this booke, which I haue written in the forme of a Dialogue, because I judge that to be the easiest way of instruction, when the Scholar may aske euery doubt orderly, and the master may answere to his question plainely.

Howbeit I thinke not the contrary, but as it is easier to blame another mans worke than to make the like; so there wil be some that wil finde fault, because I write in a Dialogue: but as I coniecture, those shall bee such, as do not, cannot, either will not perceiue the reason of right teaching, and therefore are vnmeet to be answered vnto, for such men with no reason wil be satisfied.

And if any man obiect, that other bookes haue bene writtē of Arithmetick already so sufficiently, that I needed not now to put pen to booke, except I wil condemn other mens writings; to them I answere: That as I condemne no mans diligence, so I know that no one man can satisfie euery man, and therefore like as many do esteeme greatly other bookes, so I doubt not but some will like this my booke aboue any other English Arithmeticke hitherto written, and namely such as lacke instructors, for whose sake I haue so plainly set forth the examples, as no booke (that I haue seene) hath done hitherto: which thing shall bee great ease to the rude Reader.

Therefore (gentle Reader) though this booke can be small aide to the learned sort, yet vnto the simple and ignorant (which needeth most helpe) it may bee a good furtherance, and meane vnto knowledge. And though vnto the King his Maiesty priuately I doe it dedicate, yet I doubt not (such is his clemency) but that he can be content, yea and much desirous, that all his lo-

louing subiects shal take the vse of it, and apply the same to their owne profit: which thing if I perceiue that they thankfully doe, and receiue with a good wil, as it was written, then wil I shortly with no lesse kindnesse set forth such introductions into Geometry and Cosmography, as I haue at other times promised, & as hitherto in English hath not beene enterpri ed, wherewith I dare say al honest hearts wil be pleased, and al studious wits greatly delighted. I wil say no more; but let euery man iudge as he shal see cause. And thus for this time I wil stay my penne, committing you al to that true fountaine of perfect number, which wrought the whole world by number and measure, hee is Trinity in Vnity, and Vnity in Trinity; to whom be al praise, honour and glory, Amen.

Heere followeth A Table of all the Contents of this Booke.

The contents of the first Dialogue containe the Declaration of the profit of Arithmeticke.

Unto all these are added their proofe.

The second Dialogue containeth,

The first fiue kinds of Arithmeticke wrought by Counters.

The common kinds of casting accounts after the Merchants fashion, & auditors also.

The Contents of the second part touching Fractions.

- What a Fraction is.
- Numeration in fractions
- The order of working
- Multiplication (fractions
- Diuision.

with diuers familiar questions proponed for ÿ perfect understanding, & proofe of ech of thē

Reduction of

- diuers fractions into one denomination in the varieties.
- Fractions of Fractions.
- Improper Fractions.
- Fractions to the smallest denomination with easie rules how to conuert thē thereunto.
- Fractions in other parts of things, with a table demõstratiue of their proportions.
- Fraction, and how it may be turned into any other fraction, or into what denomination you list.

Againe,

The Contents of the third Addition to this Booke.

The

FINIS.

A Collection of such Tables as are contained in this Treatise.

Lastly,

Laſtly, a Table demonſtrating the true ſolution of thrée diuers things hidden, of thrée ſeuerall perſons in paſtime.

Note that this △ Character repreſenteth the Crowne by exchange.

Item whereas I haue augmented diuers neceſſary queſtions and rules vnto the Author, the beginning of which additions are marked with this ☾ note. And where they end with this * note.

FINIS.

Before the Introduction of Arithmeticke, it were very good to haue some vnderstanding and knowledge of these figures and Notes.

j	1	one.	xx.	20	twenty.
ij	2	two.	xl.	40	forty.
iij	3	thrée.	l.	50	fifty.
iiij	4	foure.	lx.	60	sixty.
v	5	fiue.	lxx.	70	seuenty.
vj	6	sixe.	xc.	90	ninety.
vij	7	seuen.	C.	100	a hundred.
viij	8	eight.	CC.	200	2 hundred.
ix	9	nine.	D.	500	5 hundred.
x	10	ten.	DC.	600	6 hundred.
xj	11	eleuen.	M.	1000	a thousand.
xij	12	twelue.	MD.	1500	a thou. 5 hun.
&c.	&c	&c.	&c.	&c.	&c.

112 li make C. waight
56 halfe a C. waight.
28 a quarter of an hundred waight.
16 oūces make a poūd.
A poūd Troy which is the stādard of England, wherby is only wayed gold, siluer & bread, is 12 ounces
12 inches make a foot

3 foote make a yard.
The yard is parted into 4 quarters.
And each quarter into 4 nailes.
5 quarters of a yard make an elle. Which elle is also parted in 4 quarters.
And each quarter into foure nayles.

A Dialogue betweene the Maſter *and the* Scholar, teaching the Art and vſe of Arithmeticke with pen.

The Scholar ſpeaketh.

SIr, ſuch is your authoritie in mine eſtimatiō, that I am content to conſent to your ſaying, & to receiue it as truth, though I ſee none other reaſon that doth lead me therunto: whereas elſe in mine owne conceit it appeareth but vain, to beſtow any time priuately in learning of that thing, that euery childe may, & doth learne at al times and houres, when he doth any thing himſelfe alone, and much more when he talketh or reaſoneth with other.

Maſter. Loe, this is the faſhion and chance of all them that ſeeke to defend their blinde ignorance: that when they thinke they haue made ſtrong reaſon for themſelues, then haue they prooued quite contrary. For if Numbring be ſo common (as you grant it to bee)

that no man can doe any thing alone, and much lesse talke or bargaine with other, but he shall still haue to doe with nũber: this proueth not nũber to be contemptible & vile, but rather right excellent & of high reputation, sith it is the ground of all mens affaires, so ẏ without it, no tale can be told, no cõmunicatiõ without it, can bee long continued, no bargaining without it can duely be ended, or no businesse that man hath, iustly completed. These commodities if there were none other, are sufficient to approue the worthines of number. But there are other innumerable farre passing all these, which declare nũber to excéed al praise. Wherfore in all great works are Clerks so much desired? Wherfore are Auditors so richly feed? what causeth Geometricians so highly to be enhaunced? Why are Astronomers so greatly aduãced? because ẏ by nũber such things they find, which else should farre excell mans mind.

Scholar. Verily Sir, if it bee so, that these men by numbring their cunning doe attaine, at whose great works most men do wonder, then I see well I was much deceiued, and nũbring is a more cunning thing than I tooke it to be.

Master. If number were so vile a thing as you did esteeme it, then néede it not to be used so much in mens communication. Exclude Number, and answere to this question, How many yeares old are you?

Scholar.

Scholar. Mum.

Master. How many daies in a weeke? How many weekes in a yeare? What lands hath your father? How many men doth hee keepe? How long is it since you came from him to me?

Scholar. Mum.

Master. So that if Number want, you answere all by Mummes: How many miles to London?

Scholar. A poake full of Plums.

Master. Why, thus you may see, what rule number beareth, and that if number be lacking, it maketh men dumb, so that to most questions, they must answer Mum.

Scholar. This is the cause (sir) that I iudged it so vile, because it is so common in talking every while: For plenty is not daintie, as the common saying is.

Master. No, nor store is no sore: perceiue you this? The more common that a thing is being needfully required, the better is the thing, and the more to bee desired. But in numbring, as some of it is light and plaine, the most part is difficult, and not easie to attaine. The easier part serueth all men in common, and the other part requireth some learning. Wherefore as without numbring a man can do almost nothing, so with the helpe of it you may attaine to all things.

Scholar. Yea sir? why? then it were best to

learne the art of numbring first of all other learning, & then a man need learne no more, if all other come with it.

Master. Nay, not so: but if it be first learned, then shal a man be able (I meane) to learne, perceiue, and attaine to other sciences, which without it, he should neuer get.

Scholar. I perceiue by your former words, that Astronomy and Geometry depend much on the helpe of numbring: but that other sciences, as Musicke, Physicke, Law, Grammar, and such like, haue any helpe of Arithmetike, I perceiue not.

Master. I may perceiue your great clerkelinesse by the ordering of your sciences: but I wil let that passe now, because it toucheth not the matter that I intend, and I wil shew you how Arithmetike doth profit in al these, somewhat grossely, according to your small understanding, omitting other reasons more substantial.

First, (as you reckon them) Musicke hath not onely great helpe of Arithmeticke, but is made, and hath his perfectnesse of it: for al Musicke standeth by number and proportion.

And in Physicke, beside the calculation of critical daies, with other things which I omit, how can any man iudge the pulse rightly that is ignorant of the proportion of Numbers?

And as for the Law, it is plaine, that the

ma

man that is ignorant of Arithmeticke, is nei-
ther méet to be a Judge, neither an Aduocate,
nor yet a Proctor. For how can he wel vnder-
stand another mans cause appertaining to
distribution of goods, or other debts, or of sums
of money, if hee be ignorant of Arithmetike?
This oftentimes causeth right to be hindred,
when the iudge either delighteth not to heare
of a matter that he perceiueth not, or cannot
iudge it for lacke of vnderstanding. This com-
meth by the ignorance of Arithmeticke.

Now as for Grammar, me thinketh you
should not doubt in what it néedeth number,
sith you haue learned that Nounes of al sorts,
Pronounes, Uerbes and Participles, are di-
stinct diuersly by numbers: besides the varie-
tie of nounes of number, and Aduerbes. And
if you take away number from Grammar,
then is all the quantitie of Syllables lost.
And many other waies doth number helpe
Grammar. Whereby were al kinds of mée-
ters found and made? Was it not by Num-
bers?

But how néedful Arithmetike is to al
parts of Philosophie, they may soone sée, that
do reade either Aristotle, Plato, or any other
Philosophers writings. For al their examples
most, and their probations, depend of A-
rithmeticke. It is the saying of Aristotle,
that he that is ignorant of Arithmeticke, is
méete for no science. And Plato his master

wrote a like ſentence ouer his Schoolehouſe doore, Let none enter in hither (quoth he) that is ignorant of Geometrie. Seeing hee would haue all his ſcholars expert in Geometrie, much rather he would the ſame in Arithmetike, without which Geometry cannot ſtand.

And how needfull Arithmeticke is to Diuinity, it appeareth, ſeeing ſo many Doctors gather ſo great myſteries out of number and ſo much do write of it. And if I ſhould goe about to write all the commodities of Arithmetick in ciuill acts, as in gouernance of common weales in time of peace, and in due prouiſion & order of armes in time of war. For numbring of the hoſt, ſumming of their wages, prouiſions of victuals, viewing of artillerie, with other armor: Beſide the cunningeſt point of all, for caſting of ground, for encamping of men, with ſuch other like. And how many waies alſo Arithmetick is conducible for all priuate weales, of Lords and al poſſeſſioners, of merchants, and all other occupiers, and generally for all eſtates of men, beſides Auditors, treaſurers, receiuers, ſtewards, bailiffes and ſuch like, whoſe offices without Arithmetick are nothing: If I ſhould (I ſay) particularly repeat all ſuch commodities of this noble ſcience of Arithmeticke, it were inough to make a very great booke.

Scholar. No, no ſir, you ſhall not need: For I doubt not, but this that you haue ſaid, were

inough

inough to perswade any man to thinke this art to be right excellent and good, and so necessarie for man, that (as I thinke now) so much as a man lacketh of it, so much hee lacketh of his sense and wit.

Master. What, are you so farre changed since, by hearing these few commodities in generall? By likelihoode you would be farre changed if you knew all the commodities particular.

Scholar. I beseech you, sir, reserue those commodities that rest yet behind vnto their place more conuenient: and if ye will be so good as to vtter at this time this excellent treasure, so that I may be somewhat inriched thereby, if euer I shall be able, I will requite your paine.

Master. I am very glad of your request, and will do it speedily, sith that to learne it, you be so readie.

Scholar. And I to your authoritie my wit do subdue, whatsoeuer you say, I take it for true.

Master. That is too much, and meete for no man to be beleeued in all things, without shewing of reason. Though I might of my Scholar some credence require, yet except I shew reason, I doe it not desire. But now sith you are so earnestly set this art to attaine, best it is to omit no time, lest some other passion coole this great heate, and then

you leaue off before you see the end.

Scholar. Though many there be so vnconstant of minde, that flitter and turne with euery winde, which often begin, and neuer come to the end, I am none of this sort, as I trust you partly know. For by my good wil what I once begin, til I haue it fully ended, I would neuer blin.

Master. So haue I found you hitherto indeed, and I trust you wil increase rather then go backe. For better it were neuer to assay, then to shrinke and flie in the mid way: But I trust you wil not so doe, therefore tell mee briefly. What call you the science that you desire so greatly?

Scholar. Why sir? you know.

Master. That maketh no matter, I would heare whether you know, and therefore I aske you. For great rebuke it were to haue studied a science, and yet cannot tell how it is named.

Scholar. Some call it Arsemetike, & some Augrime.

Master. And what doe these names betoken?

Scholar. That if it please you, of you would I learne.

Master. Both names are corruptly written, Arsemetike for Arithmetike, as the Greekes call it, and Augrim for Algorisme, as the Arabians found it, which both betoken

the

the ſcience of numbring. For Arithmos in Grœke is called Number: and of it commeth Arithmetike, the Art of Numbring. So that Arithmetike is a ſcience or Arte teaching the maner and vſe of Numbring. This art may be wrought diuerſly, with Penne or with Counters. But I will firſt ſhew you the working with the Penne, and then the other in order.

Scholar. This I wil remember. But how many things are to bee learned to attaine this arte fully?

Maſter. There are reckoned commonly ſeuen parts or workes of it.

Numeration, Addition, Subtraction, Multiplication, Diuiſion, Progreſſion, and extraction of rootes: to theſe ſome men adde Duplation, Triplation, and Mediation. But as for theſe laſt thrœ, they are contained vnder the other ſeuen. For Duplation and Triplation, are contained vnder Multiplication, as it ſhall appeare in their place. And Mediation is contained vnder Diuiſion, as I will declare in his place alſo.

Scholar. Yet then there remaine the firſt ſeuen kinds of Numbring.

Maſter. So there doth: Howbeit, if I ſhall ſpeake exactly of parts of Numbring, I muſt make but fiue of them: For Progreſſion is a compound operation of Addition, Multiplication & Diuiſion. And ſo is the Extraction

tion of rootes. But it is no harme to name them as kinds severall, seeing they appeare to haue some severall working. For it forceth not so much to contend for the number of them as for the due knowledge and practising of them.

Scholar. Then you will that I shall name them as seuen kinds distinct. But now I desire you to instruct me in the vse of each of them.

Master. So I will, but it must be done in order: for you may not learne ẏ last as soone as the first, but you must learne them in that order, as I did rehearse them, if you wil learn them speedily and well.

Scholar. Euen as you please. Then to begin, Numeration is the first in order: what shall I doe with it?

Master. First you must know what the thing is, and then after learne the vse of the same.

Numeration.

Numeration is that Arithmeticall skill, whereby wee may duely value, expresse and read any number or sum propounded: or els in apt figures and places set downe any number knowne or named.

Scholar.

Scholar. Why? then me thinketh you put a difference betweene the value and the Figures?

Master. Yea, so do I: For the value is one thing, and figures are another thing: and that commeth partly by the diuersitie of figures, but chiefly of the places wherein they be set.

Scholar. Then I must know here thrée things: the Value, the Figure, and the Place.

Master. Euen so: but yet adde order to them as the fourth. And first marke, that there are but ten Figures that are vsed in Arithmetike: and of those ten, one doth signifie nothing, which is made like an o, and is priuately called a Cipher, though all the other sometime be likewise named. The other nine are called signifying Figures, and bee thus figured.

1 2 3 4 5 6 7 8 9.
And this is their value.
j. ij. iij. iiij. v. vj. vij. viij. ix.

But here you must marke, that euerie Figure hath two values: One alwaies certaine, that it signifieth properly, which it hath of his forme: and the other vncertaine, which hee taketh of his place.

A place is called the seate or roome that a Figure standeth in. And looke how many figures

gures are written in one summe, so many places hath that whole number. And that must be called the first place that is next to yͤ right hand, & so reckoning by order towards the left hand, so that that place is last, that is next to the left hand. As for example: If there stood before you sixe men in a row, side by side, and you should tel them as they stand in order beginning with the man that were next to your right hand: then he that were next him should be called the second, and so forth to the furthest from your right hand, which is the sixt and the last.

Scholar. Sir, I perceiue you wel: so might I reckon letters or any other thing. As if I should write eight letters after this order, a b,c,d,e,f,g,h, then must I say, h is the first, g the ii, f the iii, e the iiii, d the v, c the vi, b the vii, and a the viii.

Master. That is wel done. And after the same sort vse hereafter, that what I declare by one example, doe you expresse by another: and so shal I perceiue whether you vnderstand it or no. And so passe ouer nothing, til you perceiue it wel, and be expert therein.

Scholar. Sir, I pray you how manie of these places be there in all?

Master. There is no certaine number of them, but they are somtimes more and sometimes fewer, according to the summe that is expressed. For so many as the figures are, so

many are the places : and the last place is so called, not because it is last of al other, but it is the last of that present summe, and it may be the middle place in another summe.

Scholar. Me seemeth I perceiue this very wel, as touching the order of reckoning of the places: but as for the number of them, you say there is no certainty. Now there resteth to declare the value of the figures by the diuersitie of places, which you called the value vncertaine.

Master. But first let mee heare whether you know perfectly the certaine value.

Scholar. Yes sir, as you wrote them, so I marked them.

Master. How write you then fiue?

Scholar. By this figure 5.

Master. And how six?

Scholar. Thus, 6.

Master. Write these thrée numbers, each by it selfe, as I speake them, vii.iiii.iii.

Scholar. 7.4.3.

Master. How write you these foure other ii. i. ix. viii?

Scholar. Thus (I trow) 2.1.6.8.

Master. Nay, there you misse: Looke on mine example againe.

Scholar. Sir, truth it is, I was to blame I tooke 6 for 9, but I will beware hereafter.

Master. Now then take héede, those certaine

taine values every figure representeth when it is alone written without other figures ioined to him. And also when it is in the first place, though many other do followe: as for example: This figure 9. is ix. standing now alone.

Scholar. How is he alone and standeth in the middle of so many letters?

Master. The letters are none of his fellowes. For if you were in France in the middle of a thousand Frenchmen, if there were no Englishman with you, you would reckon your selfe to be alone.

Scholar. So it is. Then 9 without mo figures of Arithmetick, betokeneth ix. whatsoever other letters be about it.

Master. Euen so, and so doth it, if it be in the first place ioined with other, how many soeuer doe follow, as in this example, 3679, you see 9 in the first place, and doth betoken nine, as if it were alone.

Scholar. I perceiue that, and doth not 7 that standeth in the second place, betoken vij, and 6 in the third place, betoken vi? and so 3 in the fourth place betoken thrée?

Master. Their places be as you haue said, but their values are not so. For as in the first place, euery figure betokeneth his owne value certain only, so in the second place euery figure betokeneth his owne value certaine ten times: as in the example, 7 in the second

place

place is ſeuen times x, that is, lxx. And in the third place, euery figure betokeneth his owne value a hundred times, ſo that 6 in that place betokeneth vi C. And in the fourth place euery figure betokeneth his owne value a M. times, as in the aforeſaid number, 3 in the 4. place, ſtandeth for 3 M: and in the fifth place, euery figure ſtandeth for his owne value x M times; and in the vi place, a C M. times: and in the vij place a M. M. times, and in the viii place x M M. ſo that euery place exceedeth the former x. times.

Scholar. As thus: if I make this number at all aduentures, 91359684, here are eight places. In the firſt place is 4, and betokeneth but foure: in the ſecond place is 8, and betokeneth ten times 8, that is 80, In the third place is 6, and betokeneth ſix hundred. In the fourth place 9, is nine thouſand. And 5 in the fifth place is x. M. times 5, that is, fifty M. So 3 in the ſixt place, is a C M times 3, that is CCC. M. Then 1 in the ſeuenth place, a M M. and 9 in the eight place, ten thouſand thouſand times 9, that is xc, M, M. But now I cannot eaſily nor quickly read it in order.

Maſter. That ſhall you practiſe by this meanes. Firſt put a pricke ouer the fourth figure, and ſo ouer the ſeuenth. And (if you haue ſo many) ouer the tenth, thirteenth, ſixteenth, and ſo forth, ſtill leauing two figures betweene each two pricks. And thoſe two

Roomes

roomes betweene the pricks, are called Ternaries.

Then begin at the last pricke, and see how many figures are betweene him and the end, which cannot passe three, reckoning himselfe for one: then pronounce them, as if they were written alone from the rest, and at the end of their value, so many times thousand, as your numbers hath pricks.

After that, come to the next three figures, and sound them as if they were a part from the rest, and adde to their value so many times thousands, as there are pricks betweene them, and the first place of your whole number. And so doe by euery other three figures following, if you haue moe. As in example, 91359684, this was your number.

Put a pricke ouer 9 in the fourth place, and ouer 1 in the seuenth place, and then no more (for your places come not to tenne) as thus: 91359684.

Now go to the last pricke ouer 1, and take it and the figure 9 that followeth it, and value them alone.

Scholar. 91, that is, xci.

Master. So it is, then adde for the number of your pricks twice M.

Scholar. That is xci. thousand thousand.

Master. So it is. Then take the three other figures from one to the next pricke, and value them.

Scholar.

Scholar. 359. that is, CCC.lix.

Master. Now adde for the one pricke, that is betwéene them and the first place, M.

Scholar. CCC.lix thousand.

Master. Then come to the other thrée figures that remaine.

Scholar. 684, that is, vi C.lxxxiiij.

Master. Now haue you valued all. And at the end of the last number you shal adde nothing, because there remaineth no pricke nor number after it: yet proue in another number, as thus, 2308640891053 40.

Scholar. 230864089105340. I haue pricked thē as you taught me: but I am in doubt, whether I haue done well or no, because of the ciphers: for I remember, you told me that they do signify nothing, and therefore I doubt whether I should reckon them for a figure in setting of the prickes: and againe, I know not wherefore they serue.

Master. That will I tell you now. Indéed they are of no value thēselues, but they serue to make vp number of places, and so make the figure following them to be in a further place, and therefore to signify the more value, as in this example 90, the cipher is of no value, but yet he occupieth the first place, and causeth 9 to be in the second place, and so to signify ten times 9, that is xc. so that two ciphers thrusteth the figure following them into the third place, and so forth.

Scholar. Then I perceiue in the example aboue, I haue pricked wel enough: for though that cipher that is pricked, signify nothing, yet must he haue the pricke, because he came in the 13 place. Then wil I proue to number that sum. First, there is 2,0, M, M, M, M, and then followeth 864. M. M. M. And what shall I now do? There is a cipher in the third place, and no figure after him, but they that I haue reckoned.

Master. Hee did serue for them that you haue already reckoned, to make them in a place further than they should be, if hee were away: and therefore now ye shall let him go. And so doe alwaies when hee occupieth that place next before any pricke, which is the last of that Ternarie, and a cipher in the last place doth nothing.

Scholar. Then shal I say but 89 M. M.

Master. So, but go forth.

Scholar. 105 thousand. Now are all my prickes spent, and yet remaine 340, so that I must value them CCC. xl, only.

Master. Now can you reckon after this sort: and remember, that euery such number so parted, is called a Ternarie or Trinity: for haue you numbred or valued the summe most truely, and by the aide of the prickes each Denomination is distinct most plainely.

Scholar. What call you Denominations?

ons?

Master. It is the last value or name added to any summe. As when I say: a hundreth two and twenty pounds: Pounds is the Denomination. And likewise in saying 25 men, men is the Denomination, and so of other. But in this place (that I spake of before) the last number of euery Ternarie is the Denomination of it. As for the first Ternarie, the Denomination is vnites, and of the second Ternarie, the Denomination is thousands: And of the third Ternarie, thousand thousāds or Millions: of the fourth, thousand thousand thousands, or thousand Millions: and so forth.

Scholar. And what shal I cal the value of the thrée figures that may bee pronounced before the denominator? as in saying: 203000000, that is, two hundred thrée millions, I perceiue by your words, that millions is the Denomination: but what shal I call the CCiii. ioined before the millions?

Master. That is called the Numerator or valuer, and the whole summe that resulteth of them both, is called the summe, value or number.

Scholar. Now is there any thing else to be learned in Numeration: or else haue I learned it fully?

Master. I might shew you héere who were the first inuentors of this art and the reason of

all these things that I haue taught you, but that I will reserue till yee haue learned ouer all the practise of this Arte, lest I should trouble your with ouer many things at the first.

But yet this must you marke, that there are thrée kindes of Numbers, one called Digits: another Articles: and the third mixt numbers.

A digit is any number vnder ten, as this: 1,2,3,4,5,6,7,8,9.

And 10 with all other that may bee diuided into ten parts iust, and nothing remaine, are called Articles: such as are 10,20,30,40,50,&c. 100,200,&c. 1000,&c.

And that number is called mixt, that containeth Articles, or at the least one article, and a digit; as 12,16,20,21,38,107, 1005, and so forth: and for the more ease of vnstanding and remembrance, marke this: The digit nūber is neuer written with more than one figure, but the article and the mixt number are euer written with more than one figure. And thus they differ: that the Article hath euermore this Cipher 0 in the first place: & the mixt number hath euer there some digit.

Scholar. By these last words I perceiue it much better than I did before, and now (I thinke) I will neuer misse to know those thrée asunder.

Master. If you remember now all that I haue said, you haue learned sufficiently this first kinde of Arithmeticke, called Numeration. Howbeit, I wil exhort you now to remember both this that I haue said, and al that I shal say, and to exercise your selfe in the practise of it: For rules without practise, are but a light knowledge: and practise it is, that maketh men perfect and prompt in al things.

And as you haue learned to gather and expresse the value of a summe propounded, and set downe before you: so must you practise to marke, note or write downe with apt figures and in due places, any number only named or recited to you, or of your selfe imagined: as for a proofe. How note you, or write downe this summe, fiue thousand, two hundred, fifty and seuen?

Scholar. This troubleth me now, whether I should begin at the first or at the last. For reason (me thinketh) should cause me to begin at the first: and yet if I write it as you speake it, I must begin at the last.

Master. When you know your places perfectly, you may begin where you list. But the more ease for your hand is to begin with the last, that is to say, as I did speake them. Yet for the more surety, a while you may beginne with the first, repeating my words backward thus: Seuen, fiftie, two hundred, fiue thousand:

sand: or else sounding them al by their digit or value, as thus; seuen, fiue, two, fiue: for that way is easiest. But then must ye looke wel whether there be any Cipher in your summe, that he may be set in his place. As if your last value of your sum (as you cal it) bee aboue 9, then is there a Cipher in the first place. And if it bee a hundred or aboue, then is there two Ciphers, one in the first place, and another in the second, and so forth.

But because this thing is such that cannot be set forth without many words, I think best here now at the end of Numeration to adde a Table easie and readie for the first exercise of it.

Lo this is the Table.

The left side or hand.

The right side, or hand.

The names of Digits, values vncertain, or values.

T. M. of millions.	M. of millions.	C. of millions.	T. of millions.	Millions.	C. of Thousands.	T. Thousands.	Thousands.	Hundreds.	Tennes.	Unites.	The denominators of the place or value vncerten
9	9	9	9	9	9	9	9	9	9	9	Nine.
8	8	8	8	8	8	8	8	8	8	8	Eight.
7	7	7	7	7	7	7	7	7	7	7	Seuen.
6	6	6	6	6	6	6	6	6	6	6	Sixe.
5	5	5	5	5	5	5	5	5	5	5	Fiue.
4	4	4	4	4	4	4	4	4	4	4	Foure.
3	3	3	3	3	3	3	3	3	3	3	Three.
2	2	2	2	2	2	2	2	2	2	2	Two.
1	1	1	1	1	1	1	1	1	1	1	One.
0	0	0	0	0	0	0	0	0	0	0	Cipher.
Eleuenth.	Tenth.	Ninth.	Eight.	Seuenth.	Sixt.	Fift.	Fourth.	Third.	Second.	First.	The order of places.

This Table (as ye may see) hath eleuen places, & in each of them are set all the Digits, whose certaine value is written on the right hand of the Table, & the value vncertain on the left hand, so that by this Table you may learne both how to expresse any number that you list, (if that exceede not

eleuen places) that is to say, XC. thousand millions, and so may you by the help of it, value al summes proposed vnder the said number.

For example: take the summe that I proposed before, which was fiue thousand, two hundred fiftie and seuen. And if you wil expresse it, take the first number (as I speake it) which is fiue M. whose valuer or certaine value is v. and his vncertaine value or denomination is M. First you shal seeke at the right hand of the valuer 5. Then seeke along vnder the title of denomination toward the left hand til you find thousands, and vnder it right at the foote of the Table, is the number of the place, that is, in the fourth, wherein you must write your digit or valuer 5.

Afterward come to the second part of the number, two hundred, whose valuer is 2, and his denomination C. Seeke two at the right hand of the Table, and goe along vnder the denomination toward the left hand, till you come vnder C: then looke to the Table, and there you shal see the number of the place, that is to say, three, wherein you must set your digit 2.

Then doe so by your other two numbers that remaine, and you shall find 5 in the second place for your fifty, & 7 in the first place for your seuen. And thus may you doe with other numbers.

Scholar.

Scholar. Master I thanke you heartily. I perceiue you seeke to instruct me most plainly and briefly, and not to hide your knowledge with subtil words as many doe. For this rule is so plain, that I can desire it no plainer. And though it seeme somewhat long, yet I perceiue it to be a sure way.

Master. So it is, and though it be long, yet it is neither too long, neither too plain, for yong learners that lack practise: for this Table is in stead of a teacher, to them that lack one. But now I trust I haue said inough of Numeration: which after you haue wel practised, then may you learne forth.

Scholar. Yet I pray you in one thing to tel me your iudgement. Why do men recken the order of the places backward, from the right hand to the left?

Master. In that thing all men doe agree, that the Chaldees, which first inuented this Art, did set these figures as they set al their letters, for they write backward, as you term it, and so do they reade. And that may appeare in al Hebrue, Chaldee, and Arabike bookes, for they be not onely written from the right hand to the left, and so must be read, but also the right end of ẏ booke is the beginning of it, whereas the Greeks, Latines, and al Nations of Europe, do write and read from the left hād toward the right: and al their bookes begin at the left side.

Scholar. That reason doth satisfie me.

Master. It neither satisfieth mee, neither liketh mee wel, because I see that the Chaldees and Hebrewes do not so vse their owne numbers, as at another time I wil declare. But this plaine reason may best satisfie you presently: that seeing in pronouncing of numbers we keepe the order of our owne reading, from the left hand to the right: and againe, we do euer name the greater numbers before the smaller: it was reason, that the lesser places containing the lesser numbers, should bee set on the right hand, and the greater places containing the greater numbers, to procede toward the left hand.

Scholar. This reason is to me so plaine, y^t it seemeth now against reasō to make a doubt of that order. So that now for numeration I am satisfied: hoping that practise shal make me fully readie & expert in it. And in the mean season, I desire to learne the other kinds of Arithmetike.

Master. That is well said: but what should you next learne, can you tel?

Scholar. I remember you said that Addition was next.

Master. Euen so, and what that is, must you first know.

ADdition is the gathering together and bringing of two numbers or more into one ſumme: as if I haue 160 bookes in the Latine tongue, and 136 in the Greeke tongue, and would know how many they be in all, I muſt write theſe two numbers one ouer another, writing the greateſt number higheſt, ſo that the firſt figure of the one be vnder the firſt figure of the other: and the ſecond vnder the ſecond, and ſo forth in order.

When you haue ſo done, draw vnder them a right line, then wil they ſtand thus,

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160
136
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Now begin at the firſt places toward the right hand alwaies, & put together the two firſt figures of thoſe two numbers, & looke what commeth of them, write vnder them right vnder the line. As in ſaying 6 & 0 is 6, write 6 vnder 6: as thus.

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160
136
---
  6
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And then go to the ſecond figures, and doe likewiſe: as ſaying, 3 and 6 is 9 write 9 vnder 6, and 3, as here you ſee.

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160
136
---
 96
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And likewiſe do you wt the figures ye be in the third place, ſaying: 1 & 1 be 2: write 2 vnder thē and then will your whole ſum

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160
136
---
296
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appeare

appeare thus.

So that now you sée, that 160, and 136 do make in al 296.

Scholar. What? this is very easie to do: me thinketh I can do it euen since.

There came through Cheapside 2 droues of cattel: in the first was 848 shéep, and in the second was 186 other beasts.

Those two summes I must write as you taught me, thus.

848
186

Then if I put the two first figures together, saying: 6 & 8 they make 14. That must I write vnder 6 and 8, thus.

848
186
14

Master. Not so, and here are you twice deceiued. First in going about to adde together two summes of sundry things, which you ought not to do, except you séek only the number of them, and care not for the things. For the summe that should result of that addition, should be a summe neither of sheepe, nor of other beasts, but a confused summe of both. Howbeit sometimes ye shal haue summes of diuers denominations to be added, of which I wil tel you anon: but first I wil shew you, where you were deceiued in another point, and that was in writing fourteen which came of six and eight, vnder sixe and eight: which is vnpossible. For how can two figures of two places be written vnder one figure, and one place?

Scholar.

Scholar. Truth it is: but yet I did so vnderstand you.

Master. I said indeede, that you should write that vnder them, that did result of them both together: which saying is alwaies true, if that summe doe not excéede a digit. But if it bée a mixt number, then must you write the digit of it vnder your figures, as you haue said before: but if it be an article, then write 0 vnder them, and in both sortes you shall kéepe the article in your minde: and therfore when you haue added your second figures, which occupie the place of tens, you shall put that one thereto, which you kept in your mind: for though it were ten indéed, yet in that place it is but as one, because that euery one of that place is ten, for that it is the place of tennes: and in like manner, if you haue in the second place so great a number, that it amounteth aboue 9, then write the digit, and reserue the article in your mind, euer adding it to the next place following: and so of all other places, how many soeuer you haue. And if you haue a mixt number when you haue added your last figures, then write the digit vnder the last figures, and the article in the next place beyond them: so shall your number resulting of addition, haue one place more then the numbers which you should adde together.

Scholar. Now do I perceiue you, and the reason of this, is (as I vnderstand) because

that no one place can containe aboue 9, which is the greatest figure that is, and then all tens or articles must be put to the next place following: for euery place (as I may see) exceedeth the other place next before him by 10.

Now (if it please you) I wil returne to my example of cattle. But I remember you said I might not adde sums of sundrie things together, and that I might see by reason.

Master. Truth it is, if you seeke the due summe of any thing, but if you onely seeke a bare sum, and haue no respect to the thing, then were it better to name the summe onely without any thing, as in saying: 848, without naming sheepe, or any thing else. And likewise 186, naming nothing.

Now let me see: how can you adde those two summes?

Scholar. I must first set them so, ÿ the two first figures stand one ouer another, and the other each one ouer his fellow of the same place, then shal I draw a line vnder them both. And so likewise of other figures, setting alwaies the greatest number highest, thus as followeth.

Then must I adde 6 to 8 which maketh 14, that is mixt number, therfore must I take the digit which is 4, and write it vnder 6 and, 8 keeping the article 1 in my mind thus.

	848
	186

Next that, I do come to the se-

cond

cond figures, adding them together, saying, 8 and 4 make 12, to the which I put the one reserued in my minde, and that maketh 13, of which number I write the digit 3 vnder 8, and 4, and keep the article in my mind, thus.

	848
	186
	34

Then come I to the third figures, saying: 1 and 8 make 9, and 1 in my minde maketh 10. Sir, shal I write the cipher vnder 1 and 8?

Master. Yea.

Scholar. Then of 10 I write the cipher vnder 1 and 8, and keepe the article in my minde.

Master. What needeth that, seeing there followeth no more figures?

Scholar. Sir, I had forgotten, but I wil remember better hereafter. Then seeing I am come to the last figures, I must write the cipher vnder them, & the article in a further place after the ciphers thus.

	848
	186
	1034

Master. So now ye see, that of 848, and 186 added together, there amounteth 1034.

Scholar. Now I thinke I am perfect in addition.

Master. That wil I proue by this example.

There are two armies of Souldiers: in the one are 106800, and in the other 9400. How many are there in both armies, say you?

Scholar.

Scholar. First I set them one ouer another, beginning with the first numbers of the right hand thus.

106800
9400

But the nether number wil not match the ouer number.

Master. That forceth not.

Scholar. Then do I adde 0 to 0, and there remaineth 0, that must I write vnder the first place, thus:

106800
9400
0

Master. Well said.

Scholar. Then likewise in the second place I adde 0 to 0, and there ariseth 0, which I write vnder the second place, thus:

106800
9400
00

Then I come to the 3 place, saying, 4 & 8 make 12, of which I write the digit 2, & keep the article 1 in my minde, thus:

106800
9400
200

Then I adde 9 to 6, which maketh 15, to that I adde ye article 1 yt was in my mind, and it is 16. I write 6 vnder 6 & 9, and keep one in mind, thus:

106800
9400
6200

Master. Why do you not write both Figures, seeing you are come to the last couple of numbers?

Scholar. Nay reason sheweth me, that I must adde that article that is in my mind, vnto the next figure of the ouer summe, though there

there be no moꝛe in the nether ſumme.

Maſter. That is well conſidered: then doe ſo.

Scholar. Then ſay I, 0 in the ouer ſumme & 1 in the minde maketh 1, that wꝛite I vnder 0. Then followeth there yet one moꝛe in the ouer ſumme, which hath none to be added to it, foꝛ there is none in the nether ſumme, noꝛ yet in my mind: therfoꝛe I think I muſt wꝛite that euen as it is.

Maſter. Yea.

Scholar. Then doth my whole ſumme appeare thus.

106800
9400
116200

Maſter. If you marke this, you haue learned perfectly the common Addition of all ſums which are of one denomination: ſo that ye obſerue this alſo, that in Addition you muſt haue two numbers at the leaſt: oꝛ els, how can you ſay that you do adde? And euer let the greateſt number be wꝛitten higheſt: foꝛ that is the beſt way, though it be not neceſſary.

And foꝛget not this, that (if you haue many numbers to adde together) you ſhal haue oftentimes an article of a greater value than 10: ſometimes 20, ſometimes 30, ſomtimes moꝛe, yea (peraduenture) 100. Therefoꝛe, as you did with the article 10, ſo do with them, reſeruing them in your minde, and adding to the number next following, ſo many, as their

valuer or value certaine is: that is to say, 2 for 20, 3 for 30, 5 for 50, 10 for 100, 12 for 120: and so forth of other like. So that, if the article be 100, then must you set downe the 0 and kéep 10 in minde to be carried to the next row of figures or place, if any such happen to come. For your better vnderstanding take this example for all.

I would adde these xiij. sums into one, which I set after this maner: then do I begin and gather the summe of the first rowe of figures, which commeth to 107, (for I take 9 there x. times, and that is 90) then 9 and 8 is 17, that is, in all 107, of which summe I write the 7 vnder the first row of figures, and then for that 100 is x. tens, I kéepe x. in minde, which ten I must adde vnto the next rowe of figures, which are in the second place: which second rowe of figures (when they are added together with that x. that I had in my mind) make in all 125: of which sum I write the digit 5 vnder the second row, & then (for that 120 containeth xij. tens) I kéepe xij. in minde to be added to the third place or row of figures: which béeing added together, make in all 60: the cipher 0 I set downe vnder the rowe of figures in the third place.

4889
4599
2290
3699
2299
4099
1099
3298
299
699
499
899
389

And

And the figure 6 I keepe in mind to be added to the rowe of figures in ẏ fourth place: which (when they are added together) make 29. The figure or digit 9 I set downe vnder the fourth place. And, because it is my last worke, I set downe the 2 also that I haue in my mind to the 9 in the fifth place: those summes do make in all 29057.	4889 4599 2290 3699 2299 4099 1099 3298 299 699
¶ But (for your more ease in worke) when you haue an addition of so many summes to bée	899 499 389
added together, you were best	29057

part that summe into two or thrée parts, and worke them seuerall, and so put their additions together, and this were the best thing you could do when ouer manie summes fall to be added.

Scholar. This séemeth somewhat hard, by the reason of so many numbers together. Howbeit, I thinke (if I do often proue euē with this small example either by working of it alone, or els by parting it as you said euen now,) that I shall be able to do so shortly with any other summe.

Master. So shall you. For, it is often practise that maketh a man quicke and ripe in all things. But because, as well in great summes as in smal, there may chaunce to be some er-

rour, I will teach you how you shall proove whether you haue done well, or no.

Scholar. That were a great helpe and ease.

The proof of Addition.

Master. Begin first with the highest number, and then to all the other orderly, and adde them together, not hauing regard to their places, but as though they were all units: and still (as your number encreaseth aboue 9) cast away 9. Then go forth, euer casting away 9 as often as it amounteth thereto: and so do till you haue gone ouer all the numbers that you intended first to adde: and whatsoeuer remaineth after such additions & castings away of 9, write it in some void place by the end of a line for the better remembrance: and thus is the first part of your worke prooued. Then secondly, put together the Figures that result of the addition under the line, still casting away 9 also. And then, that that remaineth, write at the other end of that line: and if those two figures be like, then haue you well done: but, if they be unlike, then haue you missed. As for example, in this present sum: The first figure of the ouer line is 9, let him go: then 8 and 8 is 16, take away nine, resteth 7, and adde that 7 to 4 that followeth, and it maketh 11, from which if you take 9, there resteth 2. Then come to the next row, whose first and second numbers are 9, therefore ouerpasse them both, and take the fiue to the

the 2 which did remaine in the first rowe, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remaineth 2. Next unto that, goe to the third line, whose two first numbers you may let passe, because they are nines: then take the two figures of 2, which (with the other two that remained in the second rowe) make 6. Then, go to the fourth rowe, whose two first numbers let goe, and take the 6 to the 6 that remained, and that maketh 12, take away 9, and there resteth 3, which with the 3 that is next, maketh 6. And so go through all the other numbers, and you shal find that there remaineth 5, after you haue cast away 9, as often as you can finde it: therefore write 5 at the end of the line in a void place, thus:

5———

Then, gather all the Figures of the totall sum, which is under the lowest line, and cast away 9 as often as you can finde it: as thus: 7 and 5 make 12, take away 9, there resteth 3: to that if you adde the 2 that is last (for you may omit the 9) then doth it make 5, which 5 you must write at the other end of the line that you made in the void place, thus.

5———5

And then you see that those two figures be like. Whereby you may know that you haue done wel. And so you may proue in any other.

Scholar. (If it please you) I will proue in

another sum.

Master. With a good will.

Scholar. Then will I take one of your former examples, which was this.

First in the highest line, 8 and 6 make 14, then 9 taken away, there remaine 5, to which I adde the 1 that followeth, & that maketh 6. Then come I to the second line, where I finde first 4, which with 6 maketh 10, from that I take 9, & there resteth 1, the next figure is 0, and therfore I let him alone, so finde I one remaining which I set at the end of a line, thus. 1 ——

106800
9400
116200

Then I come to the totall sum, and there I find that all the figures put together, make ten, from which I take nine, and there resteth 1 also, which I put at the other end of the line, thus. 1 —— 1

And because they be like, I know that I haue well added.

Master. So, you know now, both how to adde two summes or more together, and also how to prove whether you haue done well, or no: and now I will teach you how to adde summes of diuers denominations together: which thing can neuer bée but when the one denomination is such, that it containeth the other certaine times. And yet you shall adde thẽ to the other, not after this sort (as you did them that were of one denominatiõ) but after

such a sort that I will now shew you, that is to say:

If you haue a sum of diuers denominations, then looke that ye set euerie denomination by himselfe, with some note or figure of his denomination, as they are wont to bee written. Then write your other sums so vnder that first, that euery one be set vnder the other of the same denominations: as for example: if your denominations be pounds, shillings and pence, write pounds vnder pounds, shillings vnder shillings, and pence vnder pence: and not shillings vnder pence, nor pence vnder pounds.

Scholar. Now that you haue spoken it, me thinketh, it needeth not to warne me of it: for it were against reason so to confound sums: but yet, if you had not spoken of it, peraduenture I should haue beene deceiued in it.

Master. If you do say it is so plaine, I will speake no more of it, but with an example make the matter to appeare euidently.

First, one man oweth me 22 l', 6 s, 8 d. Another oweth me 45 l, 16 s, 6 d. And another oweth me 4 l, 3 s. I would know what this is all together: Therfore must I first set downe my greater sum, and then the other, euerie one vnder his Denomination agreeing to the greatest summe, as heere you see with a line vn-

l'	s	d
22	6	8
45	16	6
4	3	

der them.

Then must I begin at the smallest numbers (which must alwaies be set next to the right hand) and adde them together: and (if the sum of them will make 1, or 2, or 3, of the next denomination) then must I keepe it in my mind till I come to that place, and under that first place must I note the residue (if there remain any of the same denomination:) but, if there remaine none, then need I to write under it nothing. And this is all that you must marke in this Addition: for all other things are like to the other maner of addition before mentioned. Therefore, the cheifest point of this Addition is, to know the values of cōmon coines and rated summes. As, how many shillings bee in a pound? how many pence in a shilling? of which (and of other like things) I will instruct you hereafter in teaching of Reduction. But now I may not disturbe your wit from the thing that we are about.

Therefore let us returne to that former example which I proposed of thrée debtors: which summes when I had set orderly, they stood thus wͭ a line under them.

l	s	d
22	6	8
45	16	6
4	3	
		2

Then to adde them into one sum, I must begin at the right hand, where the smallest denomination is, & adde them together, first saying: 6 and 8 make 14. Now, séeing these

14

14 are pence, which containe one ſhilling & 2 pence: the 2 pence I ſet downe vnder yͤ line of pence: and the one ſhilling I keepe in my minde to cary to the next row beeing the place of ſhillings.

l	s	d
22	6	8
45	16	6
4	3	
	6	2

Then do I adde the ſhillings together, ſaying: 1 in minde and 3 make 4, and 6 make 10, and 6 make 16, & 1 in the ſecond place which ſtandeth for 10, make 26, which is 1 pound, 6 s. The 6 s I ſet downe vnder the place of ſhillings, as appeareth in the example. And the 1 pound I keep to carie to the pounds.

l	s	d
22	6	8
45	16	6
4	3	
2	6	2

Then, come I to the pounds, adding them all together, ſaying: 1 that I keep, & 4 make 5, and 5 make 10, and 2 make 12. The figure or digit 2 I ſet downe right vnder that place or row of pounds where I gather them. And the article 1 I keepe to cary to the next place, ſaying: 1 in minde & 4 is 5, and 2 is 7, which 7 I ſet downe directly vnder that rowe alſo. And then appeareth my whole ſum thus.

l	s	d
22	6	8
45	16	6
4	3	
72	6	2

And thus muſt you do with any ſuch like ſummes whatſoeuer, whether they be money, weight

weight or measure, which (if you practise your selfe well therein, by setting downe of diuers sums) you shall be well acquainted with the feat of addition.

But now, can you tell how to proue this Addition, or such other like of diuers denominations, and to trie whether you haue well done, or no?

Scholar. I would I could.

Master. That shal you do by this meanes.

Proofe of addition of diuers denominations.

You must make a crosse, which shal haue so many lines as you haue sundry denominations in your Addition: As, if you haue but two denominations, then you may make it thus: that the ouer part and the neather part may serue for one denomination. And, if you haue three denominations (as pounds, shillings, & pence) then must you make three lines, thus. The vpright line may serue for pounds, and the highest thwart line for shillings, and the lowest for pence: as for example by the sum which wee last wrought.

l	s	d
22	6	8
45	16	6
4	3	0
72	6	2

o
6—|—6
2—|—6
o

For

For the proofe of the which, because it containeth thrée denominations, I must make a crosse of thrée lines, as in the example before. Then I reckon, first, at the right hand, the pence: 6 and 8 make 14, from which I take 12 for the next Denomination, that is to say, shilling, and there resteth 2, which I must write at one end of the nether thwart line.

After that, I gather the summs of the shillings, 3, 16, 6, which maketh 25, to whom I put 1 that I tooke of the pence, and that maketh 26, from those I take 20, the quantity of the next greater Denomination, that is to say, a pound, and there resteth 6, which I write at the end of the highest thwart line.

Thirdly, I adde together the pounds 4, 5, and 2, which make 11, to them I adde the one that came of the shillings, and they make 12, from whence I cast nine, and there resteth 3. That thrée I ioine to the 4 in the next place, and they make 7, then the 2 more make nine, which I cast away, and so there is left 0, which 0 I set at the vpper end of the crosse also: (And so should there be set also any number that were left vnder 9,) And thus is my first part of my worke proued.

That done, I come to the totall summe vnder the line, and examine it, beginning at the pence, where I find but two, and cannot take nine from him: therefore, I set him at the other end of the neather thwart line: then I come

come to the shillings, where I finde onely 6, which (because it is lesse than 9) I set it at the other end of the line of shillings, that is, the outermost thwart line.

Last of all, of the 72 s, I take eight times 9, which is 72, and there remaineth 0, which I write under the upright line: either else I may reckon them simply without any respect of their valuation or place: in saying, 2 and 7 make 9, which I take away, and so resteth 0. Then, I consider euery number, comparing it to the number that is against it: and, because I finde them to be euery one like his match, I know that I haue well done.

Scholar. This crosse I perceiue doth serue for these three denominations, pounds, shillings, pence. But, what if I had l, s, d, ob. and q?

Master. These lines (as I haue said) doe serue for three denominations, such as they be, as here three do serue for pounds, shillings and pence: but, if you haue no pounds in your sum, then may they serue for shillings, pence, and halfe penies: yea, for d, ob and q: or, in weight, for C. q. and l: or, in measure for Elles, quarters, and nailes, if you haue no greater Denomination: so that you remember that the upright line serueth for the greatest Denomination, and the highest thwart line for the next, and the lowest for the least.

And so, if you haue foure Denominations, you

ou muſt make your croſſe with
o many lines. And (if that your
umme be of moze denominati-
ns)make ſo many lines in your
roſſe. And thus will I make an
nd of Addition, ſauing that here (for the bet-
er underſtanding of this Rule) I haue ſet you
own certain exāples both of money, weight,
nd meaſures, with their workes and proofes.

Examples of Addition.

l	s	d	l	s	d
23	10	4	130	17	10
45	6	8	28	6	8
37	2	9	13	13	4
15	13	6	120	0	0
121	13	3	292	17	10

The Proofes.

	4			4	
4		4	8		8
3		3	1		1
	4			4	

C	q	l	yards	q	nailes
24	1	3	17	3	3
12	2	2	35	2	1
7	3	4	26	1	3
13	0	13	54	2	0
57	2	22	134	1	3

	3			8	
2		2	1		1
4		4	3		3
	3			8	

Subtraction.

Scholar.

THen haue I learned the two first kinds of Arithmeticke: now (as I remẽber) doth followe Subtraction, whose name (me thinketh,) doth sound contrarie to Addition.

Master. So it is indéede: for, as Addition increaseth one grosse summe, by bringing many into one: so contrariwise Subtraction diminisheth a grosse summe by withdrawing of other from it. So that Subtraction (or Rebating,) is nothing else, but an Art to withdraw and abate one sum from another, that the Remainer may appeare.

Scholar. What do you call the Remainer?

Master. That you may perceiue by the name.

Scholar. So me thinketh: but yet it is good to aske the truth of all such things, lest in trusting to mine owne coniecture, I bee deceiued.

Master. So it is the surest way. And (as I sée cause) I will still declare things unto you so plainly, that you shall not néed to doubt. Howbeit, if I do ouerpasse it sometimes (as the maner of men is to forget the small know-

ledge

ledge of them to whom they speake:) then doe you put me in remembrance your selfe, and that way is surest.

And, as for this word that you last asked mee, take you this description: The Remainer is a summe left after due subtractiō made, which declareth the excesse or difference of the other two numbers: as, if I would abate or subtract 14 out of 18, there should remaine 4, which is called the Remainer, and is the difference betweene those two numbers 14 and 18.

Scholar. I perceiue then what Subtraction is: Now resteth to know the order to worke it.

Master. That shall you do by this meanes: First you must consider, that, if you should go about to rebate, you must haue two sundrie sums proposed: the first which is your grosse sum (or sum totall;) and it must bee set highest: and then, the rebatement (or summe to be withdrawn) which must be set vnder the first, (whether it be in one parcell or in many) and that in such sort, that ÿ first figures be one iust ouer another, and so the second, and third, and al other following, as you did in addition: then shal you draw vnder them a line, & so are your summs duely set to beginne your working

Then begin you at the right hand (as you did in addition) and withdraw the neather number out of the higher, and if there remaine

main any thing, write that right under them beneath the line: and if there remaine nothing (by reason that the two figures were equall) then write under them a cipher of nought. And so do you with all the other figures, evermore abating the lower out of the higher, and write under them the Remainer still, till you come to the end. And so wil there appeare under the line what remaineth of your grosse sum, after you haue reducted the other sum from it, as in this example.

I receiued of your father 48 s, of which I haue laid out for you 36 s: now would I know what doth remaine? And therefore I set my numbers thus in order. First, I write yͤ greatest summe, and under him the lesser, so that the figures at the right side bee euen one under another, and so the other, thus.

s
48
36

Then, do I rebate 6 out of 8, & there resteth two, which I write under them right beneath the line, thus.

68
36
2

Then, I go to the second figures and doe rebate 3 out of 4, where there remaineth 1, which I write under them right, & then the whole sum & operation appeareth thus.

48
36
12

Whereby it appeareth, that if I withdraw

36 out of 48, there remaineth 12.

Scholar. Now will I proove in a greater summe, and I will subtract 2367924 out of 3468946. Those sums I set in order thus.

3468946
2367924

Then doe I begin at the right side, and deduct 4 out of 6, and there resteth 2, which I write vnder them. Then go I to the second figures, and withdraw 2 out of 4, and there remaineth 2, which I set vnder them also: then I take nine out of 9, and there resteth 0, which I write vnder them (for you say, ỹ if the figures bee equall, so that nothing doe remaine, I must write this Cipher 0 vnder them.)

Master. It was well remembred: now goe forth.

Scholar. Then I come to the fourth place, and draw 7 out of 8, and there remaineth 1, which I write vnder them also. Then, in the fift place I take 6 out of 6, and there resteth nought (for if I write vnder thẽ the cipher 0) Then, in the 6 place 3 rebated from 4, there remaineth 1, which I write vnder them; and likewise in the vij. and the last place, 2 taken from 3, there is left one, which I write vnder thẽ: so haue I done my whole working, & my summes appeare thus. Whereby

3468946
2367924
1101022

I see, that (if I doe rebate 2367924 out of 3468946) there remaineth 1101022.

Master. This is well done. And that you may be sure to perceiue fully the Arte of Subtraction, let mee see how can you subtract 52984732 out of 8250003456?

Scholar. First I set downe the greatest summe, and after that, I write vnder it the lesser number, beginning at the right side: and then my figures will stand thus.

8250003456
52984732

Then take I 2 from 6, and the rest is foure, which I write vnder them: then doe I withdraw 3 from 5, and there remaines 2, which I write vnder them. Then take I 7 out of 4, but that I cannot, what shall I now doe?

Master. Marke well what I shall tell you now, how you shall doe in this case, and in all other the like: If any figure of the neather summe bee greater than the figure of the summe that is ouer him, (so that it cannot be taken out of the figure ouer him,) then must you put 10 to the ouer figure, and then consider how much it is, and out of that whole summe withdraw the neather figure, and write the rest vnder them. Can you remember this?

Scholar. Yes, that I trust I shall. Now then in mine example where I should haue taken 7 out of 4, and could not, I put 10 to that

that 4, which maketh 14, from it I take away 7, and there resteth 7 also, which I write vnder them.

Master. So haue you done well, but now must you marke another thing also: that (whensoeuer you doe so put ten to any figure of the ouer number,) you must adde one still to the figure or place that followeth next in the neather line: as in this example there followeth 4: to which you must put one, and make him 5, and then go on as I haue taught you.

Scholar. Then shall I say, 4 and 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3, but that cannot bee: therefore I must put to it also 10, and then it will bee 13: from which I take 5, and there resteth 8 to bee written vnder them: and because of that 10 added to the 3, I must adde 1 to 8 that followeth in the neather line, & that maketh 9 which I should take out of 0, and cannot: therfore, I put therto 10 and that maketh 10, from 10 I take 9, and there remaineth 1: which I write vnder them.

Then doe I adde one likewise to the next figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure aboue, but I cannot: for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written vnder them.

Then come I to the next figure, which is 2, and to him I doe adde 1, which maketh 3, that 3 I cannot take out of nought: therefore of that nought I make 10, and thence doe take 3, so remaineth there 7 to bee written vnder them: Likewise doe I put 1 to 5 that followeth, and then is 6, that would I take out of 5 and cannot, therfore I adde ten to that 5, and make it 15, from which I rebate 6, and there remaineth 9, which I write vnder them. Now haue I spent all the neather figures, and what shall I doe more?

Master. You should haue added one to the next figure following (if there had been any) because you added ten to the last figure before of the ouer line: but seeing there is no figure following, you must adde that one to the place following, and then deduct that one from the number aboue.

Scholar. Then shall I say, because I borrowed ten to the ouer 5, I must put 1 in the next place beneath, that is vnder 2, then must I subtract that 1 from 2, & there resteth one, to bee written vnder that in the ninth place. Now I haue no more to subtract, for there is not any figure remaining beneath, neither yet any vnit to be added, because I borrowed not ten to the figure last before: and yet is there 8 remaining in the ouer line, which I thinke (by reason) should bee sette at the end of the figures in the lowest rowe, which is vnder

the

the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so in deede.

Scholar. But, Sir, I beseech you, shall I alwaies (when any number so remaineth alone as this eight did) write him under the line straight against his owne place?

Master. Yea, what else? Whether they be one or manie: and this well remembred, you haue sufficiently learned Subtraction. Howbeit, because of certaine things that might deceiue you, if you did not take good heede to your working, I will propose to you another example of many numbers to bee subtracted, as thus.

I receiued of a friend of mine to keepe 2869 crownes, of which at one time I deliuered him againe 500, at another time 368, at another time 440, at another time 80, and another time 64: now would I know, how many doe rest behinde? Therefore first I set downe my grosse summe and a line under it: and underneath it I set all the parcels thus, and under them a double line.

2869	*Crownes receiued.*
500	
368	
440	*Deliuerd.*
80	
64	

Then first I begin at the first place, and gather together the sum of

of all those lines (saue the ouermost) in their first figures: and so I doe with all the figures of the second place, and so forth, as I did in Addition, saue that I leaue out the highest row of numbers (as the line warneth mee) and that sum so gathered betwéene the double line, is the sum deliuered in all: which sum I doe afterwards subtract out of the highest row of numbers, and the remainer doe I set vnder the nethermost line: as for example.

I set the summes as before: then do I gather ỳ first figures of all the places deliuered, together: where I finde but 4 and 8, that make 12, (for thrée Ciphers increase no summe in Addition: as you learned before:) of

2869	*Crownes receiued.*
500	
368	
440	*Deliuered.*
80	
64	
1452	*Deliuered in all.*
1417	*Rest behinde.*

the 12, therefore, doe I write the Digit 2 betwéene the double line, and kéepe the Article in my minde, till I come to the second place, where I finde 6, 8, 4, 6, that maketh 24: to them I put the article in my minde, and it is 25: of which I write 5 vnder the second place and kéepe the digit 2 in my minde for the third place, where I finde 4, 3, 5, that makes 12: to the which I adde the 2 in my mind: and it maketh 14, thereof I write the 4 vnder ỳ third place

place: and because there remaine no mee figures to bee added, I write the digit, in the fourth place, as you see in the example; and so it appeareth, I haue deliuered in all a thousād foure hundred fifty two crownes.

Then come I to the subtracting of this sum betweene the lines, for by Addition it is equall to the fiue parcels ouer it. Therefore I procéede to subtract it from the ouermost sum, saying: 2 from 9, remaine 7, to be written vnder them beneath the lowest line. Then in the second place, I take fiue from six, and there resteth 1, to bee written vnder them. Then in the third place, 4 from 8 resteth 4. Last of all in the fourth place, 1 from 2 remaineth 1. And thus I sée that after those 5 sums are subtracted from 2869, the Remainer is 1417.

Master. This is a very sure and easie way for a learner: therefore I counsell you to practise it well.

Scholar. This I perceiue: but is there no shorter way and more spéedie?

Master. Yea, when you are a while exercised in it: for you may (as fast as you can gather the numbers together) withdraw them out of the highest sum. But, if in quantitie those numbers added together exceede the highest sum or vpper number, then shall you (as before hath béene taught you) imagine to borrow 10, 20 or 30 moe, as néede shall require

quire, and put them to the upper number, to helpe to further the abatement, referring or restoring the Articles that you borrowed to the next place againe: and so still go forward till you haue ended your worke: as for example: In the last summe proposed, I gather first in the first place 4 and 8, that maketh twelue, which 12 I should deduct or take out of 9 in the upper number aboue the line, but I cannot: therefore, I adde unto 9 an article of 10, and that maketh the upper number 19: from whence I take 12, then there resteth 7. Then, for the Article 10, I adde to the next place of money deliuered: saying, 1 that I bring, and 6 make 7, and 8 make 15, and 4 make 19, and 6 make 25, which 25 I should take out of 6 in the upper number, but I cannot. Therefore I adde 2 tennes of 20 unto 6 in the upper number, and that maketh 26, then 25 out of 26 resteth 1, then the two tennes which I borrowed or haue in minde, I adde to the next row or sum deliuered, saying: 2 that I bring, and 4 make 6, and 3 make 9, and 5 make 14, then 14 out of 8 I cannot take, but 14 out of 18 resteth 4. Now because there are no more places to be added, the one that I borrowed or haue in my minde, I rebate from 2 in the upper line, and there remaineth 1, which I set downe in the remainer line, and so my sum appeareth (as before) to be 147 crownes. Lo thus haue you now a shorter way.

Scholar.

Scholar. I like both waies well, and I perceiue both well: yet, as in the one the working seemeth somewhat long, so in the other it leaueth verie much (mee seemeth) to remembrance, and therefore may cause errour quickly, except a man haue a quicke and an exercised remembrance. But yet for the sharpning of my wit, by your patience (if you will giue me leaue) I will try what I can doe in such a like summe, to worke it the shortest way: whereupon I would subtract out of 40301964 these thrée parcels.

	40301964
20003428	20003428
10002432	10002432
10101461	10101461

Therfore I set the first in due order: then I gather the parcels of the first place, which are 8, 2, 1, that is 11, which I shold take or deduct out of 4, which is ouer him, but I cānot: therefore I adde an article or one ten to 4, which maketh 14, then 11 out of 14 there resteth 3 to be written vnder the first place betwéene the two lines.

Then come I to the second place, saying: 1 that I borrowed to haue in minde, & 6 make 7, and 3 make 10, and 2 make 12, which I cannot take from 6: therefore I adde 10 to 6, which maketh 16, & thē 12 from 16 resteth 4, which I write vnder the second place betwéen the

the two lines.

Then come I to the third place, saying: one that I borrowed or haue in minde, & 4 make 5, and 4 is 9, and 4 make 13, which I should take out of 9 that is ouer them, but I cannot: therefore I adde 10 to 9, which make 19, then 13 out of 19, rest 6.

Then come I to the 4 place, saying: one in minde & 1 is 2, and 2 make 4, and 3 make 7: which because it cannot be taken from one, I take it from 11, and there rests 4.

After that, I come to the fifth place, where are onely thrée Ciphers, which make nothing: vnto which I adde 1 in mind, the should I take that (that is to say) 1 from the figure our them, which is also a cipher: therefore, I must say thus: I cannot take 1 from nought, but 1 from 10, remaineth 9, so must I write 9 vnder them. Then in the 6 place I finde but 1, which 1 in mind I take out of 3 ouer him, & the remainer is 1, ỷ must be written betwéen the two lines in ỷ 6 place. So, I go to the seuenth, where I finde onely Ciphers, and in the grosse sum ouer them a cipher also: therfore, must I write the Remainer (which is nothing) with a Cipher also. Then in the eight and last place, I gather 1, 1, 2, that maketh 4, which if I take out of that 4 that is ouer them, there will nothing remaine. And that must bee noted with a cipher betwéen the two lines (as I haue often said) and so haue I ended my worke, ar

the figures stand thus.

But, Sir I remember you taught me that Ciphers should not come in the last place, for because they serue only to increase the value of other figures which follow them, and serue not those figures that go before them: & now in my example I haue set two Ciphers in the two last places.

40301964
20003428
10002432
10101461
00194643

Master. I commend you for your remembrance. And truth it is, you should not haue set them here, but onely because that I would make you plainly to perceiue the Art of Subtraction. Therefore, seeing that you doe now perceiue it, whensoeuer you would write downe a Cipher, looke whether any other figures be yet behinde: and if not; then let go the Cipher also; for it needeth not to write him in any latter places, where no other figure doth follow, except it be (as I did now suffer you) to teach the vse of Subtraction the plainer.

Therefore your figures must stand thus when the work is ended.

40301964	*Charge.*
20003428	*Discharge.*
10002432	
10101461	

Scholar. Sir, I doe thinke with that that you taught me before, and by these two summes that you

taught

taught me laſt alſo, that now I could ſubtra
any ſum.

Maſter. So may you, if you haue marke
what I haue taught you. But, becauſe thi
thing (as all other) muſt be learned ſurely b
often practiſe, I wil propound here two exam
ples to you: wherein if you often exerciſe you
ſelfe, you ſhall be ripe and perfect to ſubtract a
ny other ſum lightly, for in them is containe
all the obſeruances of whole numbers. An
becauſe you ſhall perceiue ſomewhat both
how to doe it, and alſo whether it be well don
when you haue prooued to doe it, therefor
haue I written vnder them both the Remai
ners.

30696	*Lent.*	30606	*Debt.*
10249 10249 163	*Repaid.*	10354 10249 163	*Paide.*
20661	*Paid in all*	9840	*Reſt.*
9840	*Reſt to pay.*		

Scholar. Sir, I thanke you. But I think I
might the better doe it, if you did ſhew me th
working of it.

Maſter. Yea, but you muſt prooue you
ſelfe to doe ſome things without any aide, o
elſe you ſhall not be able to do any more tha
you were taught: And that were rather t

learne

learne by rote (as they call it) than by reason. And againe, there is nothing in this example, nor any other of whole number,) but I haue taught you the rules of them already, sauing I haue not yet shewed you how you shall proue this worke.

Scholar. Sir, I pray you then shew it me also.

Master. For the performance whereof if you marke well what I said in Addition, you may easily perceiue what may be done for the proofe of Subtraction, and that is only performed by the aide of Addition, thus: Draw vnder the lowest number (which is your Remainer) a line, and then adde the summe paid in all (which before was made by the particulars) and the Remainer together. And if these twaine beeing added together in one sum doe make the contents of the vpper number aboue the line, then is this Subtractiō wel wrought, or else not.

For the triall whereof I will propone one of our examples which are done, which was of 2869 crownes receiued to keep of a friend.

Crownes 2869 receiued.

Of which (at sundry times) I deliuered these particular sums	500
	368
	440
	80
	64

which

which particulars in one Addition make, so resteth to my friend.	1452 *delivered in all.*
	1417 *rest to pay.*
	2869 *Proofe.*

Now, put or adde this Remainer, and the summe deliuered together, saying: 7 and two make 9, and 1 and 5 make 6, and 4 and 4 is 8, and 1 and 1 is 2, which in one sum amounteth to 2869: and is equall with the sum 2869 aboue the line: whereby I know the subtraction is well wrought: and this is the order and proofe hereof, and of all such like workes of Subtraction.

Scholar. Then I trust by practise to attaine the vse of it. And is this all that I shall learn of Subtraction?

Master. Yea, sauing that (as you haue seene in Addition) there are numbers of diuers denominations, in which the working is not much vnlike: yet, (without some instructions be giuen of it) it might seeme to a learner more difficult than indeede it is. Therefore I will briefly shew you that vse of it onely by one example or two.

A certaine man owed to me 14 l, 12 s, 8 d, of which he paid me at one time 4 l 6 s. 8 d: at another time 3 l, at another 2 l 3 s 4 d: and last of all 6 s 8 d.

Now, would I know what remaineth vnpaid yet: therefore I set my summes thus, e-

uery

euery one in their due place: As pounds vnder pounds, ſhillings vnder ſhillings, pence vnder pence.

l	s	d
14	12	8
4	6	8
3	0	0
2	3	4

Scholar. Sir, I pray you, why doe you write 2 l, for the common ſpeech vſeth rather to ſay, 40 s?

Maſter. We muſt here vſe the Denomination that is greateſt in any ſumme, ſo that we may not write according as we vſe to ſpeake, ſaying, 16 d, 18 d, or likewiſe, ſeuen groats, 8 groats, 24 s, 40 s, 48 s, and ſuch other: but we muſt write euery denomination that is in any ſum by it ſelfe.

Namely, ſhillings and pounds. So muſt we write for the laſt ſummes now named, 1 s 4 d 1 s, 6 d. 2 s, 4 d. 2 s, 8 d. 1 l, 4 s. 2 l, 8 s. and ſo forth of other like.

Scholar. So that wee may not write in Arithmetike pence, when the ſum amounteth to ſhillings, nor ſhillings, when the ſumme maketh pounds. Now (if it pleaſe you) end your example.

Maſter. When my ſums are ſo ſet as I ſhewed, then (according to the rules of Addition) I gather all the particular ſummes which be paid me into one totall ſumme, directly to be ſet vnder them betweene the two lines, not medling with the 14 l, 12 s, 8 d, as the line warneth me: therefore muſt I firſt begin with

the

the smallest denomination, saying, 8, 4, 8, is 20 pence; which maketh one shilling and 8 pence: the 8 d I set downe under the place of pence, and the one shilling I keepe in mind to cary to the next denominatiōs of shillings. Then come I to the shillings, and say, one ẏ I bring or haue in minde, and 6 is 7, and 3 is 10, and 6 makes 16, which, because it cōtaineth not one pound, I set directly under the place of shillings. Then come I to the pounds, whose parcels are, 2, 3, 4, that is in all 9, that 9 doe I set downe directly under the poūds: And so the total or whole Addition of all the particulars paid, amounteth to 9 l, 16 s, 8 d.

l	s	d	
14	12	8	
4	6	6	
3			
2	3	4	
	6	8	
9	16	8	
4	16	0	Rest.

Now, for the worke of Subtraction, I must rebate that totall sum of Addition out of the higher number, that is to say, from the 14 l, 12 s, 8 d.

Therfore, to performe the worke, I say 8 d out of 8 d remaineth or resteth nothing: therfore, in the place of the rest or remaine, right under that denomination I set downe 0: Then comming to the shillings, where I finde 16, which should be taken out of 12, but I cannot: therefore I imagine to borrow one of the next de-

denomination, that is of the 9 l, and put that one pound so borrowed vnto 12 s, that maketh 32 s.

Now 16 s out of 32 s, resteth 16 s, which 16 s, I set downe directly vnder the place of the rest.

Lastly comming to the pounds, saying, one pound in minde that I borrowed, and 9 make 10, then 10 out of 14, there resteth 4.

So doth my whole rest remaine appeare to be 4 l, 16 s, 0 d.

This I account the easiest way for a yong beginner to practise, though it bee something long.

Scholar. Is there any shorter way for this worke also?

Master. Yes, as in this last example I will also shew you, for you may adde together the particular summes as they are set in order, beginning with the pence, saying, 8, 4, 8 make 20 d, which 20 d, you should take out of the 8 d aboue ẏ line, but you cãnot, therefore shal you borrow 1 of the next denomination, ẏ is to say, one of ẏ shillings, and put it to the 8 d, that maketh 20, now 20 pence out of 20 d, rest 0, which cipher I sette downe directly vnder them.

l	s	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

Then one shilling that I borrowed or had

in minde, and 6 make 7, and 3 make 10, and 6 make 16, then 16 out of 12, I cannot take, therefore of the next denomination I do borrow one pound, and put it to 12 s, which maketh 32 s, then 16 s out of 32 s, resteth 16 s.

Lastly, I come to the pounds, saying: one pound in minde or that I borrowed, and 2 make 3, and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4.

So doth my remainer or rest appeare as before to be 4 l, 16 s, 0 d.

Scholar. This doe I perceiue very well, and if there be none other thing to be learned in Subtraction, then may I come to Multiplication, for that you reckoned to bee the next in order.

Master. Wee haue done indeede with the Art of Subtraction, as touching the working. But yet before we go to Multiplication, I will instruct you how to examine your work whether it be well done or not. For the performance whereof, if you marke what I said right now in the last manner of proofe, you may easily perceiue what is to bee done for this proofe, which is onely made (as before was taught you) by the aid of addition, thus:

Draw vnder the lowest number (which is your Remainer) a line, and then adde the summe paid in all, and the Remainer together. And if these twaine added together in one summe make the contents of the vpper num-

number aboue the line, which in this example is 14 l, 12 s, 8 d. then is the ſubtraction well wrought, or elſe not.

As for example, in our firſt ſumme, which ſtood thus.

Where in the title of pence I finde 8 and 0: the 8 I ſet down directly under him in that place of pence.

Then in the place of ſhillings I find 16 and 16, which make 32 ſhillings, wherein is contained one pound and 12 s: the 12 s I ſet downe directly under them in their due place of ſhillings, and one pound I keepe.

l	s	d	
14	12	8	
4	6	8	
3	0	0	
2	3	4	
	6	8	
9	16	8 *	* Paid in all.
4	16	0 reſt	
14	12	8	

Then comming to the pounds, ſaying, one that I keepe, and 4 is 5, and 9 is 14, which 14 in due order I ſet downe directly under them as this figure ſheweth. Which is alſo 14 l, 12 s, 8 d, agreeing with the upper number aboue. So I find the work is good, and the Subtraction well wrought.

Now for the proofe of the latter ſubtraction which wee reckoned for the ſhorter worke: Draw under the remainer alſo a line, then adde that remainer & all the other numbers that ye did ſubtract before together, and write that that amounteth under the loweſt line,

and if the ſumme that commeth thereof be equall to the higheſt number aboue, then is the Subtraction well wrought, or elſe not.

As for example alſo in the laſt ſums which ſtood thus.

l	s	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0
14	12	8

First in the title of pence I adde 8, 4, 8, that maketh 20 d, which containeth one ſhilling and 8 pence.

The 8 I ſet downe vnder the loweſt line in the rowe or title of pence, and that one ſhilling I keepe to cary to the next denomination or place of ſhillings.

Then returning to the ſhillings, ſaying: one in minde, or that I keepe, and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 32 ſhillings, which amounteth to one pound, 12 s, the 12 s I ſet downe vnder the title of ſhillings, and 1 pound, I keepe or haue in minde to carrie to the next denomination or place of pounds. Then come I to the pounds, ſaying, one that I bring, and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14, then do I write 14 vnder the pounds, and ſo haue I ended the Addition, and I ſee that the loweſt line is like vnto the vppermoſt line in number, wherefore I know that I haue wel done.

And thus now haue I taught you the Art of

of Subtraction, & the meanes to prove whether it be well wrought, or not.

Many other workes and proofes might bee shewed of Subtraction, but one of these whether of them you please, are most aptest & best allowed of any other worke or proofe, whether it be of l, s, d, or any other grosse summe whatsoever.

Scholar. Sir I thanke you most hartily, for now I understand wel both the feats of Subtraction and his proofe.

Master. Therfore here will I make an end of Subtraction, and will instruct you in Multiplication.

Multiplication.

Multiplication is an operation, q̃ 2 summes produce the third, which third sume so many times shall conteine the first, as there are unites in the second. And it serveth in the stead of many Additions. As for example. When I would know how many are 30 times 48: if I should adde 48 30 times it would bee a long worke. Therefore was this worke of Multiplication devised, which shall do that at once

 that

that addition should do at many times.

Scholar. I perceiue the commoditie of it, partly, but I shall not see the full profit of it, till I know the whole vse of it. Therefore Sir I beseech you teach me the working of it.

Master. So I iudge it best, but because that great summes cannot be multiplied, but by the multiplication of digits, therefore I thinke it best to shew you first the way of multiplying them: As when I say, 8 times 8 or 8 times, 9 &c. And as for the small digits vnder 5, it were but folly to teach any rule, seing they are so easie, that euery childe can doe it. But for the multiplication of the greater digits, thus shall you do.

First set your Digits one ouer the other right, then from the vppermost downeward, and from the nethermost vpwarde, draw straight lines, so that they make a crosse, commonly called Saint Andrewes crosse, as you see heere. Then looke how many each of them lacketh of 10, and write that against each of them at the end of the lines, and that is called the difference: as if I would know how many are 7 times 8, I must write those Digits, thus.

Digit difference.

8

7

Then doe I looke how much 8 doth differ from 10, and I find it to be 2, that 2

doe I write at the right hand of 8, at the end of the line thus.

8 2

7

After that I take ỹ difference of 7 likewise frõ 10, that is 3, & I write that at the right side of 7 as you see in this exãple.

8 2

7 3

Then do I draw a line vnder them, as in Addition thus.

Last of all, I multiply the two differences, saying, 2 times 3 make 6, that must I euer set vnder the differences, beneath the line: then must I take one of the difference (which I will, for all is like) from the other digit (not from his owne) as the lines of the Crosse warne me, and that that is left must I write vnder ỹ digits. As in this example, if I take 2 frõ 7, or 3 frõ 8, there remaineth 5: ỹ 5 must I write vnder the digits, and then there appeareth the multiplication of 7 times 8, to bee 56.

Digit difference.

8 2

7 3

5 6

And so likewise of any other Digits, if they be aboue 5, for if they be vnder 5, then wil their differences be greater than themselues, so that they can not be taken out of them. And againe, such little summes euerie childe can multiply, as to say: 2 times 3, or 4 times 5

and suchlike.

Scholar. Truth it is. And seeing me seemeth that I understand the multiplying of ẏ greater digits, I will prooue by an example how I can do it. I would know how many are nine times 6.

Master. It is all one in value to say nine times 6, or 6 times 9: but yet the order is best to put the lesse summe first, saying, 6 times 9, and so of all other summes.

Scholar. Then would I know how many are 6 times 9. therefore I set the digits thus, and make the crosse thus.

Then do I set their differences from 10 at the right side the differēce of 9, which is 1, against it, and the difference of 6, which is 4, against it also, as in this example.

And under them I draw a line. Then doe I multiply the digits together, saying: one time 4 maketh 4, that 4 doe I write under the differences thus.

Then take I one of the differences frõ the other digit, as one from 6, or else 4 from 9. and each waies there resteth 5, which I doe write under the digits,

Digits thus. And so appeareth the multiplication of 6 times 9 to be 54. Thus I see the feat of this maner of multiplication of Digits.

9 1
X
6 4
5 4

Master. Now might you go straight to the multiplication of greater numbers, saue that both for your ease and suretie in working I wil draw you here a Table, whereby shall appeare the multiplication of all the Digits, & this it is that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seeke your first or last digit in the greater figures, and from it go right forth toward the right hand, till you come vnder the number of your second Digit, which is in the highest rowe: and then the number that is in the meeting of the rowes of little squares (which come

come directly from both your propounded digits) is the multiplication that amounteth of them. As if I would know by this Table the multiplication of 7 times 9, seeke first 7 in the greater figures, and then go right forth toward the right hand, till you come under 9 of the highest row, in which place where you so come under the other digit (as here for example you come under 9) is alwaies contained the ofcome or product, which you seeke: and that place we terme to bee in the common angle, in respect of the two numbers so taken on the outsids, as here in that common angle, where the rowes of little squares (directly proceeding frō 7 & 9) do meet, you have 63, which 63 is the sum of the multiplication of 9 by 7.

Scholar. This is very good and ready. And so may I find the multiplication of any digits. But now how shall I do in greater summes?

Master. When you would multiplie any summe by another, you shall marke that it is the meetest order to set the greatest number highest, which is the place of the number that must bee multiplied: and likewise the lesser number under it, for that is the place of the Multiplier or Multiplicator, that is to say, the number by which multiplication is made, and is in English alwaies put before this word, Times: in such speaking when I say, 20 times 70. And the number that followeth this word times, is that which must bee multiplied

tiplied.

Therefore when I would multiply one number by another, I must write the greatest highest, and the lesser vnder it, as in Addition. And vnder them must I drawe a line. As for example: If I would multiply 264 by 29, I must set them thus.

264
29

¶ Of which numbers thus set downe to be multiplied, may bee formed a question, as thus: There are 29 men, and each man hath 264 Lambes. The question is, how many lambs they haue in all.

To the performance whereof, I must multiply euerie figure of the higher row, by euery figure of the nether row: and that that amounteth, I must set vnder the line, as thus. First I do multiply 4 by 9, saying: 9 times 4 (or 4 times 9 which is all one) & that maketh 36, as the Table before of Digits doth declare, of that 36 I must write the 6 that is the digit, vnder the 9, and the article 3 I keepe in minde to carry to the next place.

264
29
6

Then come I to the second figure of the higher row, which is 6, & say: 9 times 6 make 54, and with the 3 in my mind make 57, ÿ 7 I set downe vnder the 2, & 5 I keepe in minde.

264
29
76

After that I come to the next figure which is 2, and do multiply it by 9, and that maketh 18, and with 5 that I haue in minde maketh 23: wherefore because it is the last work of the multiplier, I set it down in order as you see:

264
29
2376

And so haue I ended the first figure of the multiplier. Wherefore I giue it now a fine dash with my pen.

Then begin I with the next figure, and multiplie it into all the higher figures, as thus:

264
29
2376
8

First, 2 times 4 make 8, that 8 doe I write vnder the second place: for euermore the digit or first figure of Multiplication that amounteth of the first figure of the higher number, must bee set vnder the multiplier of it, and the other in their order toward the left hand.

Scholar. I vnderstand you thus, that the digit of the summe amounting of the multiplication of the first figure of the higher row, by the first figure of the lower row or multiplier, must bee set vnder the first place: and that that amounteth of the same first figure by the second multiplier must bee set vnder the second place, and so of the other, if there be more multiptiers.

Master. So meane I indeed: and if there amount but a digit, then must it bee set vnder the

the Multiplyer.

And now to go forth: I multiply by the same 2, the second figure of the higher rowe, which is 6, saying: two times 6, make 12: whereof I write the digit 2 vnder the third place, and the article 1 I keepe in minde.

264
29
2376
28

Then do I multiply the last figure of the higher summe by that same 2, saying: two times 2 is 4, and with the one that I haue in minde maketh 5, which 5 I write vnder the fourth place. And so haue I ended the whole multiplication: wherefore I also giue the 2 a dash with my pen, thus: and so I doe euer as soone as I haue dispatched any Digit, by which I multiply: and the summes stand thus.

264
~~2~~9
2376
528

Then must I drawe a line vnder all those sums that amount of the multiplication, and must adde all them into one summe, as in the Example you may see.

264
~~2~~9
2376
528
7656

Where in the first place I finde but 6, and therefore write I it vnder the line. Then in the second place 8 and 7 make 15, whereof I write 5, and keepe one in my minde, and so forth as you learned in Addition. And so appeareth the whole summe to be 7656, which amoun-

amounteth of the Multiplication of 264 by 29, and that is the iust number of lambes the 29 men had.

Scholar. If there be no more to be obserued in it, then can I do it, I suppose, as by this example I shall proue.

¶ There is a peece of ground which containeth 1365 yards in length: and 236 yards in breadth, I would know how many yards square there is in all this peece of ground, which numbers I sette downe with the greater aboue, & the lesser vnder, as you see.

1365
236

Then doe I multiply 5 by 6, saying: 6 times 5 make 30: of which I write the cipher in the first place, and the article 3 I doe keepe in minde to carrie to the next place.

1365
236

Then do I by the same 6 multiply the second figure of the higher summe, which is 6, saying, 6 times 6 make 36, and 3 in my mind make 39, of which I write the 9 vnder the second place, and the article 3 I keep in minde.

1365
236
90

Then do I multiply the third figure, which is 3, by the same 6, and that maketh 18: and with 3 in my minde make 21. Then 1 I set down, and 2 keep in mind.

1365
236
190

Then

Then come I to the last figure of the higher summe, and multiply it by 6, saying: 6 times 1 make 6, & 2 in my minde make 8, that 8 doe I write vnder the fourth place. And so haue I ended the first multiplier, and dash him sleightly with my pen.

1365
236
8190

Then begin I with the second multiplier, and say first, 3 times 5, that maketh 15, of which I set the 5 vnder the second place, because that ẏ multiplier is there, & the article 1 I keepe in minde.

	1365
	236
	8190

Then come I to the second figure, that is 6, and multiplie it by 3, which maketh 18, and with one in minde maketh 19, the 9 I set downe vnder the third place, & 1 I keepe in minde.

1365
236
8190
95

Then come I to the third figure, which is 3, and multiply it by 3, saying: three times three make 9, and with 1 in my minde make 10, the Cipher I set vnder the fourth place, and the article I keepe in minde.

1365
236
8190
095

And then comming to the last figure 1, I multiply it by 3: and it maketh 3, and with the one in minde it maketh 4: which 4 I set in the fift place, & then haue I

1365
236
8190
4095

ended

endeth two of the multipliers, and the summe stand as you may see in the latter end of the page going before. And then I giue 3 his dash.

```
 1365
  236
 ----
 8190
4095
   0
```

Then come I to the third multiplier, and multiply it into euery figure of the higher sum, and first I say: 2 times 5 make 10, of which I set the cipher vnder the Multiplyer in the third place, and the Article 1 I keepe in mind.

And so multiplying the second figure 6, by that same 2, there amounteth 12 and with one in my mind make 13, whereof I write the digit 3 vnder the fourth place, and the article 1 I keepe in minde.

```
 1365
  236
 ----
 8190
4095
  30
```

Then doe I multiply the said 2 by the third figure of the higher summe, which is 3, and that maketh 6, and with the one in mind make 7, which 7 I set downe vnder the fift place, as appeareth by the example.

```
 1365
  236
 ----
 8190
4095
 730
```

Then come I to the last place, & multiply that one by 2, and there amounteth 2, which I set in yͤ first place, and then doth the sum stand thus.

```
 1365
  236
 ----
 8190
4095
2730
```

And so haue I ended the whole

whole multiplication.

But now (as you taught me) to know what this whole sum is, I must adde all those parcels together, and then under the line wil appeare, as you may see, the grosse or total summe, that is, 322140. Whereby I know there is so many yards square in that peece of ground.

```
  1365
   236
------
  8190
 4095
2730
------
322140
```

Master. This is well done, and call it workeman.

Scholar. Then, me thinketh I could call it well done, when I know, whether I had wel done or no.

Master. It is to be proued by 9, as Addition was, but the surest proofe is by diuision, and therefore I will reserue that proofe by diuision, till you haue learned the Art of Diuision. And anon I will shew you how it is commonly proued.

¶ But first, for your further instruction in this exercise of Multiplication, I will with one example more try your cunning, and so make an end; And the question is this. I would know how many daies it is since the Natiuity of our Lord and Sauiour Jesus Christ unto this present yeere 1590. Which to performe, you must multiply by the daies in one whole yeare, which are 365.

Scholar. Now, for that you haue giuen me

ſo much light into the queſtion, you ſhall ſée I will handſomely finiſh the worke, for according to your former inſtructions I ſet them downe with a line vnder them, thus.

```
1590
 365
----
```

Then ſay I, 5 times 0 is 0, the 0 I ſet downe vnder the firſt place, as here appeareth. Then ſay I 5 times 9 make 45, the digit 5, I ſet downe in the ſecond place vnder 9, and the article 4 I kéepe in minde to be added to the next multiplication. Thē ſaying fiue times 5 make 25, and 4 in minde makes 29, the 9 I ſet downe in the third place, and 2 I kéep in minde. Then comming to the laſt figure, I ſay once fiue is 5, or 5 times 1 is 5, and 2 in mind make 7, that 7 doe I ſet downe vnder the fourth place. And thus haue I ended my firſt Multiplier, and therefore I giue it a daſh with my pen.

```
1590
 365
----
7950
```

Then come I to the ſecond multiplier, which is 6, and do likewiſe multiply it into the vpper number, ſaying, 6 times 0 is 0: the 0 I ſet downe in the ſecond place right vnder his multiplier. Thē ſay I, 6 times 9 make 54, the 4 I ſet downe vnder the third place, and 5 I kéepe in mind. Then ſay I, 6 times 5 make 30, and 5 I kéepe in minde make 35, the digit 5 I ſet downe in the fourth place, and 3 I

```
  1590
   365
  ----
  7950
 9540
```

kéepe

keepe in mind. Then say I, 6 times 1 is 6: or once 6 is 6, and 3 in minde make 9, which I set downe next, and so haue I ended two multipliers: wherefore I dash the 6 with my penne.

Then begin I to multiply the third multiplier into the ouer number, saying, 3 times 0 is 0: the 0 I set downe in the third place right vnder his multiplier. Then say I, 3 times 9 make 27, the digit 7 I set downe in order next, and two I keep in mind, then say I, three times 5 is 15, and 2 in minde is 17: the 7 I set downe, and 1 I keepe. Lastly I say, once 3 is three, and 1 I keep is 4, which I set downe orderly next: And so haue I ended the multiplication, and my figures stand thus.

	1590
	365
	7950
	9540
	4770

Master. I commend you for your diligence, the worke is very perfectly done, which parcels if you now adde together into one sum, it will be 580350, which is the grosse or totall summe of that multiplication, and declareth the number of daies since our Lord and Sauiour his incarnation, vnto the end of 1590 yeares, besides 397 daies and twelue houres for leape yeares.

Scholar. This is maruellous, me thinketh, that such great matters may so easily bee atchieued by this Art, which heretofore I euer thought had bene impossible, as infinit sorts

of people are of that minde.

Master. Truth it is, that knowledge hath no greater enemie than ignorance, for this is one of the least of ten thousand things that may bee done by this Art, as hereafter you shall be able to iustifie.

Scholar. This manner of Multiplication I perceiue, if there be no more in it.

Master. Yes, there are other formes and helpes for ease & shorter labour of the works of Multiplication: but I wil remit them till you haue a little tasted diuision, where also the like helpe into diuision may bee vsed: and so therefore vnder one example for both, wil I shew you both ease in Multiplication, and also in diuision.

But sith the other formes and workings doe nothing differ from these works in effect, but onely in setting of the numbers, I will ouerpasse them til a more meete place and time. And now wil I instruct you in diuision, so that you thinke your selfe sufficiently to perceiue what I haue taught you.

Scholar. Yes, Sir, I thanke you, but I do not perceiue how to examine my worke, to try whether I haue wel done, or no: therefore as you promised mee ere-while, I pray you first shew me how I shall prooue it.

Master. That is commonly vsed by the proofe of 9, as you learned before in Addition, seeing that it differeth from that forme in

diuers

ners respects: as for example.

First, you must take a crosse after this manner.

Then must you examine your sum that should be multiplied, and looke what remaineth after casting away of 9, that set you at the one side of the crosse, then examine the Multiplier, and whatsoever remaineth in it after casting away 9 so often as you can, write that at the other side of the crosse: then must you multiply those two numbers together, & looke what amounteth thereof, if it be under 9, write at the higher part of the crosse: but, if it be above 9, then take thence 9 as often as ye can, and write the rest at the head of the crosse: as first for example, we will proove the example you put forth of the peece of ground that contained 1365 yards in length, and 236 yards in breadth.

Therefore first, I cast away all the nines from the summe to bee multiplied, saying: 5 and 6 make 11, cast away 9, rest 2: then 3 and 2 make 5, and 1 is 6: that 6 I write at the one side of the crosse thus.

Then do I examine the Multiplier, which is 236, wherein, when the 9 is cast out, there remaineth 2, that 2 therefore I set at the other side of the crosse.

Then doe I multiply 6 by 2, and it maketh

keth 12, from which 12 I withdraw 9, then resteth 3, that 3 doe I set at the head of the crosse. Then do I examine the grosse summe amounting of the Multiplication, which is 322140, where I finde 9 once, and 3 remaining, that 3 I set at the foote of the crosse, and then I sée it to agrée with the other 3 at the top of the crosse, and so know I that I haue done well: for if they two did differ, then were my worke vaine, and the multiplication false.

3
2 6
3

This is the common prowfe, but the most certaine prowfe is by Diuision, of which I wil anon instruct you.

Scholar. Sir, what is the chiefe vse of Multiplication?

Master. The vse of it is greater than you can yet vnderstand: howbeit, these plain commodities it hath, that if you would resolue any great and whole value into many small and lesse portions: as if you would change pounds into shillings, pence or any other greater or smaller parcels, by Multiplication, ye shall doe it spéedily and easily. Also if you should néed to adde one summe to it selfe, or to any other oftentimes, you shall doe it by Multiplication much more spéedily, readily, easily and surely, than by often and sundrie Additions. Take you these commodities grossely shewed for an answere at this time,

and

and hereafter I will more abundantly make you to perceiue the vse of it.

Diuision.

Scholar.

WEll, Sir, then in Diuision I pray you to instruct me. But mee thinketh by the name of it, that it should bee all one with Multiplication: for I call that diuision, when any thing is parted into diuers and many parts.

Master. You take it as it is taken commonly: howbeit, if you marke well, you shall perceiue that it is quite contrary to Multiplication, and doth not part one thing or few things into many, but contrary waies, it bringeth many parcels into few, but yet so, that these few taken together, are equall in valure to the other many: for by Diuision, pence are turned into shillings, and shillings into pounds: as for example of 120 shillings it maketh 6 pounds, so are 120 turned into 6 which is a smaller number: but then if you consider the Denominators, you shall see that

they are such, that one of the latter is equall to 20 of the first, and so in value the summes are one, though in number they doe far differ, and the latter sum is the lesser, and so it is alwaies in Diuision: howbeit, yet in the working, the sum is parted by another, and thereof doth it take the name.

Scholar. I thinke I shal better understand the reason of the name, when I know the use of the worke, therefore now would I gladly learne that.

Diuision what it is.

Master. Diuision is distributing of a greater summe by the unites of a lesser. Or Diuision is an Arithmeticall producing of a third number, in respect of 2 propounded numbers: which third number shall so often containe an unite, as the greater of the 2 propounded numbers did contain the lesser. So that, euen as Multiplication did seeme to serue instead of many Additions, so diuision may seeme to bee in place of many Subtractions: Because that third number briefly expresseth how many times the lesser of your 2 propounded numbers may bee subtracted from the greater: as in practise will more plainely appeare. Therefore (as you may perceiue) unto Diuision are required 3 numbers: the first, which should be diuided, and that must (generally) bee the greater: and the second, by which the other must be diuided, and that is (generally) the lesser, & is called the

the Diuisor. And the third, which answereth to the question (How many times?) & therefore is called the quotient.

The first must be first written, and the second so set vnder it, that the last figure of ẏ lower number be right vnder the last of the higher, contrariwise to the worke of the other kindes of Arithmeticke: for in them the two first figures were set euer meete one vnder the other: but in Diuision the last figures must bee set méete, except it chance so that the last figure of the diuisor, be greater than the last of the higher number, for then you shall set the last of the diuisor vnder the last, saue one of the higher number, as for example.

A generall rule for placing the figure.

An exception.

If you should diuide 365 (which are the summe of the daies of a yeare) by 28, which are the daies of a common moneth, then should you set them thus.

365
28

But if you would diuide those 365 daies, by 52, which is the number of wéeks in one yeare, then should you set them thus.

365
52

Likewise, if I would diuide the same 365 by 4, which is the sum of the quarters of the yéere, then must I set them thus.

365
4

Scholar. Sir, this do I vnderstand, but how now should I doe to diuide the one by the other?

Master.

Master. You must begin with the last figure next the left hand, and see how many times the last figure of the Diuisor may be taken out of the last figure of the other number, and that shall you note within a crooked line toward your right hand. As for example.

I would diuide 365 by 28, then set I those two summes thus.

365(
28

And I looke how many times I may find 2 (which is the last figure of the Diuisor) in 3, (which is the last of the nūber to be diuided) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1, and that is called the Quotient number, as I told you. Then because that when 2 is taken out of 3, there remaineth 1, I must write that one ouer 3, and deface or cancell the 3 and the 2, then will the figures stand thus.

Quotient number.

1
~~3~~65(1
~~2~~8

Then must I go to the next figure of the Diuisor, and take it likewise so many times out of ÿ figures that be ouer it, and looke what doth remaine, that I must write ouer them, and cancell them as in this example.

Therefore now do I take once 8 out of 16, and there remaineth 8, which I must set ouer the 6, and cancell or crosse out the 16, the 8 of the Diuisor: and then will the figures

gures stand thus.

And so haue I once wrought.

Scholar. So I perceiue that you take the neather figure not only out of ÿ other that is right ouer him, but out of that with the other also that remaineth before, and are writtẽ toward the left hand.

x8
365(1
28

Master. So must you do: for you must so take the Diuisor out of the ouer number, that there remaine not ouer it so great a summe as it selfe is, for then were your worke in vaine.

But yet againe here must you mark, that when you séeke how many times the last figure of the Diuisor may be found in the number ouer him, that you loke also whether you may as often find all the figures following in those that are aboue them (considering all the remainers, if there be any) if not, take your Quotient lesse by one, & then proue againe, and so stil til you find a méete quotient: and by what méet quotient must you alwaies multiply your Diuisor, and the product set vnder your Diuisor, so that the first figure stand vnder the first figure of your Diuisor, and the second vnder the second, and so forth: and then subtract that product from the number to bée diuided, that standeth directly ouer it, as you haue séene me do.

When you haue thus wrought once, then must

must you begin againe, & write your Diuisor anew, nearer toward the right hand by one place, as in this example, you shall set 2 vnder 8, & 8 vnder 5 thus.

x8
3 6 5 (1
2 8 8
2

Then (as before) seeke how many times you may take your diuisor out of the number ouer him now.

Scholar. That may I do here 4 times.

Master. Truth it is, that you may finde 2 foure times in 8: but then marke whether you can finde the figure following so many times in the other that is ouer him. Can you find 8 foure times in 5?

Scholar. No, neither yet once.

Master. Therefore take 2 out of 8 once lesse.

Scholar. That is three times.

Master. Well, then 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following make 25, in which sum I find 8 three times also: & therefore I take 3 as a true quotient, & write it within the crooked line of the Quotient before the one thus.

Mark how to cõsider this kind of Remainer.

x8
1 3 6 5 (13
2 8 8
2

Then say I, 3 times 2 make 6, then 6 out of 8 resteth 2, therefore I cancell the 8, and write ouer it the 2 that doth remaine, thus.

Then

Then do I take 8 as many times out of 25 saying: 3 times 8 make 24, and if I take 24, out of 25, there remaineth 1, so then I cancell 25 and 8, and o-uer the 5 I set 1, thus.

Or, you might (after you found 3 to be a fit Quotient) straight-way haue multiplied the whole Diuisor 28, by that 3 at once: which giueth 84, which being set vnder 28, and duely subtra-cted from 85, of the number di-uided, giueth 1, the remainer of the whole diuision, as before you had. Work which way you list, here you see also the forme.

1
281
365(13
288
2

And now haue I done with diuiding, for I can finde my Diuisor 28 no more in the o-uer summe.

Scholar. No, except you would part the 1 that remaineth into 28 parts.

Master. That is well said, and must wee do in such cases, when there remaineth any thing: but I will let that passe now, and will make you perfect in Diuision of whole num-bers, and will hereafter teach you particular-ly of broken Numbers, called Fractions.

Now if you doe perceiue the order of diui-sion, then doe you diuide this summe 136280 by 452.

Scholar. First I set downe the number

that

that should be diuided, then doe I set the Diuisor vnder it, so that the last figure of it be right vnder the last figure of the ouer number. Then will it bee thus.

136280
452

Master. Can you take the last of your Diuisor (which is 4) out of 1, which is the last of the ouer number?

Scholar. I had forgotten, because the last of the Diuisor cannot bee taken out of the last of the ouer number, in so much as it is the greater, therefore must I set the diuisor one place more forward, toward ẏ right hand thus.

136280
452

And then must I looke how often I may finde ẏ last figure of the diuisor (that is 4) in 13, which thing I may doe 3 times; therefore doe I say: 3 times 4 is 12: which I take out of 13, and there remaineth 1. Then doe I make at the right hand of my summes a crooked line, and write before it my quotient 3, and I cancell 13 and 4, and ouer the 3 I set the 1 that remaineth, & then the figures stand thus.

1
~~13~~6280(3
~~4~~52

Then do I multiply the same quotient into euery figure of the diuisor, and withdraw the summe that amounteth out of the numbers

bers ouer them, as first I say: 3 times 5 make 15, which I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remaineth, thus.

```
  x1
 xx6280(3
  4x2
```

Thus do I say likewise, 3 times 2 make 6, which I take out of 12, and there resteth 6, therefore I cancell the 12 and the 2, and ouer the 2 I write the 6 that remaineth, thus.

```
   x.x6
 xxxx80(3
   4xx
```

Then should I set forward the diuisor into the next place toward the right hand, thus.

```
  xx6
 xxxx80(3
  4xx2
   45
```

Master. But you may see that ouer the 4 is no figure, therefore I must set the Diuisor yet forwarder by another place.

And marke, whensoeuer it chanceth so, that you should set forward the diuisor, and that it cannot stand there, because there is no number ouer the last place, or if there bee any it is lesser than the last figure of the diuisor, then must you remoue the diuisor yet once again: and because that his first place of reckoning serued not to subtract him so much as once, therefore you shal write in the quotient a cipher 0. And if you should by chance neede to doe so oftentimes, for euery time write a cipher

cipher in the Quotient. The reason of this wil I shew you hereafter.

Scholar. Then must I set my summes thus.

```
  226
236280(3
  45352
   4
```

And, because I remoued ẏ Diuisor, so that I ouerskipped one place, I must write a Cipher in the Quotient: & then must I seeke a new quotient, as in this example I must say. How many times 4 is there in 6? (and sith it can be but once) therefore do I write 1 in the Quotient: and then say I, 1 times 4 taken out of 6, remaineth 2, I cancell the 6 & the 4, and write 2 ouer them thus.

```
   2
  226
236280(301
  45252
   4
```

```
   2
  226
236280(301
  45252
   4
```

Then say I againe, once 5 out of 28, remaineth 23. I let the 2 stand as it did, and ouer that 8 I set 3, cancelling the 8 and the 5 vnder it, thus.

```
   2
  2263
236280(301
  45252
   4
```

Master. You might as well haue said, once 5 out of 8, and so remaineth 3, but now go forth.

Scholar.

Scholar. Then once 2 out of 0 cannot be: What shall I now doe?

Master. Borow of the next number that is behinde (for there is 230) and doe as you learned in Subtraction in a like case.

Scholar. Then must I borrow 1 of the 3 comming behinde next, and make that 0 to be 10, and then take I 2 out of 10, and there resteth 8. And because I borrowed one of the 3, I must cancell the 3, and write 2 over it: then doth the figure stand thus:

```
     22
   ̶1̶1̶6̶3 8
  ̶1̶3̶6̶2 8 ̶0  (301
   4̶5̶2̶2̶2
     4 5 ̶5
      4
```

Master. Now haue you done, and yet remaineth 228, and your quotient sheweth you, that if you diuide 136280 by 452, you shall finde your Diuisor in your greater number 301, that is CCC times and once, and 228 remaining.

And in the other example (where I diuided 365 by 28) the Quotient was 13, and 1 remained, whereby I knew that in a yeere (which containeth 365 daies) there are 13 moneths, reckoning 28 daies (or 4 weekes) iust to a moneth, and 1 day more.

Scholar. Why then doe wee call a yeere but twelue moneths?

Master. Of that at a more conuenient time will I fully instruct you: but now it is not conuenient to intangle your minde with o-

ther things, then doe directly pertaine to your matter. Therefore if you remember what you haue heard, you haue learned a short manner of Diuision, which I would haue you often to practise, so that you may be perfect in it, and heereafter I will shew you certaine other proper points touching it.

Scholar. Then I pray you yet tell me how I shall examine and trie my worke, whether I haue done well or no, that though no man be by me to tell me, yet I may perceiue it my selfe.

Master. Some men (yea and commonly most) doe trie it by the rule of 9, as in all the other kindes, saue that their order is: First, they cast away 9 as often as they can out of the Diuisor, and that remaineth they set at one side of a crosse, as in our first example the Diuisor was 28, from which you may take 9 three times, and 1 remaineth: which they set by a crosse, thus:

 1

Then doe they likewise examine the Quotient (which in our example is 13) and from thence they cast away 9 as often as they can, and the remainer they set at the other side of the crosse, and then multiply they together those two remainers: and to it that amounteth they adde the remainer of the Diuision, if there were any: from that whole summe they withdraw 9 as often as they can

and

and the rest they set at the head of the crosse, as in our example the quotient is 13, from which take 9, and there remaineth only 4, and therefore must you set 4 at the other side of the crosse, thus:

4 ✕ 1

Then multiply 4 by 1, and it yeeldeth but 4: thereto adde the remainer of the Diuision (which was 1) and it will be 5, which summe doth not amount to 9, and therfore must be set wholly at the head of the crosse, as you see heere:

5
4 ✕ 1

And this number on the head of the crosse is the first proofe, to which if you finde another like in the number that was diuided, then haue you done well.

Therefore now shall you likewise examine the whole summe that was diuided, and take away 9 as often as you can, and that that remaineth, set at the foot of the crosse: and if it be equall to that in the head of the crosse, then haue you well done, else not.

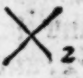
As in our example the whole summe was 365, which maketh 14, from that take 9, and there resteth 5, which set at the foot of the crosse, thus:

5
4 ✕ 1
5

And you shall see that they agree: therefore haue you well done.

Now will I likewise examine

our ſecond example, where the Diuiſor was 452, which maketh 11, from thence I take 9, & the 2 that remaineth I ſet at the right ſide of the croſſe, thus:



Then examin I the Quotient, which was 301, where I finde but onely 4, that doe I ſet at the other ſide of the croſſe, thus:

Then doe I multiply 4 by 2, and it maketh 8: to that doe I adde the remainer of the diuiſion, (which was 228, and maketh 12) & they two make 20: wherein I finde twice 9, and 2 remaining: that 2 muſt I ſet at the head of the croſſe, thus:

Then doe I examine the whole number to be diuided, which was 136280, where I finde twice 9, and 2 remaining, which I ſet at the foot of the croſſe, thus:

And becauſe that it doth agree with the figure at the head of the croſſe, I know that the diuiſion was well wrought.

Againe of the proofe of diuiſion.

Maſter. This is the common proofe. Howbeit, the more certaine working is by the contrary kinde: as, to proue Diuiſion by Multiplication, thus:

Multiply the Quotient by the Diuiſor, and

it

if the summe that amounteth be equall to the summe that should be diuided, then haue you well diuided: else not.

Howbeit, this must you mark, that if there remained any thing after the Diuision, that must you adde to the summe that amounteth of the Multiplication. As in our first example our quotient was 13, and the Diuisor was 28: Now multiplie the one by the other, and the summe will be 364: to that if you adde the one that remained after the Diuision, then will it bee 365, which was the summe that should be diuided: and therefore I know that I haue well done.

Scholar. Now will I prooue the same in the second example, whose Diuisor was 452, and the quotient 301: these doe I multiplie together, and there amounteth 136052; to which if I adde the 228 that remained, then will it be 136280, which was the whole sum to bee diuided: and therefore I perceiue that I haue well done.

Master. This is the surest way, to examine Diuision by Multiplication: and contrariwise the surest proofe of Multiplication is by Diuision.

And therefore (according to my promise) now will I shew how you may prooue Multiplication by Diuision.

A proofe of Multiplication by Diuision.

When you haue ended Multiplication, and would know whether you haue well done

or not, set the crosse summe that amounteth of the multiplication ouermost, and diuide it by the multiplier: and if the quotient bee the same number that should be multiplied, then haue you well wrought, else not, as in that example where we multiplied 264 by 29, the crosse summe was 7656.

Now if you will know whether that multiplication be true, you shall diuide that 7656 by the multiplier 29, and you shall perceiue that the quotient will be 264, and that is a token that you haue well wrought.

Scholar. By your patience I will prooue that, and first set downe the grosse summe and the multiplier, not after the rule of Multiplication, but after the rule of Diuision, for now that number is become the Diuisor, that was before the Multiplier, I shall set them therefore thus:

7656
29

Then shall I seeke how many times 2 in 7: that may bee 3 times, and 1 remaineth: but then may not 9 bee found so often in 16, therefore must I take a lesser Quotient, that is to say 2: then say I, twice 2 maketh 4, which I take out of 7, and there remaineth 3: then doe I cancell 7 and 2, and ouer 7 I write 3, and in the Quotient I set 2: so the figures stand thus:

3
~~7~~656 (2
~~2~~9

Then say I forth, two times 9 make 18, which

which I abate out of 36, and there resteth 18: then cancell I 3, and ouer him set 1, and likewise I cancell 6 and 9, and ouer them I set 8, so that thus stand the figures:

```
 1
38
7656  (2
29
```

Then doe I set forward the Diuisor by one place, and seeke a new Quotient, that is to say, how many times 2 are in 18, which I finde to be 9 times: but then can I not finde 9 so many times in 5, therefore I take a lesser Quotient, as to say, 8: but yet that is too great: for if I take 8 times 2 out of 18, there remaineth but 2, and I cannot finde 8 times 9 in 25: therefore yet I take a lesser Quotient, that is 7, which is also too great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 25, therefore yet I seeke a lesser quotient, as to say, 6: then say I, 6 times 2 make 12, that I take out of 18, and there remaineth 6: so I cancell 18, and the 2, and write 6 ouer 8 thus:

```
 16
 38
7656  (26
299
```

Then say I forth, 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the figures stand thus:

```
  1
 16
 381
7656(26
 299
```

Then must I set forth the diuisor againe, and seeke a new quotient, which will bee 4: for

```
  2
```

 though

though I may finde 2 in 11 fiue times, and 1 remaine, yet I cannot finde 9 ſo often in 6: therefore I ſet the figures thus:

1
163
381
7656(264
2999
22

And the 4 in the Quotient I multiply into the figures of the diuiſor, ſaying, 4 times 2 maketh 8, which I take out of 11, and there reſteth 3: therefore I cancell the 11 & the 2, and ſet 3 ouer the firſt place of 11, thus:

1
163
381
7656 (264
2999
22

And then do I ſay forth: 4 times 9 maketh 36, which I take from 36, and there remaineth nothing: ſo that the quotient of this diuiſion alſo (where 7656 is diuided by 29) is 264. Which doth declare, that if 264 bee multiplied by 29, the ſum will be 7656. And thus I perceiue now how both Multiplication is prooued by Diuiſion, and Diuiſion alſo by Multiplication.

Maſter. Now haue I ended the fiue common kindes of Arithmetik. For (as touching Mediation, Duplation, Triplation, and ſuch other) they are no ſeuerall kindes of Arithmetik, but are contained vnder the other. For Mediation is contained vnder Diuiſion, and is nothing elſe but dimiding by 2: and ſo are Duplation & Triplation cõtained vnder Multiplication. For Duplation is nothing elſe but multiplying by 2: & Triplation is multiplying by

by 3: of which I will onely propose an example for the rules you haue heard already.

If you would mediate or diuide into 2 this summe 4531010, you shall set two for the Diuisor, and work as you learned before, as thus:

4531010 (
2

An example of mediation.

Then I finde 2 in 4 two times: therefore my Quotient must be 2: so I cancell 4 and 2, and remoue the Diuisor forward thus, as ẏ work requireth, and as before in Diuision hath béene declared.

4531010 (2
22

Which mediation or diuision by 2 being finished, you shall haue for your Quotient 2265505, which is the halfe of 4531010, as you may trie by Duplation: for, double that Quotient, or multiply it by 2, and the same number will amount.

Duplicatiõ

I will no longer tarry about these, seeing they are but members of the other kinds. But héere now (according to my promise) I will teach you certain easy forms both of Multiplicatiõ & of Diuision. And first of Multiplication.

If you would therfore multiply any summe by 10, you shall need to doe no more but adde a cipher before his first place: as for example, 36 multiplied by 10 make 360.

Easy forms of Multiplication.

Likewise if you would multiply any summe by 100, put two ciphers at his beginning. So if you would multiply any summe by a thousand, adde thrée ciphers to the beginning of it.

Scholar.

Scholar. This doe I well perceiue, and also the reason of it.

Master. I will omit all reasons till our next meeting, when I shall tell you the reason of all other parts of Arithmetik also: and as to our matter now, looke, as I haue told you, that you both remember it, and also often practise it.

And now haue you learned how to multiply easily by 10, 100, 1000: and of like maner may you doe with any other of that sort.

But now if you will multiply by 20, 30, 40, and so foorth, or by 200, 300, and such like, where there is one cipher in the first place, or many orderly in the first places, you shall take away those ciphers, and multiply the summe only by the other figure, or figures (if they bee many) & then at the beginning of the summe that amounteth you shall set so many ciphers as you tooke away.

Example of 2873, which I would multiply by 300. First I omit the 2 ciphers from the multiplier, and I multiply the summe by onely thrée that is left, and it amounteth to 8619: before which I put the two ciphers that I before omitted or tooke away, and then is it 861900. And that is the sum that amounteth when 2873 is multiplied by 300.

Scholar. And if there were two or more figures besides the ciphers, I must onely take away the ciphers, and multiplie by the other

figures

figures, as I learned before: as if I would multiplie 93648 by 25000, I should take away the three ciphers, and multiply the same by 25, and then at the beginning of that totall summe should I adde the three ciphers againe.

Master. Euen so: but and if it chance the number that should be multiplied, or both the summes, as well the number that should bee multiplied, as the multiplier, to haue ciphers in their first places, euermore omit the ciphers, and worke by the rest. But remember to restore as many ciphers to the amounting summe as you bated before, as in this example: 30200 shall bee multiplied by 206, I shall onely take away the two ciphers from the greater number, and then multiply 302 by 206, and afterward adde the two ciphers againe. But if I would multiply the same 30200 by 2060, I shall not onely take away the two ciphers from the number that should bee multiplied, but also I may take away the one cipher from the multiplier, and then must I adde 3 ciphers to the summe that amounteth: but take heed that you take away no cipher that commeth after any signifying figure, as in this last example, you may not take away that in the fourth place of the higher number, neither that in the third place of the multiplier: howbeit, yet this you may doe: If one cipher or more come in the midst

of

of your summes, you may multiplie by the o-ther figures, and ouerskip them: but so, that you giue euery figure his due place : as thus : I will multiply 3026 by 2004: therfore I set them thus:

```
     3026
     2004
    -----
    12104
6052
```

And thus I doe multiply them: First, foure times 6 make 24 : I set the 4 vnder the first place, and keepe the 2 still in my minde. Then say I againe, 4 times 2 maketh 8, and the 2 that is in my minde maketh 10 : I set downe the cipher 0, & keepe the article 1 in my minde: Then 4 times 0 is 0, and the one in my minde maketh 1 : I set downe the figure 1, and say againe, 4 times 3 is 12 : I set downe 2, and keeping the 1 still in my mind (hauing no more places of yͤ vpper number to multiply it withall) I put it downe next 2 in the fifth place.

But now when I come to the next place (being a cipher 0) I let it goe, because it multiplieth nothing: and likewise the second cipher.

But then, when I doe come to the 2, and multiply into the 6 of the ouer number, you must take heed (according as I taught you in Multiplication) that the first number amounting of the Multiplication bee set right vnder the Multiplier, and the other orderly toward the left hand, according as you may see in this example:

```
     3026
     2004
    -----
    12104
 6052
```

Which

which being finished with the addition thereof gathered together, will stand as this example sheweth.

Which is indeed wrought much sooner and shorter by ouerskipping of the 2 ciphers: which otherwise (if the same example were wrought at length) it would haue had 2 workings more, as by the same example heere also set downe doth appeare.

```
   3026
   2004
-------
  12104
 6052
-------
6064104
```

Scholar. Sir I thanke you: for I see great ease in this way of multiplication: and (if you can shew me such like in Diuision) you shall greatly further me.

Master. Yes, I will teach you some easie waies in Diuision also, and first this: If you would diuide any summe by 10, you shall only with your pen make a square line betweene the first figure of your summe and the second, and then haue you done: for the whole number that followeth the line standeth for the Quotient, and the figure that is before the line, is the remainer: as for example, 3648 diuided by 10, wil stand thus:

364 (8

Where 364 is the Quotient, and betokeneth that so many times are 10 in 3648, and the 8 after the line is the remainer, which cannot be diuided into 10, but by breaking it into

If I diuide 4 by 365 what wilbe the que

into fractions, wherewith I will not meddle yet.

And so likewise if you would diuide any sum by 100 with your pen, you shall cut away the two first figures, and if you would diuide by 1000, you must cut away the 3 first figures, & so of any other diuisor, whose last figure is 1, and the other ciphers, looke how many ciphers the diuisor hath, and so many figures at the beginning shall you cut away with the square line, and they stand alwaies for the remainer, because they are lesse then the diuisor, and cannot be diuided by it, and the other figures that are behinde the line stand for the Quotient.

But now if your diuisor haue any other figure in his last place then 1, and in all his other places haue ciphers, looke how many ciphers they be, cut away so many of the first figures of the number that should be diuided, and diuide the rest that followeth the line by that figure that is in the last place, as if it were the whole diuisor.

Example of 64284, which I would diuide by 300: heere must I cut away the two first figures (for so many ciphers my diuisor hath) and must diuide the rest by 3, which is the figure in the last place of the diuisor: First therefore I part away the two first figures, and the sum standeth thus:

642 [48

Then doe I diuide 642 by 3, and the Quotient

tient is 214: for in 6 I finde twice 3, and in 4 once, and 1 remaining, which 1 with the 2 next before doth make 12: wherein I finde 3 foure times: and this is a ready way to turne shillings into pounds: for sith one pound doth containe 20 shillings, I must diuide the whole number of shillings by 20. Therfore easily to doe it, I see that my diuisor hath one cipher, and therefore I cut away one figure from the beginning of the whole summe of shillings, and then I do mediate or diuide by 2 the other figures or summe that followeth.

Scholar. I will put an example.

If you would diuide 64287 shillings by 20: that is to say, If I would turne so many shillings into pounds, I must cut away the first figure, that is 7, and diuide the rest, that is 6428 by 2, so shall the Quotient bee 3214, whereby I knowe that 64287 shillinges make 3214 pounds, and 7 shillings remaining.

Master. Now proue by Multiplication whether you haue well done or no.

Scholar. The quotient is 3214, which I doe multiply by the Diuisor 2, and it doth amount to 6428.

Master. Heereby you may perceiue not onlie that you haue well done, but also how by Diuision you may turne shillings easily into pounds: and contrariwise by multiplication you may turne pounds into shillings.

But

But héere shall you sée amongst diuers men diuers formes of such diuision: but if you mark what I haue told you, you shall perceiue easily all their waies. For some men doe not cut away so many of the first figures of the summe that they would diuide, as there are ciphers in the first places of their diuisor: but they set all their ciphers orderly vnder the first places of the number that they would diuide; and then with the other figure or figures (if there be many) they diuide the rest of their sum. Example. If they would diuide 725931 by 3400, they doe set their summes thus:

Another maner of ye abridgement.

725931
3400

And then doe they diuide orderly till they come to the ciphers: for there they stay and end their worke, as in this example. They séeke how often 3 may bee found in 7, which is two times, and one remaining: therefore they set 2 in the quotient, and cancell 3 and 7, and ouer 7 they set the 1 that remained, thus:

1
725931 (2
34 00

Then doe I goe forth, saying, Two times 4 maketh 8, which they take out of 12, and there remaineth 4, thus:

4
725931 (2
34 00

Then renew they the diuisor forward, and séeke how often 3 may bee found in 4, which is

is but once, and 1 remaineth, then set they 1 in the Quotient, and cancell 3 and 4, and ouer them they set that 1, thus.

1
14
725931 (21
34400
3

Then take they once 4 out of 15, and there resteth 11. Or else more easily: Take once 4 out of 5, and there resteth one, so they cancell the 4 and 5, and set one ouer them, thus.

1
141
725931 (21
34400
3

Then set they forth the Diuisor againe, and seeke how many times three, are in 11, which they finde three times, and 2 remaining: so they set 3 in the Quotient, and cancell 11 and 3, and ouer them set 2, thus.

12
1411
725931 (213
344400
33

Then doe they multiply 4 by 3, which maketh 12, that withdraw they out of 29, and there resteth 17, of which the 7 must bee set ouer the 9, and the 1 ouer the 2, thus.

1
12
14117
725931 (213
3444400
33

And now are the two ciphers next ensuing, so that ye Diuisor can no more bee set forward, and therefore is the diuision ended, and the

 Remainer

Remainer is 1731.

Now the Quotient, which is 213, doth declare, that if you diuide 725931, by 340, you shall finde it therein 213 times, & there remaineth 1731: so shall you finde it, if you worke as I taught you, by cutting away the 2 first figures, because of the 2 ciphers.

But this must you marke (as you may perceiue by this last example,) that if there be left any other Remainer in the summe that was behinde the squire line, that the Remainer must bee set to the latter end of the first Remainer, which was cut away wt the squire line: as if you would diuide 725931 by 3400, after the forme that I taught you, thē would your summes appeare thus.

```
   |1
   12|
  141|7
  7239|31 (213
  3444
   34
```

So that 17 which remaineth after the line, must be set to the 31 (that was cut away with ye line) in higher places, as you see here: where that 17 with the 31, do make 1731.

¶ And here will I make an end of Diuision, (sauing that I doe request you to exercise your selfe well herein by many summes, till you haue attained some expertnesse therein.) For the reasons and conclusions thereof are so many, and so auaileable for all sorts of men whatsoeuer, that if I would speake of the infinite

finite vses therof, I should rather lack wordes then matter. And therefore recommending it to your iudgement hereafter, vpon your further trauell into the Art, I will here end this Treatise, representing vnto you one example or simple question of Diuision and Multiplication in stead of many, which is this.

There are 4 brasse péeces: The first of them at a shot spendeth 9 pounds of powder, the second spendeth 5 pounds, the third 4 pounds, & the fourth 2 pounds. They are all appointed against the batterie of a hold, and there is allowed by the Maister gunner 700 pounds of powder to bee spent by these 4 péeces in this assault. The question is two fold: The first how many shot each péece shall iustly make about with this 700 pounds of powder? And lastly, how many pounds of powder ought iustly to be allowed to each péece for his true proportion?

Scholler. Why sir, you make me smile, to beare me in hand, that these two demaunds may bee simply resolued by Multiplication & Diuision.

Maister. Truly that they may: and that may you by and by worke your selfe with a little labour: First adde together their quantities of powder, that is 9 pounds, 5 pounds, 4 pounds, and 2 pounds, al which make 20: Diuide the 700 pounds of powder by that 20, & your Quotient giueth 35, as here appeareth,

which sheweth for most certaintie that they shal make iust 35 shottes about.

$\cancel{1}$
$\cancel{7}\cancel{0}\cancel{0}$
$\cancel{2}\cancel{0}\cancel{0}$ (35
$\cancel{2}$

Scholler. Sir, all this haue I done, and I see it is so; but whether it be true or not, I cannot tell.

Maister. To trie the truth of the same, multiply the first peece that spends 9 pounds by 35, and you shall see his allowance, which is 315 pounds of pouder. Multiply also the second peece that spends 5 pounds by 35, and you shall find 175 pounds his allowance: then 4 by 35, and you shall find 140 pounds his allowance. Lastly multiply 2 by 35, and you shall find 70 pounds his allowance. All which foure particular summes you shal adde together by Addition, as here appeareth, and it maketh iust 700 pounds, and so is the question truly absolued.

315
175
140
70
———
700

Scholler. Truly Sir, these excellent conclusions doe wonderfully more and more make me in loue with the Art.

Maister. It is an Art that the further you trauell, the more you thirst to go on forward. Such a fountain that the more you draw the more it springs. And to speake absolutely in one word (excepting the study of Diuinitie, which is the saluation of our soules) there is

no

no study in the world comparable to this for delight in wonderfull and godly exercise: For the skill hereof is well knowne immediately to haue flowed from the wisedome of God into the heart of man, whom hee hath created the chiefe image & instrument of his praise & glory.

Scholler. The desire of knowledge doth greatly encourage me to be studious herein: and therfore I pray you ceasse not to instruct me further into the vse hereof.

Maister. With a good will. And now therefore for the further vse of these two later, that is, Multiplication and Diuision, I will briefly shew you the feate of Reduction.

Reduction.

REduction is, by which all summes of grosse denomination may bée turned into summes of more subtile denomination. And contrariwise, all summes of subtile denomination, may bee brought to summes of grosser denomination.

Scholler. What call you grosse denomination and subtile denomination?

Maister. That I call a grosse denomination

which doth containe vnder it many other subtiller or smaller: As a pound (in respect to shillings) is a grosse denomination; for it is greater then shillings, and containeth many of them. And shillings (in comparison to pou͂ds) are a subtile denomination, for because they are lesser then pounds, and many of them are contained in one of the other: and so likewise of other things, whatsoeuer thing is compared to other, if it bee a greater, and containeth many of them, it is a grosser denomination: but if it be lesser (so that many of them are in the other (then are they called subtile denominations: whereby you may perceiue that one denomination may bee called a grosse denomination, and also a subtile (that is to say, a great and a small) in diuers comparisons. For shillings compared to pounds, are a subtile or small denomination: but compared to pence, they are a grosse or great denomination.

Scholler. Now I vnderstand the name, I pray you teach me the vse.

Maister. The vse is easily learned, if you remember what you haue learned before. For if you will reduce any summe of a grosse denomination into a summe of a smaller or subtiller denomination, you must consider how many of that subtiller denomination do make one of the grosser denomination, and by that number or Numerator doe you multiply the other sum: as if you would reduce 20 pounds into

into shillings, you must consider that in a pound are included 20 shillings, therfore multiply the one 20 by the other 20, and there will amount 400, whereby you may know ỹ in 20 pounds are contained 400 shillings. Likewise, if you would reduce 30 shillings into pence, considering that in a shilling are 12 pence, you must multiply 30 by 12, and it will bee 360: whereby you finde, that in 30 shillings are contained 360 pence. And thus may you reduce any grosse denomination into a more subtiller, By multiplication, if you know how many of the lesser doe make the greater: of which thing I will anon giue you a briefe table for the most accustomed kindes of money, weights, measures, and time, and such like: whereby you may know how often each subtile denomination is contained in ỹ grosser, when you shall neede it for the foresaid kinde of Reduction. And also the same shall serue you, if you would reduce any sum of a subtiller denomination, into a summe of a grosser denominatiõ. For in such Reductiõ you must consider (as in the other forme) how many of the smaller do make the greater, and by that number must you diuide ỹ other sum, and the Quotient will declare how many of the greater denomination are comprehended in that summe, as for example. If you would know how many shillings are contained in 3240 pence, consider that 12 pence doe make

1 shilling: you must diuide that 3240 by 12, and your Quotient will be 270, whereby you knowe that so many shillings are in 3240 pence. But, and if you would know further, how many pounds are in those 270 shillings, séeing that euery pound containeth 20 shillings, diuide that 270 by 20, and it will be 13, and 10 remaining, whereby you may know that in 3240 pence, (or 270 shillings) are 13 pounds and 10 shillings. For euermore the remainer must bee named by the name or denomination of that sum that was diuided, which in this place were shillings. And thus may you doe with any other kinds of denominations.

¶ Wherefore to the intent you may haue certaine light or knowledge in most common coines weights, & measures (which is ẏ most chiefe and principallest thing in trafficke to be known) I haue in each reductiō as they come in order, set down certain instructions incidēt therunto. And first I haue herunto added this Table, wherin is comprehended not only our currant and common coines, but also the most part of the vsuall coins of Christendome, with their iust weights and value currant in this Realm of England, intēding at the latter end of my Addition to this booke, to write of ẏ ordinary money vsed in diuers places, and their common values currant for trafficke, with the maner of their exchāges frō place to place, &c.

A Table of the names and valuation of the most vsuall Gold-coins throughout Christendome, with their seuerall weight of *Pence and Graines: and what they* are worth of currant money English.

The names and titles of the Gold	The weight Pence.	Grains	The value. Shil.	pence
Royall.	4	23	15	0
Halfe Royall.	2	11	7	6
Old Noble.	4	6	13	4
Halfe old Noble.	2	4	6	8
Angell.	3	8	10	0
Halfe Angell.	1	16	5	0
Salute.	2	5	6	4
2 parts of Salute.	1	11	4	2
George Noble.	3	0	9	0
Half Georg Noble	1	12	4	6
First Crowne K.H.	2	9	6	4
Base Crowne K.H.	2	0	5	0
Great Souereigne.	10	0	30	0
Souere. K.H. best.	3	14	10	8
Souere. K.H.	4	0	10	0
Edward Souer.	3	14	10	0
Elizabeth Souer.	3	14	10	0

The names and titles of the Gold.	The weight Pence.	Grains.	The value. Shil.	Pence.
Elizab. Crowne.	1	19	5	0
Vnicorne of Scot.	2	10	6	0
Scottish crowne.	2	5	6	0
French Noble.	4	16	13	4
All sorts of French crownes.	2	5	6	0
Old French crown	2	5	6	0
Flaunders Rider	2	6	6	6
Gelders Rider.	2	2	3	6
Phillips Royal.	3	10	10	0
Phillips crowne	2	5	5	0
Collen Gilden.	2	2	4	8
New Andr. Gild.	2	3	5	0
Flaunders noble.	4	10	12	0
Halfe Flaun. Noble	2	6	5	0
Flem. Angel best.	3	6	9	0
Flād Royal orkey.	3	10	10	0
Carolus Gilden.	1	21	3	6
Flaunders Royal.	2	6	5	0
Saxon Gilden.	2	2	4	8
Flaunders crowne.	2	5	6	0
Philips Gilden.	2	3	4	2
Halfe Phil. Gilden.	1	1	2	1

The names and titles of the Gold	The weight Pence.	Grains	The value. Shil.	pence
Golden Lyon.	2	16	7	8
3 parts of gold. Li.	0	21	2	5
$\frac{2}{3}$ parts of gold. Li.	1	19	4	11
Dauids Gilden.	2	2	4	0
Horne Gilden.	1	12	4	11
Old Andre. Gild.	2	3	4	10
Crusa. long crosse.	2	6	6	0
Crusa. short crosse.	2	6	6	2
Mil rayes.	4	20	13	4
Halfe Mil rayes.	2	10	6	8
Portigue 1 ounce.	2	16	68	0
Golden Castilio.	2	23	8	10
Ducket of Aragon.	2	6	6	6
Hungarie Ducket.	2	7	6	4
Double Pistolat.	4	8	11	8
Single Pistolat.	2	4	5	10
Ducket of Floren.	2	5	6	4
Double Ducket.	4	11	13	0
Single Ducket.	2	6	6	6
dou. duc. of Rome.	4	13	21	8

Of Siluer Coines currant in this Realme.

The Edward Crowne of 5 s.
The Edward halfe Crowne of 2 s 6 d.
The Edward Shilling, halfe shilling, and the
three pence.
Phillip and Maries Shilling, & halfe Shilling.
The Mary Groat, and Mary two pence.
Queene Elizabeths Shilling, 6 d, 4 d, 3 d, 2 d,
1 d, three farthings, and halfe peny.

It is to be vnderstood (gentle Reader) that whereas the weight is called by the name of a peny, it is not meant a peny of siluer money, but a peny of Goldsmiths weights, which is 24 barly cornes drie. And 20 of those pence make an ounce: and 12 of those ounces make a pound Troy. So that if a man haue not the weight wherewith to weigh any peece that may come to his hand, he may doe it with the Barly graines or cornes being dry, and taken out of the middle of the eare.

Here would I now expresse the values of sundry other coynes of diuers countries, but for three causes I now refraine. The first and chiefest is, because they are not currant by the statutes of this Realme. An other cause is, by reason they are so vncertaine, that they be neuer long at one rate. And againe, they are so different in so many places, that it were mat-

matter inough for a great booke, to speake sufficiently of them all. Howbeit yet because you shall not be altogether ignorant of thẽ, I will shew you the values of some that are most in vse, and first of France.

The most common money are Deniers, Soulx, and Frankes. 12 Deniers make 1 s̄, 20 Soulx make one Franke: so that as you see these thrée kinds are like in the rate to pence, shillings, and pounds with vs, but that this is the difference, that their Denier is but the ninth part of our penie, and so their Soulx, (commonly called Sowses) go 9 to our shilling, and 9 of their Frankes to an English pound of money. So that 3 of their Frankes make a Noble. And by those thrée may you practise how to reduce Frẽch mony into English mony, according as I haue set forth here following.

2160 Deniers make 240 d, or 20 s̄.
3240 Deniers make 360 d, or 30 s̄.
8352 Deniers make 929 d, or 2 l, 17 s, 4 d.
2160 Soulx make 240 shillings. And so of other in like rate. As for the rest of their coines I omit them till hereafter that you haue some vnderstanding in brokẽ numbers.

But now as for the coynes of Flaunders, they bee so changeable, that you must know them from time time: else you cannot reduce them into our money certainly. But yet that you may haue an example of their money

money to exercise you withall, you shall take those that be most common: as Stiuers both single & double, Groates Flemmish, Carolus, and Gildens. A Flemmish Groate is a little aboue 3 Farthings English. A single Stiuer is 1 d. ob. *p.* halfe farthing. The double stiuer is 3 d farthing. The double stiuer Carolus is 4 d ob, halfe farthing. Then there is also the Carolus Gilden, which is worth 20 stiuers. And the Flemmish Noble is worth 3 Carolus Gildens and 12 Stiuers.

So that if you would conuert Flēmish money, or any other kinde of money whatsoeuer it be, iustly into sterling, you must reduce it first into the smallest part of English money, that is in that coine, as for example. If I would reduce 368 double stiuers into English money, (considering that a double stiuer conteineth 3 d farthing,) you shall first looke how many farthings be in the double stiuer, & you shall finde them 13: therfore multiply the sum of the stiuers by 13, & then haue you their value in farthings, which is 4784. Now, if you diuide that by 4, then there will appeare the number of pence: but better it were to diuide it by 48 (for so many farthings are in 1 shilling) and then will the Quotient declare the sum of the shillings.

Likewise if you would reduce any sum of single stiuers into English money, you must multiply the sum first by 13, and then haue you

you reduced them into a certaine summe, that is to wit, halfe farthings, which sum if you diuide by 8, then will amount the sum of pence: or if you diuide it by 96, the sum of shillings will appeare.

But marke this in all Diuision: when yée do reduce to bring one denomination into another, if there be any Remainer after the Diuision, that must bee named by the denomination of the grosse sum that was diuided: as for example. I would bring 254 farthings into pence, therefore I do diuide that 254 by 4 (for so many farthings make a peny) & the Quotient is 63, which is the sum of the pence, and thē remaineth yet 2, which are farthings still, as one may proue by diuiding. And this must be marked in all Diuision, namely, when it is done for Reduction.

¶ Touching Danske mony, they haue their Soulx, whereof 20 is a Liuer: which is two s sterling. They haue also their Grash, whereof 80 make a Gilden, which is foure s sterling. They haue also Dollors, and their common or old Dollor is 35 Grash. New Dollors they haue, which bee diuers, some valued at 24 Grash, some at 26, and some at 30. And thus much I thought good to adde to the Authour, touching Danske money.

Concerning Spanish money, whereof the most common are Cornados, Maruides, Ryals and Duckets: sixe Cornados make a

Marueide,

Marueide, 4 Marueides make 1 Riall, and 11 Rials make one Ducket, so the Ducket conteineth 374 Marueides, which is about 5 s, 10 d sterling. Therfore if you would conuert 124 l, 5 s sterling into Duckets, consider that pence is the least value or denomination named in this question: therefore reduce 124 l', 5 s into pence, and it maketh 29820 pence: which if you diuide by the pence that a Ducket is worth, (which is 70.) you shall haue for your Quotient 426 Duckets, your desire.

Thus much haue I sayd of money: Now will I shew you in like sort, the distinction of weights, after the Statutes of England, where the least portion of weight is commonly a Graine (meaning a Graine of Corne, or wheate, dry and gathered out of the middle of the eare.) Of these Grains in times passed, 32 weighed iust 1 peny of Troy, & then was but 20 pence in an ounce. But now are there 46 pence in an ounce, so that there are not fully 14 Graines in 1 penie. But now of Ounces after Troy rate (which is the standard of England,) 12 do make 1 pound.

But commonly there is vsed another weight called Haberdepoise, in which 16 Ounces make a pound. Therefore when you would reduce Ounces into pounds, you must consider whether your weight be Troy weight or Haberdepoise; and if it be Troy weight, you must

must diuide your Ounces by 12, to bring the to pounds, but if it be Haberdepoise, you must diuide them by 16. Now again, there be greater waights which are called an hundred, half a hundred, and a quarterne, and also a halfe quarterne, &c. A hũdred weight.

Scholar. Why? so there may be reckoned 20 pound, 40 pound, 200 pound, and such innumerable.

Master. All these are numbers of weight, but they haue not common weights made to their rate, as the other haue. And againe, these that I did name are not iust in number as they seeme by their name, for an hundred is not iust 100, but is 112 pound. And so the halfe hundred is 56: the quarter 28, and the halfe quarter 14. And these be the common weights vsed in most things that are sold by weight.

Howbeit there are in some things other names, as in wooll, 28 pound is not called a quarterne but a Todde: and 14 pound is not named halfe a quarterne but a Stone, and the 7 pound halfe a stone. Other names because they differ in many places, and agree in few, I let them passe. Wooll weights. Todde. Stone.

But a Sacke of Wooll by the statutes, is limited to be 26 stone. Sacke.

¶ Now in Cheese, though it be sold by the hundreth, and by the stone in some places, yet the verie weights of it are Cloues and Cheese-waights.

L Weyes

Weyes. So that a cloue containeth 8 pound: and a Wey 32 Cloues, which is 256 pound, that is, twelue score and sixteene pound: and so much weigheth the Wey of Suffolk cheese, and the like weight is or should be the barrell of Suffolke butter.

The Wey of Essex cheese containeth sixscore, and sixteene pound: and so much is also the Barrell of Essex butter.

Measures for liquor.

Now of weights are made other measures both for graine and liquor. For a pound in waight maketh a pint in measure, so that 8 pound or 8 pints do make a gallon: halfe a gallon is named a pottle: a halfe pottle is called a Quart, which containeth two pints. Now aboue a Gallon the next measure is a Firken: then a Tertian, a Kilderkin, or halfe Barrell: and a Barrell. And by these measures are sold commonly Ale, Beere, Wine, and Oyle, Butter and Sope: Solmon, Herrings, and Eeles.

A Pint. Gallon. Pottle. Quart.

Firken. Tertian. Kilderkin. Barrell.

Ale measures.

But as these be vnlike things, so the measures of their Ale are, as followeth.

Of Ale	the Firkin	conteineth	8	gallõs.
	ye Kilderkin		16	
	the Barrell		32	

Of Beere	the Firkin	conteineth	9	gallõs
	ye Kilderkin		18	
	the Barrell		36	

Sope measures, both Firken, Kilderkin, and Barrell should be equall to Ale measure. Moreouer the statutes doe limite the weight of euery of those thrée vessels being emptie. Sope measures.

A Barrell	to	26		
Halfe a Barrel	weigh	13	pounds	
A Firken	empty	6 ½		

Herrings also be sold by the same measures that Ale and Sope be sold by. Herring.

Herrings also are sold by the tale, 120 to the hundred, ten Thousand to the Last.

Salmon and Eeles haue a greater measure. Salmon & Eeles.

	the Butte		84	
Salmon	the Barrell	hol-	42	
& Eeles.	halfe Bar.	deth	21	gallons
	the Firken		10½	

Howbeit, some statutes did limite Eele vessels equall with Herring vessels.

Now as for wine vessels they are seldome smaller then Hogsheds, which are of 63 gallons: euery Hogshead, is two Barrels: yet there are many other wine vessels, but of thē ll sée this Table and marke the measures one y another. Wine measures.

Of wine and oyle.	the Rondlet	hol- deth	18	Gal- lons.
	the Barrell		31	
	the Hogshed		63	
	the Tertian		84	
	the Pipe		126	
	the Tunne		252	

Tertians. But you shal marke that there bee other kindes of Tertians: for there be Tertians (ỹ is to say) Thirdles of Pipes, of Hogsheds, & of Barrels, as well of other things as of wine.

Butte. Also Malmeseyes, and Sacke, &c. the half Tun is not called a Pipe, but rather a Butte.

And thus much haue I thought meet to tel you at this time.

Scholar. And is that alwaies true?

Master. I haue told you how it should be, but how it is, I may not say: how they do differ daily from their iust measure, that Gagiers can tell you better then I. But I wil let this passe now, & speake briefly of the other measure.

Dry measures. And as of weights there did spring the liquid measures, (whereof I spake last) so of the same springeth drie measures, as Pecks, Bushels, Quarters, and such like; whereby are measured corne and like grains, also salt, lime, coales, and other like. And this is the order and quantitie of them.

A pecke. A Pecke is the measure of two Gallons.

A

A Bushell containeth foure Pecks. A Bushell.
A Quarter holdeth eight bushels. Quarter.
A Wey containeth sixe quarters. Wey.

These are the common names and measures, but in diuers places there bee diuers sorts.

The Bushell in many places is two bushels: but then is that bushell there called a Strike, and in some places halfe a quarter is called a Cornoke. But these diuersities are too many to tell you briefly them all, & again, sith they are against the law and statutes, I count them vnmeete to be vsed. Strikes.

But now remaineth yet another kinde of measures, whereby men meate length and breadth, and thicknesse, & those are, an Inch, a Foot, and such other: whose names and quãtities this table sheweth. Measure to meate length, breadth, and thicknesse.

3 Grains of barley in length make an inch. An inch.
12 Inches make a foote. Foote.
3 Foote make a yard. Yard.
3 Foote and 9 inches make an Elle. Elle.
5 Yards and a halfe, make a perch. Perch.

1 Perch in breadth and 40 in length, doe make a rodde of Land, which some call a rood, some a yard land, and some a Farthendele.

2 Farthendels make half an acre of groũd.
4 Farthendels make an acre. Acre.

More, 40 rods in length do make a Furlong; 8 furlongs make a mile, which containeth 320 perches.

So that an English mile grounded upon the statute, is in length 1760 yards, 5280 foote and 63360 inches.

Somewhat greater than the Italian mile of 1000 paces and 5 foote to a pace.

Here might I tell you many things else touching measure, and also how to reduce strange measures to our measures, but because it can not be well done without the knowledge of Fractions, which as yet you haue not learned, I will let them passe till an other time that I haue taught you the knowledge of broken numbers.

The parts of time.

Scholar. But yet Sir of the parts of time, I pray you tell me somewhat.

A day. An houre. Weeke. Moneth. Yeare.

Master. You know that a naturall day hath 24 houres, and euerie houre hath 60 minutes. It needeth not to tell you, that 7 daies make a weeke, and 4 weekes make a common moneth, and 13 moneths make a yeare, lacking one day, and certaine houres, and minutes: But of that I shall instruct you hereafter.

Here will I make an end of Reduction for this time, which though it be counted no kind seueral of Arithmetick, yet you see it is no lesse needfull to be knowne, or easier to bee done, than any of the other.

Scholar. Mary Sir, it seemeth unto me much harder then any other sort, for it requireth the knowledge of so many things: but

now

now sir whē you sée time, I am ready to learn forth: for as much of Reduction, as you haue taught me, I remember; but and if I do at any time forget, I shall haue recourse to the tables which you set forth for me.

Master. So do you, for it will not bee remembred without exercise. But in as much as you vnderstand so much as wee haue intreated of, I wil now instruct you in Progression.

Progression.

ALthough vntil this day the most part of writers haue defined Progression as a compendious kinde of Addition, yet truely it is not so: for progression (as the verie nature of the word doth informe any man) is a going forward & procéeding in numbers, & that regularly and orderly, whose place is aptly chosen to be very néere, or rather next after the exposition of the 4 principal parts of Arithmeticke, for in it after a most easie maner, are all the 4 former parts exercised and practised: and not only Addition, as customably is done. Which custome hath bin the cause, why it hath so specially bene named a kinde of Addition, and defined to be a quick

and briefe Addition of diuers summes proceeding by some certaine and reasonable order.

You shall also understand, that there are infinite kinds of Progressions, but for you (as yet) two are sufficient to be exercised in: of which the one I call Arithmeticall, and the other Geometricall.

Arithmeticall Progression.

Arithmeticall Progression is a rehearsing or placing downe of many numbers, number after number, in such sort that betweene euerie two next numbers rehearsed or placed downe, the difference, diuersitie, or excesse, be equall and alike.

Scholar. Sir, I thanke you for that you haue both opened unto me what progression is, truly, and also why it is here placed. But I pray you with an example make plain your definition.

Master. Examples cannot want, seeing all reasonable creatures naturally use the order of one kinde of Arithmeticall Progression, (which therefore is also named Naturall) whensoeuer they distinctly do count or number any multitude by one, saying: 1, 2, 3, 4, 5, 6, wherby the proceeding from number to number, and every one surmounting and exceeding his fellow next before by a like quantitie, (which here is 1) declareth the same to be Arithmeticall Progression. And for the more plainnesse, I set it downe in this manner.

The

The common excesse, 1 1 1 1 1

The Progression. 1 2 3 4 5 6

Scholar. This is most euident. And I thinke that I am able to tell you now of any Progression Arithmeticall propounded, what is that common excesse or difference whereby it procœdeth, if this order be kept in it.

Master. What say you of 3, 6, 9, 12, 15?

Scholar. They excœde each other by 3. And that may I set downe in such euident order, as you did your example of naturall Progression in this wise.

The common excesse. 3 3 3 3

3 6 9 12 15

Master. And do you not also now perceiue, that the whole Table of Multiplication may be made by the order of Progression Arithmeticall: either if you will begin at the first number of any of them on the left hand, and so procéede right ouerthwart: or at any of the first numbers of the vpper row, and go directly downeward.

Scholar. I pray you let mee consider the thing a little, and I wil answere you.

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The

The common excesse,		1	1	1	1	1
The Progression.	1	2	3	4	5	6

Scholar. This is most euident. And I thinke that I am able to tell you now of any Progression Arithmeticall propounded, what is that common excesse or difference whereby it procædeth, if this order be kept in it.

Master. What say you of 3, 6, 9, 12, 15?

Scholar. They excæde each other by 3. And that may I set downe in such euident order, as you did your example of naturall Progression in this wise.

The common excesse.		3	3	3	3	
	3	6	9	12	15	

Master. And do you not also now perceiue, that the whole Table of Multiplication may be made by the order of Progression Arithmeticall? either if you will begin at the first number of any of them on the left hand, and so procede right ouerthwart: or at any of the first numbers of the vpper row, and go directly downeward.

Scholar. I pray you let mee consider the thing a little, and I wil answere you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceiue it now very well for the common excesse or difference betwæn anie two next, is continually as much as the [illegible]t number of euerie row, either from the left [illegible]d ouerthwart taken, or from any of the vppermost ouerthwart rowes downeward.

To know the totall summe of an Arithmeticall progressiō.

Master. Now then if of any such Progression you would spéedily know the totall sum, much quicklier then by common rules of Addition: first tell how manie numbers there are (which numbers here wee call places or parcels) and if they be odde, write their sum downe by it selfe, as in this example, 2, 4, 6, 8, 10, 12, 14, where the numbers are 7 as you may sée, therfore set down 7 in a place alone, then adde together the first number and the last, as in this example; adde 2 to 14, and that maketh

maketh 16, take halfe of it, & multiply by the 7 which you noted for the number of the same places, and the summe that amounteth, is the summe of all those figures added together, as in this example: 8 multiplied by 7 maketh 56, and that is the summe of all those figures.

Scholar. That will I worke by another example. I would know how much this sum is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tel the places and there are 9, that I note. Then I put the first number 5, and the last 29 together, & they make 34, I take the halfe of it, that is, 17, and multiply by 9, and it maketh 153. That you say is the summe of all the numbers.

Master. So shall you find it if you trie it.

Scholar. How shall I trie it?

Master. By your common addition, fo[illegible] you adde all the parcels together, you shall [illegible] the same summe amount, if you did worke well. And that manner of Addition trieth all kinds of summing any Progression.

Scholar. Then can I summe a Progression, if the numbers of the parcels be odde. But what if they be euen: as in this example, 1, 2, 3, 4, 5, 6, 7, 8?

Master. When the number of the parcels is euen, then note that also as you did before, and likewise adde the first summe to the last, and by the halfe of the number of the places so you multiply it: as in your example, the parcels

parcels are 8, that I note: then adding the first summe to the last, there amounteth 9, that doe I multiply by the halfe of parcels, that is, by 4, and maketh 39, which is the summe of the parcels.

But if you wil take one rule for these both, doe thus, multiply the halfe of the one, by the other whole, and the summe wil amount all one. For sometime it chanceth that the number of the parcels be odde, so that their halfe cannot bee taken: and sometime it chanceth the Addition of the first number and the last, to bring forth an odde number, so that the halfe of it cannot be taken: but they will never be both odde.

A generall rule.

Scholar. Then I perceiue this, if there bee no more belonging to it.

Master. This is enough for Progression Arithmeticall: howbeit there is another maner of Progression called Geometricall, when the numbers increase by a like proportion, that is, if the second number containeth the first, 2, 3, or 4 times, and so forth: then the third containeth the second so many times also: and so the fourth the third, and the fift, the fourth, wherefore I set these three examples.

Progression Geometricall.

3, 6, 12, 24, 48,
1, 3, 9, 27, 81,
2, 10, 50, 750.

Here in the first example you see, that euery number containeth the other (that goeth next

next before him)two times: and in the ſecond example three times, and in the third example fiue times. Now if you wil know how to find eaſily the ſum of any ſuch numbers, doe thus: Conſider by what number they be multiplyed, whether by 2, 3, 4, 5, or any other, and by the ſame number doe you multiply ẏ laſt ſum in the Progreſſion.

Scholar. I pray you worke it by this example, 2, 8, 32, 128, 512, 2048, which I haue framed by proceeding from 2, and continually multiply by 4.

Maſter. Then muſt I multiplie the laſt ſumme (which is 2048) by 4 alſo, and it will be 8192. Now muſt I abate from this ſumme the firſt number of Progreſſion, which here is 2, then reſteth 8190, which ſumme I muſt diuide by 1 leſſe then was the number that I multiplied by. Seeing then I multiplied by 4, I muſt diuide by 3, ſo diuiding 8190 by 3, the Quotient wil bee 2730, which is the ſumme of al the Progreſſion. And now to proue whether you can doe the ſame, I giue you theſe numbers to adde by this rule, 3, 15, 75, 375, 1875, 9375, 46875.

Scholar. I cannot well tel by what number this Progreſſion doth increaſe.

Maſter. In any ſuch doubt do thus: Diuide the ſecond number by the firſt, and the Quotient wil ſhew you the number that engendreth

gendreth the Progression.

Scholar. Then is that number in this exãple 5, for so many times is 3 in 15.

Master. So is it. Now work as I taught you.

Scholar. The last number is 46875, which I multiply by 5, and it yeeldeth 234375, from which I abate the first number of the Progression, that is 3, and there resteth 234372, which I diuide by 4, for that is one lesse than 5, and the quotient is 58593, which is the whole summe of the Progression.

Master. If you remember well this, you haue learned the Art of Progression both Arithmeticall, and also Geometricall, which you may prooue either by subtracting of each number alone from the summe, and so will there nothing remaine: or else by adding togither of all the parcels, for so will the same summe amount.

¶ And now for the vse and better vnderstanding of both these rules, I will propone vnto you certaine pleasant and necessarie questions Arithmeticall and Geometricall, and to the performance of their workings such necessarie rules and documents, as are requisite for the vnderstanding of them, or any such like.

A certaine mercer sold 20 yards of Veluet to bee paid in 12 weekes, by Arithmeticall proportion: that is to wit, to receiue the first

weeke

wéeke 6 shillings, the second wéeke 12 shil-
lings, the third wéeke 18 shillings, & so forth,
encreasing the nũber of wéekes by 6 shillings,
till the twelfth and last wéeke were expired.
The question is how many pounds hée had
for 20 yards of veluet.

To the performance of this question and
such other the like, I set forth the 12 paiments 6
in such sort, as for example here appeareth in 12
the margent. 18

Then touching the adding together of these 24
summes without the aide of Addition accor- 30
ding to the rules I taught you in Progression 36
Arithmeticall, I note the number of the pla- 42
ces, which are 12, then adding the last num- 48
ber of the progression, which is 72, and the 54
first number together make 78, and multi- 60
plying 78 by halfe the number of the places, 66
which is 6, amounteth to 468 shillings, and 72
in pounds maketh 23 l, 8 s. And so much hath
the Mercer for his 20 yards of veluet, which
is nie about 23 s, 5 d, a yard.

Scholar. I understand this worke very
well: but is there any proofe for the iustifying
thereof as you haue of other workes?

Master. The worke of it selfe (being so per-
fectly wrought) that in your procéeding and
going forward from number to number, each
number excéeding his fellow by an equall or
like quantitie, is all that is demanded for iu-
stifying of the same: yet notwithstanding be-
cause

cause your request is reasonable, I will propone an example for the proofe hereof.

A certaine man is bound to pay for 20 yards of veluet, the summe of 23 l, 8 s, and it is to be paid weekely in 12 weeks or termes, by Arithmeticall Progression. The question is therefore to know with what number the same progression is to be begun and continued in such equall proportion Arithmeticall, that in 12 weekes the same may iustly be accomplished.

The proof of the last question.

For the solution whereof, and of all such other like, reduce 23 l, 8 s, all into shillings, which maketh 468 s.

A generall rule.

Then adde 1 unto 12 the number of the termes, it maketh 13, which 13 you shall multiply by halfe the number of the termes, which is 6, it maketh 78, then diuide 468 by 78, and you shall find 6 in the quotient which is the true number that shall begin and continue the said Progression. That is to say, the first weeke 6 s, the second 12 s, ẏ third weeke 6 s more, which is 18 s, and so euery weeke as they rise, 6 s more than the weeke before, as is manifest in the question aforesaid.

A Farme is to bee sold to bee paid by the weekes in a yeare, the first weeke to pay 4 s, the second weeke 8 s, the third weeke 12 s, and so forth increasing each number by 4, till the number of 52, which are the number of weekes in a yeare, be expired. The question is

what

what the price of the farme commeth to.

Scholar. I doubt not, but by that you haue already taught me, to end this question very wel, wherefore I set forth the Progression with his excesse 52 times.

Master. Nay stay a while. And heere for your further ease, (to abridge you of great labour that appeareth to fall out in this question, and so may doe in any other the like, if a question were proponed of 100 or 200 places or more, and that this question nor any other the like can be ended, vnlesse you know absolutely what the last number of the Progression at the 52 place is or ought to bee) I wil giue you a general rule how to know the last number of any Progression Arithmetical, aswel as if you had ordinarily procéeded by continual addition, til you had come to the last worke, which is this. A generall rule.

Multiply the excesse by a number lesse by one than the number of the places, and thereto put the first number of the Progression, and you shal haue your desire.

Scholar. This rule is wel worth the noting: for if I vnderstand you aright, I consider that my excesse is 4, which I multiply by 51, which is one lesse than the number of the places, and it maketh 204: whereunto I adde the first number of the Progression which is 4, and then it is 208: which you say is or should be the last number of the Progression.

Master. This is a most approued truth if there were neuer so many places.

Scholar. This rule is so easie, that I were much to blame, if I do not remember it. For by the benefit hereof I haue such an ease and light into this excellent Art, that my first entrance doth seeme to passe a great many mens further study and longer continuance.

Master. Many mo considerations could I propound you in these Arithmeticall Progressions: but these are sufficient for a tast, to giue you occasion to think that rules of knowledge and Artes are infinite capable of enlargement.

Scholar. Happy were I, if I did but well understand that which is already inuented & written. But these things, in my simple fantasie, offer themselues to be greatly beneficiall unto the ayde of Progression. Therefore now I wil go forward with your question.

Now considering that the 52 and last place is 208, I adde thereunto the first number of the Progression, which is 4, it maketh 212, which I multiply by halfe the number of the places, which is 26, and it amounteth to 5512 shillings. And so much is the totall summe or addition of this progression: which maketh 275 l, 12 s, as appeareth here by my tables.

Master. I like well your labour and commend you for your diligence. I will here propone one example more, and therewithall fo[...]

thi[...]

this time will end progression Arithmetical.

A certaine man bought 20 elles of Holland to be paid in 17 weekes or termes by progression Arithmetical. And the first weeke to pay 1 s, 8 d, the second weeke 3 s, 4 d, the third weeke 5 s, the fourth week 6 s 8 d, and so forth each weeke succeeding 20 pence more than the week before. The question is, what the sum of his 20 elles commeth to.

Scholar. Because here is mention made both of shillings and pence, I feare there is some harder matter contained herein, than in the other before: therefore I pray you work it your selfe, and I wil diligently marke your labour.

Master. There is no more to bee done in this, then in the other before, but because your request is so reasonable, bee attentiue vnto me.

First by the generall rules I seeke to finde out the last number of the 17 place what this progression ought to bee. Therefore here in my Tables multiplying the excesse 20 by 16, which is one lesse thē the number of the terms or places, and it commeth to bee 320, and thereunto adding the first number of the progression which is 20 pence, all is 340 pence, or 28 s, 4 d: for so much ought the last number of the payments to be.

Then finally to know what the whole 17 places amount vnto, I adde the first number

of the Progression and the last together, both which make 360. Now because 17 is an odde number, whose halfe cannot be taken, I take the halfe of 360, which is 180: and multiplying 180 by 17, commeth to 3060 pence, which maketh as you see by Diuision 12 l 15 s. And so much is the buyer to pay for his 20 elles of Holland. Which 3060 pence if you diuide by 20, the number of elles that was bought, you shal finde 12 s, 9 d, and so much paid he for an elle one with another.

The Proofe.

A certaine man doth owe 12 l, 15 s, to bee paid in 17 weekes or termes by Arithmetical Progression. The question is, to know with what number he shal begin and continue the Progression in such equal proportion, as the same may bee truely paid and satisfied in 17 weekes.

The Answere.

First I reduce 12 l 15 s al into pence: which as you see here in my Tables make 3060 pence, that I let stand by a while.

Then I adde 1 to 17, the number of the places or termes, which maketh 18, which I should multiply by halfe the number of the weekes or termes, which is 8 $\frac{1}{2}$, which $\frac{1}{2}$ multiplied

tiplied by 18 cannot wel be done unlesse you were acquainted with Fractions or broken numbers, therefore you shal let that passe and multiply 17 by the halfe of 18: which is 9, (for that is al one with the multiplication of $8\frac{1}{2}$: and the multiplication of 9 into 17 maketh as you see 153, with which number you shal diuide the 3060 pence beforesaid: and the quotient bringeth forth 20 pence, which is the first number or payment to begin the progression with al: and so each weeke succeeding to rise 20 d more then the weeke before, and thereby in 17 weekes shal 12 ℓ 15 s, be paid: as before was sufficiently declared. Thus much for progression Arithmetical.

Scholar. Certainly sir, I know not how to render you condigne thanks for these benefits shewed me, which me thinketh are so easie, delightful and pleasant, that I count my selfe happie to be in your company.

Master. I am glad you delight so wel herein, which is an art of wonderful dexteritie to al sorts of men of what degree or profession soeuer they be. And now wil I propone a question or two of Progression Geometrical.

A Mercer hath 12 yards of satten which he valueth at 16 s the yard, and selleth the same 12 yards to another man to be paid, as followeth. That is to wit, for the first yard to haue one shilling, for the second yard 2 s:

for the third yard 4 s, for the fourth yard 8 s and so forth doubling each number following, til the twelfth and last yard. The question is, who hath made the better bargain of the buyer or the seller.

First you may set downe 12 the number of the yards, as you se here in this example. And against each number the number of shillings due to be paid, as the order of Duplation or the Multiplication by two teacheth.

Then resorting to the adding vpp or summing of this progressiõ, where I consider that the increase of this sum proceeded by the multiplication of 2, and therefore after I haue drawne a line vnder the 12, I worke and multiply the last summe by two also, and it yeeldeth 4096: from whence I abate the first number of the progression which is one, and then resteth 4095: which I should diuide by one lesse than I did multiply by, but seeing it is one, I neede not to diuide it: for 1 (as I haue said before) doth neither multiply nor diuide, therfore I take that summe 4095 for the whole summe of the shillings, which by reduction amount

s	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12

amounteth to 204 l, 15 s, and so much hath the Mercer for his 12 yards of satten: which is 17 l, 1 s, 3 d, 1 yard. But I thinke you wil buy none so deare.

Scholar. No sir by the grace of God this yeare.

Master. Then what say you to this question? If I sold unto you a horse having 4 shoes, and in every shooe 6 nailes, with this condition, that you shall pay for the first nayle 1 ob: for the second nayle 2 ob: for the third naile 4 ob: and so forth, doubling untill the end of all the nayles. Now I aske you how much would the price of the horse come unto.

Scholar. First to know the number of the nayles, I must multiply 6 by 4, and it maketh 24. Then wil I doe thus: I wil write the number of the nayles everie one in order from 1 to 24, and against each number of the nailes the summe of halfe pence duely, as the order of Duplation or Multiplication by two teacheth, and as in this next figure following appeareth.

Then do I resort to the rule of summing by this Progression, where I consider that ye increase of this summe proceedeth by the multiplication of two, as the last example did. And therefore multiplying the last summe by two also, and it yeeldeth 16777216, from which I abate the first number which is 1, and then resteth 16777215, which I should

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24

I diuide by one lesse thã I did multiply: but seeing yͤ it is 1, I need not to diuide it, for 1 (as you haue before said) doth neither multiply nor diuide, therefore I do take yͤ number 16777215 for the whole summe of the halfe pence, which by Reduction I find to be 699050 shillings and 7 d, halfe peny: that is 34952 pound 10 s, 7 d ob.

Master. That is wel done, but I think you wil buy no horse of the price.

Scholar. No sir, if I bee wise.

Master. Wel then answere mee to this question.

A Lord deliuered to a bricklayer a certaine number of loads of bricke, whereof hee willed him to make twelue walles, of such sort, that the first wall should receiue two thirdels of the whole number: & the second two thirdels of that which was left: and so euery other 2 thirdels of that that remained: and so did the

Brick-

Bricklayer: and when the 12 walles were made, there remaineth 1 load of bricke.

Now I aske you, how many load went to the wall, and how many loade was in the whole?

Scholar. Why sir, it is vnpossible for me to tel.

Master. Nay it is verie easie if you marke it wel. Marke wel that I said, that euerie wal should receiue 2 thirdels of the summe that was left. Now take away two thirdels from any summe, and you must needs grant that that which remaineth is 1 thirdle of the summe last before: example of 9, from which if you take 2 thirdels, there wil remaine three, which is one thirdle of 9. Likewise from three take 2 thirdels, and there wil remaine 1.

Scholar. This is true, and now I perceiue, that the least wall had but two load of bricke.

Master. And by the same reason may you know how many loade euery wall had, according as this figure following doth shew, and likewise what the whole summe of bricks was: for if you make 12 summes, multiplying by 3, stil from the last remainer, as you may see here on the left side of the Table, there wil appeare al the remainers of the whole wall: and if you multiply the last of those 12 summes by 2 also, then wil that bee the sum of the loades which was deliuered to the

Bricklayer

bricklayer.

Againe, if you doe double every remainer, as you may see at the right side of this Table, those numbers wil shew the sum of the loads that went to each wall, whereby you may perceive, that each wall was three times so great as the next lesser.

Scholar. Lo, now it appeareth easie enough. Now surely I see that Arithmeticke is a right excellent Art.

Master. You wil say so when you know more of the use of it: For this is nothing in comparison to other points that may bee wrought by it.

Scholar. Then I beseech you, cease not to instruct mee further in this wonderfull cunning.

The Remainer after euery wall.			Loads due to each wall.
	1	12	2
	3	11	6
	9	10	18
	27	9	54
	81	8	162
	243	7	486
	729	6	1458
	2187	5	4374
	6561	4	13122
	19683	3	39366
	59049	2	118098
	177147	1	354394

Summe of the 531441 loads deliuered.

Th

The Golden Rule.

Master.

By order of the science (as men haue taught it,)there should follow next the extraction of rootes of number, which because it is somwhat hard for you yet, I will let it passe for a while, and wil each you the feate of the Rule of Proporti-ns, which for his excellencie is called the olden Rule: Whose vse is, by three numers knowne to find out any other vnknown, hich you desire to know, as thus. If you pay r your boord for three moneths sixteene shilngs, how much shall you pay for eight moeths?

To know this and all such like questions, u shall consider which two of your numers be of one denomination, & set those two e one ouer the other, so that the vndermost e it that the question is asked of: as in my estion, 3 and 8 be both of one denomination; r they both be moneths: and because 8 is the mber that the question is asked of, I set em one ouer the other, and 8 dermost, thus, with such a oked draught of lines, Then doe I set the

3
8

other

other number which is 16, against 3 at the right ſide of the line, thus.

3	16
8	

And now to know my queſtion, this muſt I do: I muſt multiply the lowermoſt on the left ſide, by that on the right ſide, and the ſumme that amounteth, I muſt diuide by the higheſt on the left ſide. Or in plainer words, thus, I ſhall multiply the number, of which the queſtion is aſked (which is called the third number) by the number of another denomination (which is called the ſecond,) and the ſumme that amounteth, muſt I diuide by the ſumme of like denomination, which is called the firſt. Then for the knowledge of this queſtion I multiply 8 into 16, and there amounteth 128, which I diuide by 3, and it yeeldeth 42 s, and 2 s remaineth, which I turne into pence, and they be 24 pence, of which the third part is 8 pence, ſo the third part of 128 s, is 4 s, 8 d, which ſum I write at the right hand of the figure againſt 8 thus.

3	16 s.
8	42 s. 8

Hereby I know that if three moneths boording do come to 16 s, that 8 months boordin wil come to 42 s, 8 d: and likewiſe of any other like queſtion.

But here muſt you marke, that the fi number and the third be of one denomina on, and alſo the ſecond and the fourth, t

whi

which you seeke: or else be of such denominati-
ons, that you in working may bring them in-
to one. As if a man should aske me this que-
stion.

¶ Twelue weekes iourneying cost me 14 French crownes at 6 s the peece, how many pounds is that in one yeare? Heere you see no two numbers of one denomination, but yet in working you may turne them into like denomination: as thus, turne the one yeere into 52 weekes, and the fourth summe will bee French Crownes, by the order of the working: Then to know this question, multiply the third summe 52, by the second 14, and the summe wil be 728: that diuide by your first number 12, and the quotient will bee 60 Crownes, and 8 Crownes remaining: which if you turne into shillings, they wil bee 48 s; which if you diuide by your first number 12, the quotient will be 4, which signifieth 4 s: put those 60 French Crownes, which make pounds with the 4 shillings, for the sum that answereth to the question: and it is the ...t expences of a ...re: And the sum ...l be thus.

12 — 14
52 — 60 — 4 s.

And take this euermore for a generall rule touching this whole Arte, that the doubtfull vnknowne number that you would be resolued of, shall alwaies be set in the third place. Note also the first number and the third must

muſt euer be of one nature and denomination or elſe muſt in working be brought to like denomination, and then of neceſſity muſt the other number be in the ſecond place.

Remember alſo, that the place of the firſt number is the higheſt on the left ſide: and the place of the ſecond right againſt it on the right ſide: the place of the third number is vnder the firſt, as by thoſe examples you haue ſeen.

Scholar. This I truſt I can doe.

Maſter. But and if the queſtion be aſked thus: In 8 weekes I ſpend 40 s, how long wil 105 ſhillings ſerue me? Heere you ſee that 8 weekes anſweres himſelf, and ſaith 40 ſhillings. But how long time 105 ſhillings wil ſerue you know not. Therefore you ſhall ſet 105 in the third place, according as I told you euen now. And the firſt place muſt alwaies be of the ſame nature or denomination that the third is of, which here is 40. Then muſt 8 needs be that other. Now multiply 105 by 8, and it wil bee 840, which if you diuide by 40 it wil yeelde 21, which is the fourth number, & ſheweth how many weekes 105 ſhillings wil ſerue, if you ſpend 40 s in 8 weekes.

The figure of this queſtion is this: as if you ſhould ſay: If 40 s ſerue for 8 weeks, 105

ſhillings.	Weekes
40	8
105	21

ſer

serue for 21 weekes.

Other diuersities there bee of working by this rule, but I had rather that you would learne this one well then at the beginning to trouble your minde with many formes of working; sith this way can doe as much as all the other, and hereafter you shall learne the other more conueniently:

¶ And for your further aid and instruction, to make you better acquainted with this golden Rule, I haue here proponed 6 questions and their answeres, which I thinke most conuenient and meete to preferre the desirous to perfect vnderstanding. The first foure are all branches of one question sprung out of the best tree, (for a yong learner to taste of) that groweth in this Ground of Artes: for that no manner of question in the Rule of 3 whatsoeuer it be, can be proponed, but it must be comprehended vnder the reason or stile of one of these foure.

The questions be these.

If 15 elles of cloth cost 7 l, 10 s: what comes 27 elles to at that price? Answere: 13 l 10 s.

If 27 elles cost 13 l 10 s: what are 15 elles worth? Answere: 7 l, 10 s.

If 27 elles cost 13 l, 10 s; how many elles shall I haue for 7 l, 10 s? Answere: 15 elles.

If

If I sell 15 elles for 7 l 10 s: how many elles are to bee deliuered for 13 l,10 s? Answ. 27 elles. If 8 pound of any thing cost 16 s,6 d: what money is to bee receiued for 49 pound? Ans. 5 l,1 s,0 d.

If 4 l of any thing cost 17 d: what money wil 8765 pound of that commoditie cost? Answere. 155 l,4 s,3 d,q̄.

Of all which questions I omit the work of purpose, that you shal whet your wit thereby at conuenient leasure, to clime each branch and gather the fruit of them, and doe minde now, before we make an end of this rule, to giue you some instructions of the backer rule of 3, whose order is quite contrarie to this that you haue learned.

For in this rule hitherto, euermore looke how much the third number is greater than the first, so much the fourth number is greater than the second. And contrariewaies: looke how much the first summe is greater than the third, (if it do chance so) so much is the second sum greater than the fourth. But in this rule there is a contrarie order, as this: That the greater the third summe is aboue the first, the lesser the fourth summe is beneath the second, and this rule therefore you may call the Backer rule, as in example.

The backer rule. Questions of buying cloth.

If I haue bought 30 yards of cloth of yards breadth, and would haue canuas of yards broad to line it withall, how many yard

yeards should I neede?

Scholar. Why, there is none so broad.

Master. I do not care for that, I doe put this example onely for your easie understanding: For if I should put the example in other measures, it would bee harder to understand. But now to the matter: If you would know this question, set your numbers as you did before: but you shal multiply now the first number by the second, and that ariseth thereof, you shal diuide by the third: which thing if you doe here, I meane if you multiply 30 by 2, it wil be 60: which summe if you diuide by 3, there wil appeare 20: whereby I know, that if 30 yards of cloath of two yards broad, should be lined with canuas of three yards broad, 20 yards of canuas would suffice, as this figure sheweth.

Breadth.		Length.
2	Z	30
3		20

And now because ye found fault with my example, how say you, perceiue you this?

Scholar. Yes sir, I suppose.

Master. Then answere me to this question: how many elles of canuas of elle breadth, will serue to line 20 yards of Say, of three quarters broad?

Scholar. In good faith sir, I cannot tell, for I know not how to bring the summes to the denominations.

Maſter. Then wil I tell you, ſith there is mention heere of quarters, and againe euerie one of the meaſures both elles and yards may be parted into quarters, do you part them ſo both in the breadth and length, and then put forth the queſtion by quarters.

Scholar. Then I ſhall ſay thus. How many quarters of canuas of fiue quarters broad, wil line 80 quarters of three quarters broad?

Maſter. Now anſwere to the queſtion.

Scholar. Firſt I will ſet them downe in their forme thus, for 5 is ioined with the queſtion, 5 is therefore the third number:

Breadth	Length
3	80
5	

then is 3 the number of the ſame denomination, I meane becauſe they bee both referred to breadth. Now I multiply 80 by 3, and it is 240, which I diuide by 5, and it yeeldeth 48. Then ſay I that 48 quarters of 5 quarters broad, will ſuffice to line 80 quarters of 3 quarters broad.

Maſter. Turne the quarters againe into elles and yards.

Scholar. Then I ſay, that 9 elles and three quarters of a yard of elle broad wil ſerue to line 20 yards of three quarters broad, as this figure ſheweth.

Breadth	Length
3	80
5	48

Maſter. Now what ſay you to this queſti-

question: I lent my friend 400 l for 7 moneths, how much money ought hee to lend me againe for 12 moneths, to recompence my courtesie shewed him? Can you answere to this?

Scholar. Yes sir, I suppose, for I wil set downe my Numbers thus: where I multiply 7 into 400, and it maketh 2800, which I diuide by 12, and it yéeldeth 233 l, and there is 4 l remaining of my Diuision, what do I therewith?

Moneths		Pounds
7	Z	400
12		

Master. Turne the same 4 pound into s, & then diuide it by 12 as you did before.

Scholar. Well sir, it shall be done, so haue I 6 s for my quotient, and yet remaineth 8 s vpon my diuision.

Master. You must also reduce that 8 s into pence, which maketh 96, and diuide that also by your diuisor.

Scholar. So haue I done, and I finde 8 d for my quotient, and nothing is left.

Master. This must you alwaies doe, when any thing remaineth vpon your diuision: whether it bee money, waight, measure, or any kinde of thing whatsoeuer. This rule is so profitable for all estates of men, that for this rule onely (if there were no more but it) all men were bound highly to estéeme Arithmeticke.

By this rule may a Captaine in warre worke many things, as Master Digges in his Stratiaticos doth declare: Onely now in this my simple addition, for a taste and incouragement, I wil inlarge the Author with a question or two more, wishing you and every my countrimen or Gētlemen whatsoeuer, that by nature be any thing giuen to Militarie affaires, to be familiar and acquainted wͭ this excellent Art, the which hee shall finde not onely at the Sea, but also in the Campe and field seruice, aboundantly to ayde him, either in fortification, paying of soldiers wages, charges of ordinance, pouder, shot, munition, and instruments whatsoeuer, as for example.

If it should chance a Captaine which hath 40000 souldiers to bee inclosed with his enemie, that hee could haue no fresh purueyance of victuals, and that the victuals which hee had would serue that Army but onely 3 moneths, how many men should he dismisse to make the victual to suffice the residue eight moneths.

Scholar. As you taught mee, I sette the numbers thus, saying: If three moneths suffice 40000, to how manie will eight moneths suffice?

Moneths.		Men.
3	Z	40000
8		

To know this, I multiply the first number

ber 3 into the ſecond 40000, and it yeeldeth 120000, which ſumme I diuide by 8, and there wil be in the quotient 15000, which if I doe ſubtract from 40000, the remainer wil declare that hee muſt diſmiſſe 25000 as this figure ſheweth.

Moneths. Men
3 Z 40000
8 15000

¶ Maſter. Now anſwere me to this queſtion: If 136 Maſons in a moneth bee able to build a Fort to preſerue the ſoldiers from the enemy, and ſuch expedition requireth that I wold haue the ſame finiſhed in eight daies; How many workemen ſay you is there to bee appointed?

Schol. As you taught me, I ſet the numbers thus, ſaying: If 28 daies require 136 Maſons, what number of men by the like proportion will 8 daies require.

28 Z 136
8

To know this, I multiply the firſt number 28 into 106: and it yeeldeth me 3808: which I diuide by 8. And my quotient is 476, which is the iuſt number of Maſons that ſhall ſupply this worke. And now me thinke theſe queſtions are verie eaſie.

Maſter. Truely if you take delectation herein, you ſhall finde this Art not onely eaſie, but wonderfull pleaſant and profitable. Now therefore one queſtion more I wil pro-

poſe, and ſo leaue off this rule in whole numbers, vntill we come to the vſe of it in broken numbers: for had you the vnderſtanding of broken numbers perfectly, not onely in this rule, but in all other, the queſtion that in ſight or apparant ſeemeth to bee 100 times more harder to abſolue, may thereby be wrought as ſoone or ſooner than this.

Scholar. Your words doe greatly encourage me to be ſtudious to attaine whole numbers: but might I once attaine to be a practitioner in broken numbers, I ſhould think my ſelfe happy.

Maſter. What ſay you then to this queſtion: If 48 Joiners in 2 daies make 200 light horſemens ſtaues, (eſteeming they worke but 12 houres a day) and ſuch need requireth that 384 Joyners are ſet to the finiſhing of thoſe 200 ſtaues; in what time ſay you, will they make them vp?

Scholar. I ſee heere that I muſt turne my 2 daies into houres. And ſo doing I ſet my numbers thus.

48 Z 24
384

Saying, if 48 men are 24 houres, 384 men wil make an end quickly. For it is grounded vpon an olde prouerbe, many hands make quick ſpeede.

I multiply 48 into 24, and it amounteth to 1152, which I diuide by 384: and my quo-

tient

tient is thꝛee houres, which is my deſire.

I take this foꝛ a note woꝛthy the marking, either in the rule of thꝛee, foꝛward oꝛ backward, when the two numbers are multiplied together, the pꝛoduct is of the ſame nature and denomination, that the ſecond number is of.

Note.

Maſter. Well, ſith you perceiue now the vſe of this rule, I wil ſhew other which enſue of the ſame, & firſt the double rule, which is ſo called, becauſe there is in it double woꝛking, by which thing onely it differeth from this.

The double rule.

Scholar. Then by an example I ſhall vnderſtand it well enough.

Maſter. So ſhall you, and let this be the example: If the carriage of 1 C weight (that is to wit 1 1 2 pound) 30 miles do coſt 1 2 d, how much wil the carriage of 5 C waight coſt, being carried 100 miles?

Of carriage.

Scholar. I pꝛay you ſhew me the woꝛking of it.

Maſter. You muſt make two woꝛkings of it: the firſt thus. If 1 C waight coſt 12 d, how much will 5 C. waight coſt? Set your figures thus.

C waight.	Pence.
1	12
5	

And multiply 5 by 12, and thereof amounteth 60, which if you diuide by one, the quotient will be ſtill 60, that is the pꝛice of fiue C

 waight

weight for 30 miles.

Then begin the second worke, saying, If 30 miles cost 60 d, how much will 100 miles cost? Set your figures thus.

Miles.	Pence.
30	60
100	

Then multiply 100 by 60, whereof amounteth 6000, which beeing diuided by 30, will yeeld 200 d. Then you may say, that so many pence shall cost the carriage of 500 pound waight 100 miles, after the rate of 12 pēce for the 100, caried 30 miles.

Question of prouision touching an army.

Scholar. Now I perceiue it also.

Master. These and such other like questions of the double Rule of three, are to bee answered much sooner, at one onely working by the rule of three composed of 5 numbers, which anon I wil shew you; and then when you haue the vse thereof, you may vse which way you thinke good.

Scholar. Sir, I thanke you much for your courtesie, and I long now til this rule bee ended, that I may see how I shall behaue my selfe with that new rule of fiue numbers: for that I haue euer sinc you taught me hitherto, in the Golden Rule both forward and backwarde, wrought but with three numbers onely.

Master. But yet a while wee will go on forward with this double rule of 3: therefore answere to this question; thirty bushels of

wheat

wheat sowed, yéelded in one yeare 360; how many will 80 bushels yeeld in 7 yeares? I meane sowing euery yeere of those seuen, still 80 bushels.

Scholar. First I say, that if 30 bushels wil yéeld 360 in one yeare, then 80 bushels will yéelde 960 in one yeare. Then for the second worke I say: If one yeare yéelde 960, then seuen yéeres wil yéelde 6720: as these two figures do shew.

Seede	Encrease	Yeare	Encrease
30	360	1	960
80	960	7	6720

But now sir, if I set forth 30 bushels of corne to another man for 7 yéeres, agréeing so that hee shall sow euerie yéere the whole increase of the corne, and I at the end of those seuen yeares to haue the halfe of the whole increase: I would know how many bushels will there amount to my part, supposing the increase to be after the rate of the last questiõ, 30 bushels in one yeare to yéeld 360.

Master. In such a question you must haue as many seuerall workings as there be yeares, for example: in the first yeare 30 bushels yeelde 360: then to know the yéelding of the second yeare, I must say: If 30 yéeld 360, how many yéeldeth 360? Worke by your rule, & you shall finde 4320. Then say for the third yeare:

yeare. If 30 yeelde 360, how many wil 4320 yeeld? you shall haue 5 1 840, and so euerie yeere multiplying the whole increase by 360 and diuiding it by 30, the increase of the next yeere will amount, as these 7 figures following do orderly declare: where I haue set 7 letters for the 7 yeares, of which the first is set without art, because that is ẏ increase which you do presuppose: and the last number of each other doth shew the increase of the yeare that it standeth for, which the letters doe declare, so that the increase of the seuenth yeare, is 1074954240 bushels: how many quarters that is, and also how many waies, you may by Reduction soone finde.

a

30—360

b

30 360
360 4320

c

30 360
4320 51840

d

30 360
51840 622080

e

30 360
622080 7464960

f

30 360
7464960 89579520

g

30 360
89579520 1074954240

Now

Now with one question more I wil proove you. If 6 Mowers doe mow 45 acres in fiue daies, how many mowers will mow 3 hundred acres in 6 daies?

Scholar. If 45 acres require 6 mowers, then 300 acres require 44. Now againe: if fiue daies require 40 mowers, then 6 daies neede but 33 mowers.

Master. Why do you not make mention of the 2 that remaineth in the last Diuision? for the last part of the question is wrought by the backer rule, where the first number fiue is multiplied into the second that is 40, whereof amounteth 200, which if you diuide by the third number 6, the quotient will be 33, as you said: but then will there remaine two, which cannot well be diuided into 6 partes: howbeit you may vnderstand by the sixt part of 2, the third part of one mans worke, which you must put to the 33: or else you must say, that 33 workemen will end all the 300 acres in 6 daies, saue two mens worke for one day, or 2 daies worke for one man. But such broken numbers called Fractions, you shal hereafter more better perceiue, when I shal wholy instruct you of them.

Master. Yet one question more of field matters I wil propone, and so I will make an end of this double Rule of 3.

Scholar. With all my heart sir I thanke you, and I wil dispatch it as soone as I can, because

because I would faine ſee the order of the nex rule of 5 numbers.

Maſter. If a Captaine ouer a band of me did ſet 300 Pioners a worke, which in eigh houres did caſt a trench of 200 Rods: I demaund how many Labourers will bee ab with a like trench in three houres to intrench camps of 3400 Rods.

Scholar. I thinke I am now in the Bac houſe ditch, for I know not well which wa to goe about it. And beſides that, truly I thinke I ſhall neuer come to preferment th way, my growth is ſo ſmall.

Maſter. You know not how God may ra you hereafter by knowledge and ſeruice in the fauour of your Prince, for the auail your Country.

Example for Nauigation: Sir Franc Drake, a man greatly honoured for his know ledge, is not the talleſt man, and yet ha made as great a venture for the honour his Prince and Country, as euer Engliſhma did.

Scholar. Sir, I thanke you for your g encouragement. My mind, though I be litt is as deſirous of knowledge, as any other: I haue pondred now a little of it, and thus I ſet forth the worke.

Rod.	Me
200	30
Rod	
3400	

Saying, if 200 rod require 300 men, wh

[sh]all 3400 rods require? I multiply 3400 by [3]00, and it yeeldeth 1020000, which I divide [b]y 200, and my quotient is 5100 men.

Then must I say for my second worke, if [in] 8 houres 5100 men be able to discharge it, [h]ow many shall performe the same in three [h]oures? Now if I would worke by the Gol[de]n rule of proportion forward, I should find [a] lesse number of men: because three houres is [les]se than 8 houres: but because reason tea[ch]eth me that the lesser the time is, wherein [th]e trench must be made, the more labourers [I] ought to haue, thereupon I vse now the [ba]cker rule, as in example. And I haue in [m]y quotient 5000. So many Pioners [m]ust I haue to entrench the campe in three [ho]ures.

Master. You haue answered the question [ve]ry artificially: And truely I commend you [for] your diligence and apt vnderstanding: and [no]w according to my promise, I will (in whole [nu]mber) giue you a little taste of the rule of [co]mpounded of 5 numbers.

The

The Rule of 3 compound of 5 Numbers.

This Rule of 3 composed, is distinct for most needful questions, into two severall parts of workings: And there belongeth unto it alwaies five numbers, whereof in this rule being the first part, the second number and the fift are alwaies of one nature and like denomination, which rule is to be wrought thus; you must multiply the first number by the second, and that shall be your divisor: Then againe, multiply the other three numbers, the one by the other, and their product shall be your dividend.

And now according to my promise, we will first worke the question of waight & carriage, which I delivered you in the double rule of 3, to be absolved by this rule, which was this.

If the carriage of 1 C waight 30 miles cost 12 d; what will the carriage of 5 C waight stand me in being carried 100 miles?

C. waight.	Miles.	Pence.	C. weight.	Miles.
1	30	12	5	100

Now marke well how these five numbers stan

stand: Then multiply the first number by the second, as 30 by 1, which maketh but 30, and that number keep for your diuisor. Then multiply the other three numbers the one into the other: that is to wit, 12 by 5, which maketh 60: lastly 60 by 100, which as you see herein our Tables, ariseth to 6000, which 6000 you shall diuide by the product of the two first numbers, which here is 30. And as you see there is found 200 pence, which is the duetie that you ought to pay for the carriage of 5 C. waight 100 miles after the rate of 12 d a hundred, and agreeth with the conclusion of the double rule of three.

Scholar. Sir I thanke you, it is euen so.

Master. Yet note this for a generalitie in this rule, looke what nature or denomination your middle number is of (which heere are pence) and of the like denomination or nature is alwaies your quotient.

Scholar. Well now and if it please you, by your patience, I wil see how I can end the question next following of thirty bushels of wheat sowed, which in one yeare yeelded 360: how many the wil 80 bushels yeeld in 7 yeere, sowing euery year of those 7 yeare wil 80 bushels, and according to your reasons I set my numbers thus.

Bushels,	Yeare.	Bush.	Bush.	Yeer.
30	1	360	80	7
		80		
		28800		
		7		
		201600		

Where

Where I multiply 30 by 1, and it maketh 30 my diuisor: then multiplying the other 3 numbers the one into the other, as héere appeareth in my tables, they make 201600: which I diuide by 30: and my quotient is 6720 bushels my desire, for so much also it came to at two workings by the rule of 3.

Master. Yet one question more I wil propound vnto you, and so leaue this rule, till it please God hereafter, that I may make you worke it in broken numbers.

What comes the interest of 258 l for fiue moneths to, after the rate of 8 l taken in the 100 l for 12 moneths?

Scholar. Sir, this is yet within the compasse of some reasonable vsance. Therefore to minister equitie in this case, I will see how I can worke.

The same which I set downe thus, praying you if I haue not done well, to shew me mine errour.

l	moneths.	l	l	moneths.
100	12	8	258	5

Master. Procéed, you haue done very well.

Scholar. Then I doubt not by the grace of God but to end it: I multiply 100 by 12, it yéeldeth 1200, and the thrée other numbers multiplied together produceth 10320: which I diuide by 1200: and my quotient is eight pounds. Then according as you haue taught me heretofore, I turne the 720 l that I left into

into shillings: and dividing it by the first number, my quotient is 12 s. So I answere that the loane of 258 l. for 5 moneths, after the rate of 8 l in the 100 l for a yeare, comes to 8 l, 12 s.

Master. You say true, I commend your diligence: now behold the manner of the second part of this rule.

Master. In the second part of this rule of 3 omposed, the third number is like unto the irst. And the rule is to bee wrought as thus: ou shall now contrary to the last rule multi-ly the third number and the fourth toge-her, and that product shall bee your Divisor. hen multiply the first by the second, and the oduct thereof by the first: and that is the umber that shall be divided. For example, I opound this question, for a proofe of my last uestion of interest. A merchant hath recei-ed 8 l, 12 s, for interest of certaine money 5 moneths terme, which he received after e rate of 8 l, in the 100 l, for a yeare. The uestion is now, how much money was deli-ered to raise this interest: Behold therefore e manner,

The 2 part of the rule of 3 compound.

The proof of the last question.

w the que-	l	moneths.	l	moneths.	l	s
on is set	100	12	8	5	8	12

th.

Scholar. Sir I perceive it very well: and ording to the doctrine which you prescribed the working thereof: if it please you now it

it is set downe, I thinke I can follow the worke.

Master. Nay stay a while, and before you worke, marke well how I deliuer a reason for the perfect vnderstanding of this rule, which is thus: If 8 l in 12 moneths doe yeelde me 100 l, to take 8 l, 12 s, for 5 moneths; must needs yeelde a great deale more.

The 2 part of the rule of 3 compound.

So vpon the knowledge that I haue in this Art, the first part of this rule is answerable to the rule of 3 forward: And this latter part accordeth to the rule of 3 backward.

Scholar. Sir, I yeelde you most heartie thankes for these your last instructions, they haue giuen mee great light into these two rules, whereby I may the better by deliberation conceiue how to vse them hereafter, when occasion shall require.

The proofe of the last question.

Master. You say well, goe to now if you will, and trie your cunning in the question: But this note take with you by the way, in as much as heere is mention made of shillings, turne all your money as you worke into s, for your more ease in worke.

Scholar. If it please you to beholde me a little, I will quickly end it: for I haue but my first, my second, and my last number to bee multiplied together for my diuidend: And my third into my fourth for my diuisor.

Moneths. l, Moneths. l, s.

100 —— 12 —— 8 —— 5 —— 8 —— 12

20	20	20
2000	160	172
12	5	
4000	800	
2000		
24000		
172		
48000		
168000		
24000		
4128000	4128000 / 8 / 60	

Which 4128000 I diuide by 800, and my quotient is 5160 s, which in pounds yeldeth 258 my desire.

Master. I wil here for this time in whole numbers end this rule, and I wil instruct you in the rules of Fellowship. You may at your conuenient leasure for your exercise worke the same by the rule of 3 at twice. And for your aide and incouragement therein, I set downe here a proffer how to apply it.

Moneths A l, s Pound. B l

5 Z 8 12 8 Z 100

12 Z 413 $\frac{4}{5}$ s 412 $\frac{4}{5}$ Z 258 l

The Rule of Fellowship.

BUt now wil I shew you of the Rule of Fellowship or Company, which hath sundry operations, according to the diuers number of the company. This rule is sometime without difference of time, and sometimes there is in it difference of time. First I wil speake of that without difference of time, of which let this be an example.

Foure Merchants of one company made a banke of money diuersly, for the first laid in 30 l, the second 50 l, the third 60 l, and the fourth 100 l: which stocke they occupie so long, till it was increased to 3000 l. Now I demaund of you, what should each receiue at the parting of this money.

Scholar. I perceiue that this rule is like y^e other, but yet there is a difference which I perceiue not.

Master. Then will I shew it to you. First by Addition you shall bring all the particular summes of the Merchants into one summe, which shall be the first summe in your working by the Golden rule, and the whole sum of the gaines by that stocke shall be the second summe. Now for the third summe you shall

set

set the portion of each man one after another, and then worke by the Golden Rule, and the fourth sum will shew you each mans gaines: as in example.

The parcels of those foure Merchants make in one summe 240 pounds: set that in the first place, the gaines in the second, & the first mans portion of stock in the 3 place thus.

240 3000
30

Now multiply the second by the third, and it will be 90000, which you shall divide by 240, and there will appeare 375 l, thus. And that is the gains for the first man.

240 3000
30 375

Now for the second man, set the 50 l that hee brought, in the third place, and worke as before: and his part wil be 625 l: as this figure sheweth.

240 3000
50 625

Likewise for the third man, set his money which was 60 l, and his part of gaines will bee 750 l, as heere appeareth.

240 3000
60 750

And so for the fourth man, if you set his sum which is 100 l, his gaines will bee 1250 l, as the worke wil declare.

240 3000
100 1250

Scholar. This I perceive: but is there

any way to examine whether I haue well done or no?

Master. For the triall hereof, adde together all their foure portions, and if their addition make the whole summe of their gaines, then is the worke well done.

Scholar. That will I trie by and by, the foure parcels are these, which added together, make 3000, which is the iust sum of money that they gained, wherby I know the work is well done.

375
625
750
1250
3000

Master. Well, now another example will I put to you, not of gaines, but of losse: for one reason serueth for both.

If thrée merchants in one shippe, and of one fellowship, had bought Merchandise, so that the first had laid out 200 l, the second 300 pound, and the third 500 l, and it chanced by tempest that they did cast ouer board into the Sea merchandise of the value of 100 l, how much should each man beare in this losse?

Scholar. If I shall doe in this, as you did in the other question, then must I ioine their thrée portions together, 200, 300, and 500, which maketh 1000. Then say I, If 1000 loose 100, then shall 200 loose 20, and 300 shall loose 30, and 500 shall loose 50, as by the

these thrée figures it doth appeare plaine.

1000	Z	100	1000	Z	100
200		20	300		30

1000	Z	100
500		150

Master. Well, sith now you haue done these, I will propound a question of more importance, which shall make you not onely the abler to vnderstand this rule, but also it will greatly aid you in the next rule of Fellowship with time, if such néeds be that your money be of diuers denominations.

For this may not be forgotten in all such questions: if the number be of diuers kindes, you must by Reduction bring it into one kind, that is to say, to the least value that is named in the question. And likewise shall you doe, if the time be of diuers kindes, as some yeares, some moneths, wéekes & daies, you shall make all moneths, wéeks or daies, according as the last name of time in the question is: as for example. Note.

First in diuersitie of money. Thrée companions bought 2000 shéepe, and paid for them 241 l—13 s—4 d, of which sum one paid 101 l—10 s. The second paid 82 l-17 s 10 d. And the third paid 57 l-5 s-6 d: How many shéepe must each of them haue? Answere: The first shall haue 840. The second 686. And the third 474. And that must you work thus. Question of corne.

First conſidering that your money is of diuers denominations, you ſhall (by Reduction) bring it all into the ſmalleſt denomination which is in it, that is to ſay, pence; and ſo will the total ſumme be 58000 pence.

Now, if you turne each mans money into pence alſo, the firſt mans ſum will bee 24360 pence: The ſecond mans ſum 19894 pence: And the third mans money will bee 13746 pence.

Now to knowe how many ſhéepe euerie man ſhall haue, let the whole ſum of money, that is, 58000 pence be ſet in the firſt place, & in the ſecond place ſet the number of ſheepe, and then orderly in the third place ſette each mans money, & then multiplying the third & the ſecond ſums together, and diuiding that that amounteth by the firſt, there will appeare the number of ſhéepe that each man ought to haue: as theſe thrée figures do ſhew.

a

58000 — 2000
24360 — 840

b

58000 — 2000
19894 — 686

c

58000 — 2000
13746 — 474

Scholar. Why do you ſet the money in the firſt place, ſéeing in the queſtion you ſay,

2000

2000 sheepe cost 58000 pence, and not thus, 58000 pence, cost 2000 sheepe.

Master. You remember I taught you at the beginning of this Golden Rule, that the first and third numbers must be of one name, and of like things: and euermore the number that the question is asked of, must be set in the third place. Now is the question plainely this: If foure men bought 2000 sheepe for 58 thousand pence, how many sheepe shall each man haue.

But seeing in this question, there ought more respect to be had to the summe of money than to the summe of the persons, (for in the summes of money is there proportion toward the sheepe, and not in the number of persons) therefore must we turne the question thus.

If 58000 pence bought 2000 sheepe, how many did 24360 buy? Againe, how many did 19894 pence buy? And how many bought 13746 pence?

Scholar. I perceiue it reasonablie, and so shall I doe in all like questions.

Master. Euen so. But for easinesse of the worke, marke this: Whensoeuer the first and second numbers haue ciphers in the first places, you may both in the multiplication and in the diuision leaue out those ciphers, so that you leaue out like many out of both summes, as in this question, the first number 58000 hath three ciphers, and so hath the second, that

in

is 2000: therefore caſt away their ciphers, and ſo will the firſt number be 58, and the ſecond 2: ſet them in their places, and worke according to the rule, and you ſhall perceiue that it wil be al one, ſauing that this is the ſhorter and eaſier way, as theſe thrée figures doe ſhew.

a

58 2
24360 840

b

58 2
19894 1676

c

58 2
13746 474

And this you ſee is both eaſier, and alſo the more certaine way to know the anſwere to this queſtion.

Scholar. Truth it is as you ſay: but ſir me ſéemeth I might aſke a further queſtion here, not onely how many ſhéepe each man ſhould haue, but alſo what euery ſhéepe coſt.

Maſter. That queſtion doth not onely belong to this rule, but may alſo be diſcuſſed by Diuiſion, eſpecially if the queſtions number be one onely: as thus: Diuide the totall ſum 58000 pence by 2000 (or 58 by 2, omitting the ciphers) and the quotient wil bee 29 pence, that is 2 s, 5 d; howbeit, by this rule you may doe it, and beſt when the number of the queſtion doth excéede 1: as if I ſhould aſke this

this question, 2000 shéepe cost 58000 d, how much do 20 cost? Then shall I set my figure thus.

200 Z 58000
20

And doing after the rule, there wil amount 580 pence, that is 2 l—8 s—4 d, the price of one score: But if you wil vse that easie way yͤ I did teach you, you may change the first and second number thus.

2 Z 58
20

Thus do you perceiue the vse of the rule without time. And that you may as well perceiue the same with diuersitie of time, I propose this example.

The rule of fellowship with time.

Foure Merchants made a common stocke, which at the yeares end was increased to 55145 l. Now to know what shall be each mans portion of gaines, you must know each mans stocke and time of continuance.

Question of a bank.

The first man of these foure laid in 669 l, which he did take from the stocke againe, at the end of 10 moneths. The second man laid in 81 l for eight moneths. The third laid in 900 l for 7 moneths. And the fourth laid in 1040 l for 12 moneths.

Note. A generall rule.

This question shal you examine as you did yͤ other before, sauing that whereas in yͤ third place of yͤ figure you did set ech mãs sũ alone, here you shal set the same being multiplied by the number of their time, & likewise in yͤ first place of yͤ figure you shall set the nũber which

mounteth of their whole ſummes ſo multiplied by their time, and added into one whole ſum, as thus.

The firſt mans ſumme is 669 l, which I multiply by 10 (that was the number of his time) and it maketh 6690. The ſecond mans ſumme 810 l, multiplied by by 8, (which was his time) maketh 6480. The third mans ſumme 900 l, multiplied 7 (for that was his time) yéeldeth 6300. The fourth mans ſumme was 1040 pound, and his time 12: multiply the one by the other, and it will bee 12480.

Theſe foure ſummes thus multiplied by their time, muſt bee ſet orderly in the third place of the figure, and in the firſt place muſt be ſet the whole ſumme of all foure, which is 31950, and the gaine muſt be in the ſecond place, which is 35145. Now to end the queſtion, I ſay firſt, If 31950 did get 35145, what did 6690 get? Anſwere. 7359 l, as by this figure appeareth.

a

31950 Z 35145
6690 Z 7359

Likewiſe the ſecond man had to his part 7128 pound, the third muſt haue 6930 l. And the fourth man ſhall haue for his parts 13728 pound, as theſe Figures do partly declare.

Scholar,

b		c	
31950	35145	31950	35145
6480	7128	6300	6930

d	
31950	35145
12480	13728

Scholar. This I like very wel: but what proofe is there of this worke?

Master. The summe that I taught you for the other to adde all the portions togither, and it may agree to the whole summe, then is your worke well done.

Scholar. Then will I proue in this example. The foure parcels are these, which if I adde together, there wil amount 35145, and that was the whole sum, whereby I perceiue the worke is well done.

7359
7128
6930
13728
35145

Master. If it fall out otherwise, be sure it is not well.

Scholar. Then doe I vnderstand this worke also very well; but what haue I now to learne?

Master. There are many other excellent parts behinde, of which I wil not as now make mention, because that without the knowledge of Fractions they cannot be duely taught, and much lesse vnderstoode. There will

fore will I propoſe to you two or three queſtions more, (that thereby you may better perceiue the vſe of this rule and all other like) and ſo may make an end for this time.

¶ Three partners by ſome ill aduenture ſuſtained the loſſe of 160 l, whereof the firſt laide into the common ſtocke 200 l for tenne moneths. The ſecond laid in 350 l, and the third 100 l, but for how long the two latter is vnknowne. But breaking off their partnerſhip, the firſt found himſelfe a loſer, 80 l, the ſecond 56 l, and the third 24 l. The queſtion is, for how long time was the money of the two latter in company?

For the ſolution hereof, and of ſuch other like, you muſt alſo multiply the firſt mans 200 l, that he put into the ſtocke by his time of continuance, which was 10 moneths, and it maketh 2000: wherefore now I affirme, if his money that loſt 80 l, multiplied by his time, make 2000: what ſhall his money make that loſt 56 l, & his that loſt 24 l; which 2 numbers I commit to the triall of the rule of 3 at 2 workings, thus: If 80 giue 2000, what giueth 56? And againe, if 80 giue 2000, what giueth 24?

80	2000
56	1400

80	2000
10	600

To conclude, if you now diuide 1400 the ſecond mans portion by 350, which was his

his stocke that he laid into company, you shall finde in your quotient 4 moneths, and for so long time did the second man put his money into the common stocke.

Lastly, if you diuide the third mans new laying in, which was 600 by 100, which was his stocke that he put into company: the quotient declareth his time of continuance, which was 6 moneths. And thus is the question resolued.

Scholar. Sir, I haue attentiuely beheld your working, and the more we trauell herein, the more me thinke I am in loue with this excellent art.

Master. Then what say you to this question?

There is in a Cathedrall Church 20 Cannons, and 30 Vicars, those may spend by year 2600 l, but euery Canon must haue to his part 5 times so much as euery vicar hath: how much is euery mans portion, say you?

Scholar. I pray you make the answere your selfe also, so shall I perceiue best the meanes to answere to such other like.

Master. In this question, you must do as in those before said, that haue diuersitie of time, for here is diuersitie of portions: Therefore shall you multiply the number of the persons by their difference of portions (as you did in the other by time.) Then must you multiply the 20 (which is the number of Cannons) by

by 5 (for that is the number of their portion) so will it be 100: Then 30 (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put those two summes together, and they make 130: then say thus: If 130 spend 2600 l, what may 100 spend? The rule sheweth 2000 pound.

Againe for Vicars: If 130 spend 2600 l, what may 30 spend? Answere, 600 l, as these figures shew.

130	Z	2600	130	Z	2600
100		2000	30		600

But if euery Cannon should haue so often times 4 l, as the Vicar should haue 3 l, then should I multiply 20 by 4 (that were 80) and 30 by 3 (that were 90) and then both were 170. Then should the figures be set thus.

		l	s	d			l	s	d
170	Z	2600			170	Z	2600		
80		1223	10	7	90		1376	9	5

But this sort is too hard for you by reason of the fractions, therefore I will let it rest to that place.

And by this rule you see what the 20 Canons may spend, which summe if you diuide by 20, you shall see each Canons portion: and so of the Vicars, if you diuide their summe by 30, the quotient will declare euery Vicars portion.

The

The ſecond Dialogue.

The accounting by Counters.

Maiſter.

NOW that you haue learned the common kinds of Arithmeticke with the pen, you ſhall ſee the ſame Arte in counters: which feat doth not onely ſerue for them that cannot write & reade, but alſo for them that can doe both, but haue not at ſometime their pen or tables ready with them. This ſort is in two formes commonly. The one by lines, and the other without lines. In that that hath lines, the lines do ſtand for the order of places: and in that that hath no lines, there muſt be ſet in their ſtead ſo many counters as ſhall néed, for each line one; and they ſhall ſupply the ſtead of lines.

Scholler. By examples
I ſhould better perceiue —— 100000 ——
your meaning. —— 10000 ——
Maiſter. For example —— 1000 ——
of the lines, loe here you —— 100 ——
ſee ſixe lines, which —— 10 ——
ſtand for ſixe places, ſo —— 1 ——

that ye neathermost standeth for ye first place, and the next aboue it for the second, and so vpward till you come to the highest, which is the first line, and standeth for the first place. Now what is the value of every place or line, you may perceiue by the figures which I haue set on them, which is according as you learned before in Numeration of figures by the pen: for the first place is the place of vnits or ones, and euery counter set in that line, betokeneth but one: and the second line is the place of 10, for euery counter there standeth for 10. The third line the place of hundreds; the fourth of thousands, and so forth.

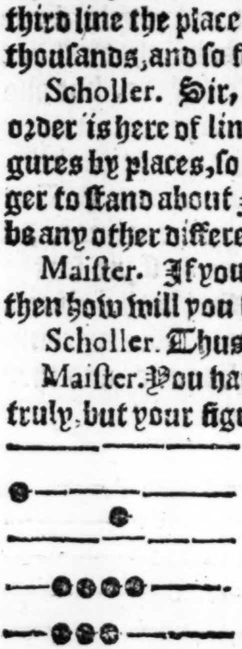
Scholler. Sir, I do perceiue that the same order is here of lines as was in the other figures by places, so that you shall not need longer to stand about Numeration, except there be any other difference.

Maister. If you doe vnderstand it, then how will you set 1543?

Scholler. Thus, as I suppose.

1
5
4
3

Maister. You haue set the places truly, but your figures be not meet for this vse: for the meetest Figures in this behalfe, is the Figure of a Counter round, as you see here, where I haue expressed that same summe.



Scholler.

Scholler. So that you haue not one figure for 2, nor 3 nor 4, and ſo forth, but as many Digits as you haue, ſo many Counters you ſet in the loweſt line, and for euery 10 you ſet one in the loweſt line: and ſo of other. But I know not by what reaſon you ſet that one Counter for 500 betwéene two lines.

Maiſter. You ſhall remember this, that whenſoeuer you néede to ſet downe 5, 50, or 500, or 5000, or ſo forth any number whoſe Numerator is 5, you ſhall ſet one counter for it in the next place aboue the line that it hath his denomination of: as in this example of that 500, becauſe the numerator is 5, it muſt be ſet in a void ſpace, and becauſe the Denominator is a hundred, I know that his place is the void ſpace next aboue hundreds, that is to ſay, aboue the third line.

And further you ſhall marke, that in all working by this ſort, if you ſhall ſet downe any ſumme betwéene 4 and 10, for the firſt part of that number you ſhall ſet downe 5, and then ſo many Counters more, as there reſt numbers aboue 5. And this is true both of Digits and Articles. And for example I will ſet downe this ſumme 297965, which ſumme if you marke well, you néede none other examples for to learne the nume-

ration of this forme.

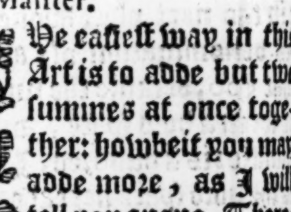
But this shall you marke, that as you did in other kinds of Arithmeticke, set a prikce in the places of thousands, in this work you shall set a Starre, as you see before.

Scholler. Then I perceiue Numeration: but I pray you how shall I do in this Art to adde two summes or more together.

Addition.

Maister.

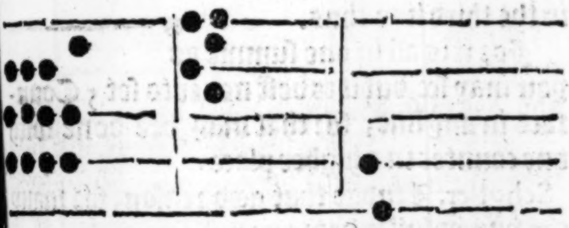
THe easiest way in this Art is to adde but two summes at once together: howbeit you may adde more, as I will tell you anone. Therefore when you wil adde two summes, you shall first set downe one of thē, it forceth not which, & then by it draw a line crosse the other lines. And afterward set downe the other summe, so that the line may be betweene them: as if you would adde 2659 to 8342, you must set your sums as you see here.



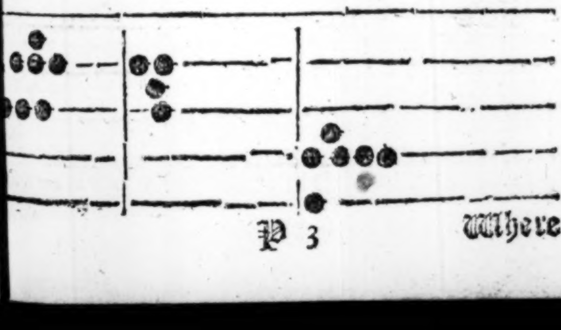
And then if you list, you may then adde th one to the other in the same place: or else yo ma

may adde them both together in a new place: which way becauſe it is moſt plaine, I will ſhew you firſt.

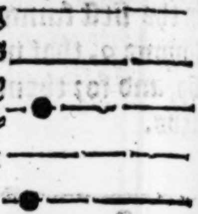
Therefore will I begin at the vnits which in the firſt ſumme is but 2, and in the ſecond ſumme 9, that maketh 11. Thoſe doe I take vp, and for them I ſet 11 in the new roome, thus.



Then doe I take vp all the Articles vnder a hundred, which in the firſt ſumme are 40, and in the ſecond ſumme 50, that maketh 90: or you may ſay better, that in the firſt ſumme there are foure Articles of 10, and in the ſecond ſumme 5, which maketh 9, but thẽ take heede that you ſet them in their right lines, ſee here.



Where I haue taken away 40 from the firſt ſumme, and 50 from the ſecond, and in their ſtead I haue ſet 90 in the third roome, which I haue ſet plainly that you might well perceiue it: howbeit, ſeeing that 90 with the 10 that was in the third roome already doth make 100, I might better for thoſe 6 Counters ſet 1 in the third line thus.



For it is all in one ſumme as you may ſee, but it is beſt neuer to ſet 5 Counters in any line, for that may bee done with one counter in a higher place.

Scholler. I iudge that good reaſon, for many are vnneedfull where one will ſerue.

Maiſter. Well, then will I adde forth of hundreds: I finde 3 in the firſt ſumme, and 6 in the ſecond, which maketh 900, them doe I take vp, and ſet in the third roome, where is 100 already, to which I put 900, and it will be 1000: therefore I ſet one Counter in the fourth line for them all, as you ſee here.

Then

Then adde I the thouſands together, which in the firſt ſumme are 8000, and in the ſecond 2000, that maketh 10000: them doe I take vp from thoſe two places, and for them I ſet one counter in in the fifth line, and then it appeareth as you ſee to bee 11001, for ſo many doth amount of the Addition of 8342 to 2659.

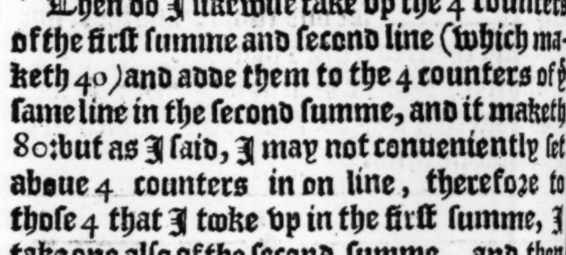
Scholler. Sir, this I do perceiue: but how ſhall I ſet one ſum to another, not changing them to a third place?

Maiſter. Marke well how I do it: I will adde together 65436 and 3245, which firſt I ſet downe thus.

Then do I begin with the ſmalleſt denomination, which is 1 in the ſecond ſum, and ſet it in his place: then do I find 5 in the firſt ſum, and 5 in the ſecond ſum, which I put together ſaving that 2 counters cannot be ſet in a void place of 5, but for them both I muſt ſet 1 in ÿ ſecond

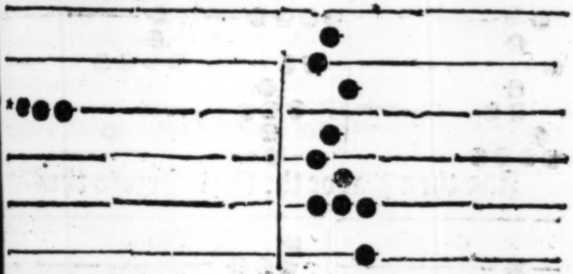
ſecond line, which is the place of 10, therefore I take vp the fiue of the firſt ſumme, and the 5 of the ſecond, and for them I ſet 1 in the ſecond line, as you ſee here.

Then do I likewiſe take vp the 4 counters of the firſt ſumme and ſecond line (which maketh 40) and adde them to the 4 counters of ye ſame line in the ſecond ſumme, and it maketh 80: but as I ſaid, I may not conueniently ſet aboue 4 counters in on line, therefore to thoſe 4 that I tooke vp in the firſt ſumme, I take one alſo of the ſecond ſumme, and then haue I taken vp 50: for which fiue Counters I ſet downe one in the ſpace ouer the ſecond line: as here doth appeare.

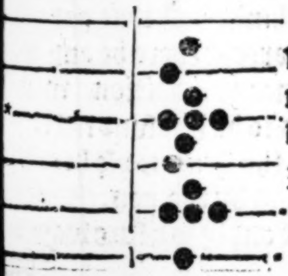


And then is there 80, as well with thoſe 4
counters, as if you had ſet downe the other 4
alſo:

Now do I take the 200 in the firſt ſumme, & adde them to the 400 in the ſecond ſum, and it maketh 600, therefore I take vp the two counters in the firſt ſumme, and thrée of them in the ſecond ſumme, and for them 5, I ſet 1 in the ſpace aboue, thus.



Then take I the 3000 in the firſt ſumme, vnto which there are none in the ſecond ſum agréeing, therefore I doe onely remoue thoſe thrée counters from the firſt ſumme into the ſecond, as here doth appeare.

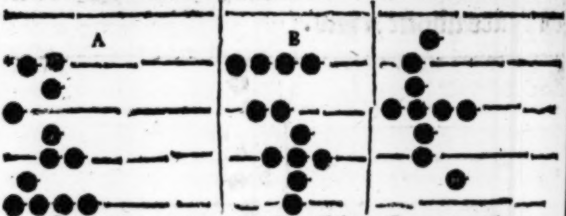


And ſo you ſée the whole ſumme that amounteth of that Addition of 65436 with 3245, to be 68681.

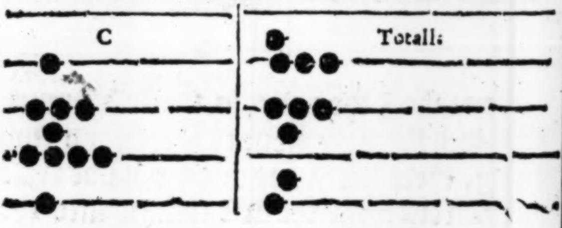
And if you haue
marked

marked these two examples well, you neede no further instruction in Addition of 2 onely sums: but if you haue more than two sums to adde, you may adde them thus.

First adde two of them, and then adde the third and fourth, or more if there be so many: as if I would adde 2679 with 4286, and 1391. First I adde the two first sums thus.



And then I adde the third thereto thus.



And so of more, if you haue them.

Scholler. Now I thinke best that you passe forth to Subtraction except there be any way to examine this manner of Addition, then I thinke that were good to be knowne next.

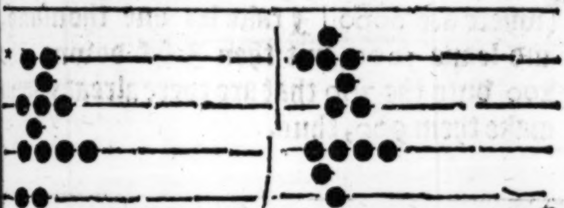
Maister. There is the same proofe here that is in the other Addition by the pen, I meane Subtraction; for that onely is a sure way: but

con

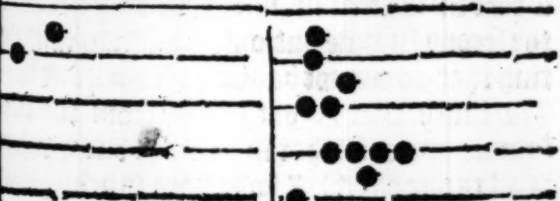
conſidering that Subtraction muſt bee firſt knowne, I wil firſt teach you the Art of Subtraction, and that by this example.

Subtraction.

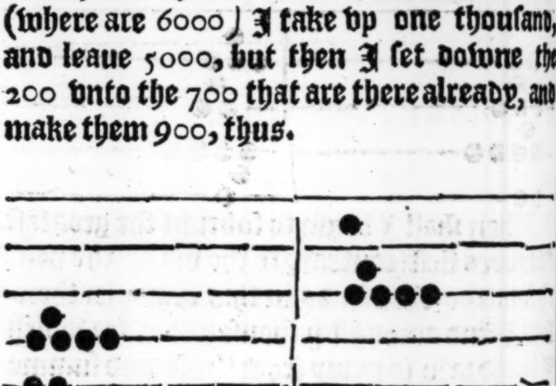
I Would ſubtract 2892 out of 8746. Theſe ſummes muſt I ſet downe as I did in Addition: but here it is beſt to ſet the leſſer number firſt, thus.



Then ſhall I begin to ſubtract the greateſt numbers firſt (contrary to the vſe of the pen) that is, the thouſands in this example: therefore I find amongſt ỹ thouſands 2, for which I withdraw ſo many from the ſecond ſumme (where are 8) and ſo remaineth there 6, as this example ſheweth.



Then doe I likewiſe with the hundreds, of which in the firſt ſumme I finde 8, and in the ſecond ſumme but 7, out of which I can- not take 8, therefore this muſt I doe: I muſt looke how much my ſum differeth from 10, which I finde here to be 2, then muſt I abate for my ſumme of 800, one thouſand, and ſet downe the exceſſe of hundreds, that is to ſay, 2, for ſo much as 1000 is more than I ſhould take vp: therefore from the firſt ſumme I take that 800, and from the ſecond ſumme (where are 6000) I take vp one thouſand, and leaue 5000, but then I ſet downe the 200 vnto the 700 that are there already, and make them 900, thus.

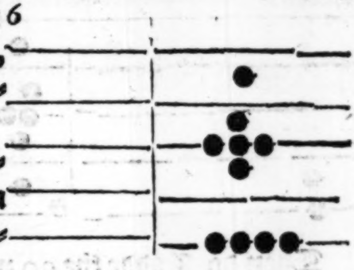


Then come I to the Articles of tennes, where in the firſt ſumme I finde 90, and in the ſecond ſumme but only 40. Now conſide- ring that 90 cannot be abated from 40, I looke how much that 90 doth differ from the next ſumme aboue it, that is 100, or elſe (which is all to one effect) I looke how much 9 doth differ from 10, and I finde it to be 1: then i[...]

th[...]

the ſtead of that 90, I do take from the ſecond ſumme 100: but conſidering that is 10 too much, I ſet downe 1 in the next line beneath for it, as you ſee here. Sauing that here I haue ſet one counter in ẏ ſpace in ſtead of 5 in the next line.

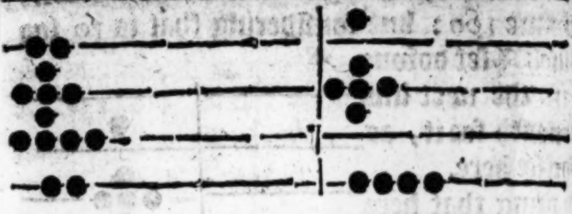
And thus haue I ſubtract all ſaue 2, which I muſt bate from 6 in the ſecond ſum, and there will remaine 4, thus.



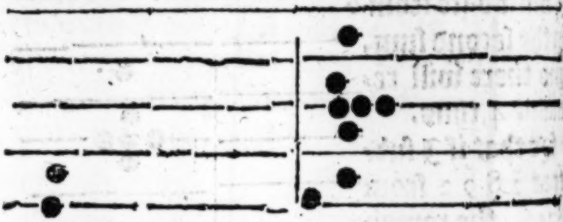
So that if I ſubtract 2892 from 8746, the remainer will be 5854.

And that this is truly wrought, you may proue by Addition: for if you adde to this remainer the ſame ſumme that you did ſubtract, then will the former ſumme 8746 amount againe.

Scholler. That will I proue, and firſt I ſet the ſum that was ſubtracted, which was 2892 and then the remainer 5854. thus.



Then doe I adde the first 2 to 4, which maketh 6: so take I vp 5 of those counters, and in their stead I set 1 in the space, and one in the lowest line, as here appeareth.

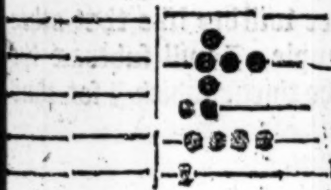


Then do I adde the 90 next aboue to the 50, and it maketh 140, therefore I take vp those 6 counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line thus.

Then

Then doe I come to the hundreds, of which I finde 8 in the first summe, & 8 in the second, that maketh 1600, therefore I take up those 8 counters, and in their stead I set 1 in the fourth line, and 1 in the space next beneath, and 2 in the third line, as you see here.

Then is there left in the first summe but onely 2000, and in the second 5000, which is 7000, which I shall take up from thence, and set in the same line in the second summe, to the one that is there already: and then will the whole summe appeare, as you may well see, to be 8746, which was



the first grosse sum, and therefore I doe perceive that I had well subtracted before,

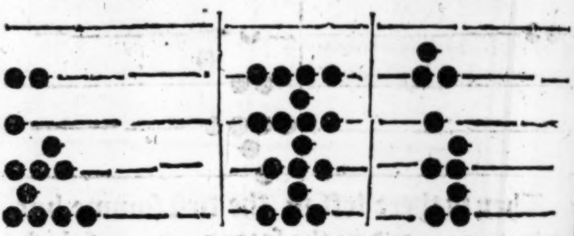
And thus may you see, how Subtraction may be tried by Addition.

Scholler. I perceive the same order here with Counters, that I learned before in figures.

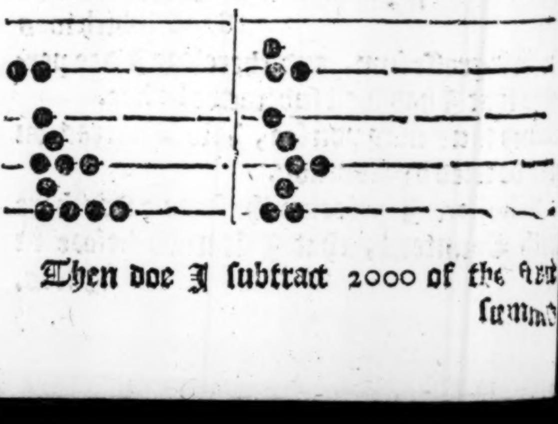
figures.

Maiſter. Then let me ſee how you can try Addition by Subtraction.

Scholler. Firſt I wil ſet forth this example of Addition, where I haue added 2189, to 4988. And the whole ſumme appeareth to be 7177.



Now to try whether that ſumme bee well added or no, I wil ſubtract one of the firſt two ſummes from the third, And if I haue well done, the remainer will bee like that other ſumme, as for example, I will ſubtract the firſt ſumme from the third, which I ſet thus in their order.

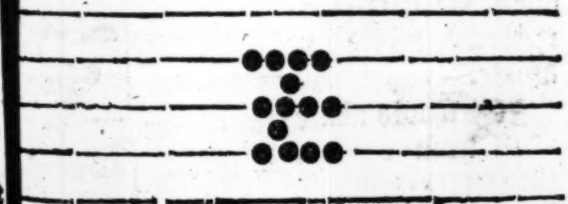


Then doe I ſubtract 2000 of the firſt

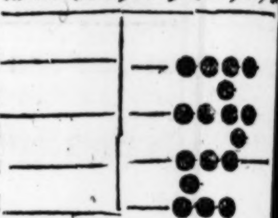
ſumme from the ſecond ſumme, and then remaineth there 5000, thus.

Then in the third line I ſubtract the 100 of the firſt, from the ſecond ſumm, where is onely 100 alſo: and then in the third line reſteth nothing as you may ſee in this example following

Then in the ſecond line with his ſpace ouer him I find 80, which I ſhould ſubtract from the other ſumme: then ſeeing there are but onely 70 I muſt take it out of ſome higher ſumme, which is heere onely 5000: therefore I take vp 5000: and ſeeing that is too much by 4920, I ſet down ſo many in the ſecond roome, which with the 70 being there alreadie, do make 4990, and then the ſums doe ſtand thus.



Yet remaineth therein the first summe 9, to be abated from the second summe, wherein that place of vnities doth appeare onely 7: then must I abate a higher sum, that is to say 10, but seeing that 10 is more then 9 (which I should abate) by 1 therefore shall I take vp one counter from the secōd & set downe the same in the 1 or lowest line, as you see heere.

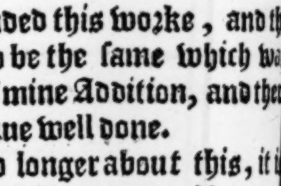


And so haue I ended this worke, and th summe appeareth to be the same which wa the second summe of mine Addition, and there fore I perceiue I haue well done.

Another way of Addition

Master. To stand longer about this, it i but folly, except that this you may also vnde stand, that many do beginne to subtract wit counters, not at the highest summe, as I hau taught you, but at the nethermost, as they d vse to adde, and when the summe to be abate in any line appeareth greater then the other then doe they borow one of the next higher roome, as for example.

If I should abate 1846 from 2378, they set the summes thus.



Fi

First they take 6, which is the lower line, and his space from 8 in the same roomes in the second summe, & yet there remaineth 2 counters in the lowest line. Then in the second line must 4 bee subtracted from 7, & so remaineth there 3. Then 800 in the third line and his space, from 300 of the second summe cannot bee, therefore doe they bate it from a higher roome, that is, from 1000: and because that 1000 is too much by 200, therefore must I set downe 200 in the third line, after I haue taken vp 1000 from the fourth line. Then is there yet 1000 in the fourth line of the first summe, which if I withdraw from the second summe, then doe all figures stand in order, thus. 532.

So that (as you see) it differeth not greatly whether you begin Subtraction at the higher lines or at the lower.

Howbeit, as some men like that one way best, so some like the other: therefore you now knowing both, may vse which you list.

Multiplication.

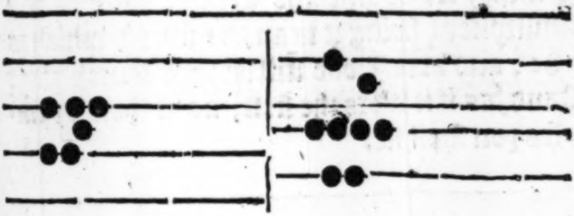
But now touching Multiplication: you shall set your numbers into two roomes (as you did in those other kinds) but so that the Multiplier be set in the first roome, then shall you begin with the highest numbers of the second roome, and multiply them first after this sort.

Take the ouermost line in your first working, as it were the lowest line, setting on it some moueable marke (as you list) and looke how many counters be in him, take them vp, and for them set downe the whole multiplier so many times as you tooke vp counters: reckoning (I say) that line for the vnites. And when you haue done with the highest number, then come to the next line beneath, and do so euen with it, and so with the next, till you haue done all. And if there be any number, in a space, then for it shall you take the Multiplier 5 times: and then must you recken that line for the vnites, which is next beneath that space. Or else after a shorter way, ye shall take onely halfe the multiplier, but then shall you take the line next aboue the space for the line of vnites. But in such working, if by chance your Multiplier bee an odde number, so that you

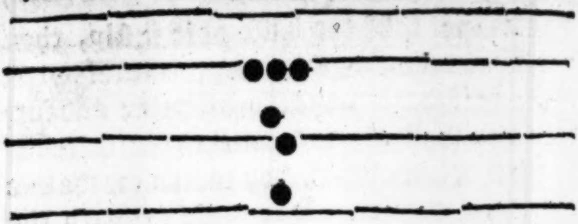
you cannot take the halfe of it iuſtly, then muſt you take the greater halfe and ſet downe that, as if that it were the iuſt halfe: and further you ſhall ſet one counter in the ſpace beneath that line, which you recken for the line of vnites, or elſe onely remoue forward the ſame that is to be multiplied.

Scholar. If you ſet forth an example hereof, I thinke I ſhall perceiue you.

Maſter. Take this example: I would multiply 1542 by 365, therefore I ſet my nūbers thus.

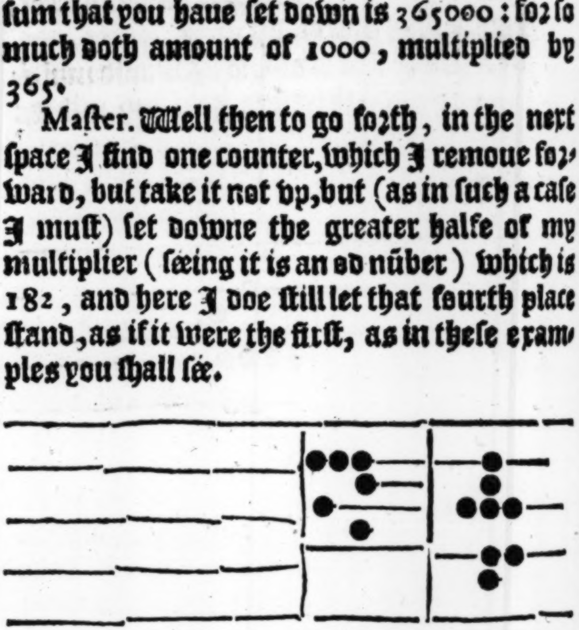


Then firſt I begin at the 1000 in the higheſt roome as if it were the firſt place, & I take it vp ſetting downe for it ſo often (that is once) the multiplier, which is 365, thus as you ſee here: where, for the one counter taken vppe from the fourth line, I haue ſet down other 6, which make the ſum of the multiplier, reckning the fourth line, as if it were yͤ firſt, which thing I haue marked by the ſtarre ſet at the beginning.



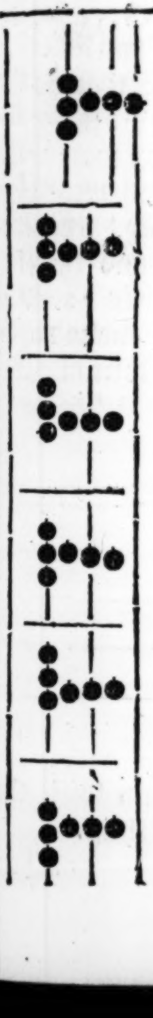
Scholar. I perceiue this wel, for indéed this summe that you haue set down is 365000: for so much doth amount of 1000, multiplied by 365.

Master. Well then to go forth, in the next space I find one counter, which I remoue forward, but take it not vp, but (as in such a case I must) set downe the greater halfe of my multiplier (séeing it is an od nũber) which is 182, and here I doe still let that fourth place stand, as if it were the first, as in these examples you shall sée.



Where I haue set the multiplication with other; but for the ease of your vnderstanding, I haue set a little line betwéene them. Now should they both in one summe stand thus.

How



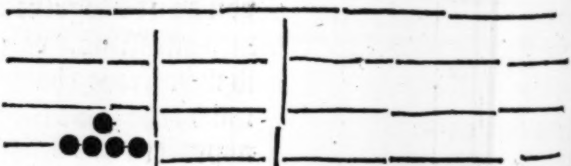
Howbeit another forme to multiply such Counters in space is this: first to remoue yͤ finger to the next line beneath the space, & then to take vp yͤ Counter, & to set downe the multiplier 5 times: as heere you see.

Which summes if you do add together into one sūme, you shal perceiue that it wil be the same that appeareth of yͤ other working before, so that both sorts are to one intent: but as yͤ other is shorter,

so this is plainer to reason, for such as haue had small exercise in this Art.

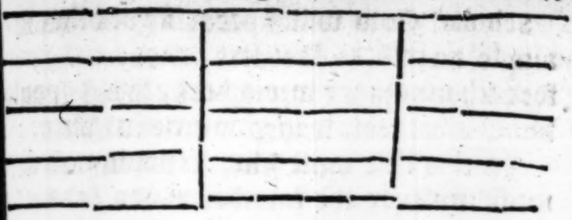
Notwithstanding you may adde thē in your mind before you set them down: as in this example you might haue said, 5 times 300 is 1500, & 5 times 60 is 300, also 5 times 5 is 25, which al put together do make 1825, which you may at one time set downe if you list.

But now to goe forth, I must remoue the hand to the next counters which are in the second line, & there must I take vp those foure counters, setting down for them my multiplyer 4 times seuerally, or else I may gather the whole summe in my mind first, and then set it downe: as to say, 4 times 300 is 1200. 4 times 60 are 240: and 4 times 5 make 20, ỹ is in all 1460: that shall I set downe also, as here you see.

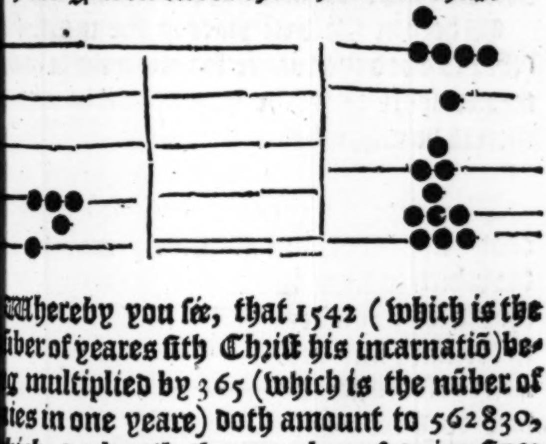


Which if I ioyne in one sum with the former numbers, it will appeare thus.

Then



Then to end this Multiplication, I re- moue the finger to the loweſt line, where are nely 2, them doe I take vp, and in their ſtead o I ſet downe twice 365, ẏ is 730 for which I ſet one in the ſpace aboue the 3 line for 500 1 more in the third line with that one that there already, and the reſt in their order, & haue I well ended the whole ſum thus.



Whereby you ſée, that 1542 (which is the uber of yeares ſith Chriſt his incarnatiõ) be- g multiplied by 365 (which is the nũber of ies in one yeare) doth amount to 562830, hich declareth the number of daies ſince riſts incarnation vnto the end of 1542 ars, beſide 385 daies and 12 houres for p yeares.

Scholar

Scholar. Now will I proue by another example, as this, 40 laborers) after 6 d the day for each man) haue wrought 28 days, I would know what their wages do amount vnto.

In this caſe muſt I worke doubly: firſt I muſt multiply the number of the laborers by the wages of a man for one day, ſo will the charge of euery day amount.

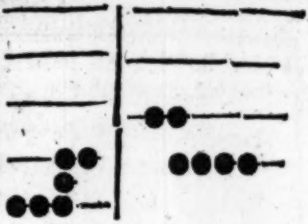
Then ſecondarily ſhall I multiply the charge of 1 day by the whole nũber of dayes, & ſo will the whole ſum appeare: firſt therefore I ſhall ſet the ſummes thus.

Wherein the firſt place is the multiplier (that is one days wages for one man) & in the ſecond ſpace is ſet the number of the workemen to be multiplied.

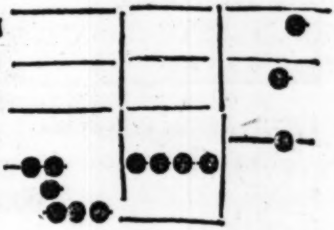
Then ſay: If 6 times 4 (reckoning that ſecond line of the line of Units) maketh 24, for which ſumme I ſhould ſet 2 coũters in the third line and 4 in the ſecõd, therefore do I ſet 2 in ỹ third line, and lette the foure ſtand ſtil in the ſecond line thus.

So appeareth the whole dayes wages

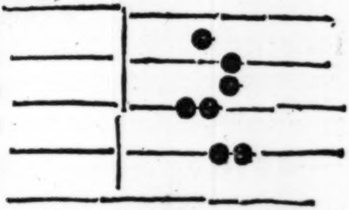
s 240 d, that is 20 s.
Then do I multi-
y againe the same
m by the number
dates, and first I
tte the numbers
us, then because



ere are Counters
diuers lines I shall begin with the high-
t, and take them
p, setting for them
e multiplier so
any times as I
oke vp counters,
at is twice, then
ill the sum stand
us.



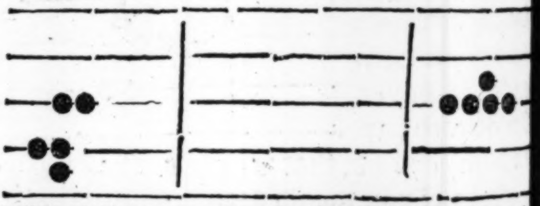
Then come I to the second line, and take
p those foure counters, setting for them the
ultiplier foure times,
will the whole
m appear thus.
So is the whole
ages of 40 work-
en for 28 dayes
fter 6 d each day



a man) 6720 d, that is, 560 s, or 28 l.
Now if you would proue Multiplicati-
n, the surest way is by Diuision, therefore
ill I ouerpasse it, til I haue taught you the
rt of Diuision, which you shall work thus.

FIrst set dow the Diuisor, f feare of forget ting, & then set number th shall be diuid at ẏ right side far from the d uisor, that t Quotient ma be set betwe them, as for example.

If 225 sheepe cost 45 l', what did euery sh cost? To know this I would diuide the who sum, that is 45 l', by 225, but that cannot be therefore must I first reduce that 45 l'. into lesser denomination, as into shillings; then multiply 45 by 20, and it is 900: that sum sh I diuide by the number of shéep which is 22 these two numbers therefore I set thus.

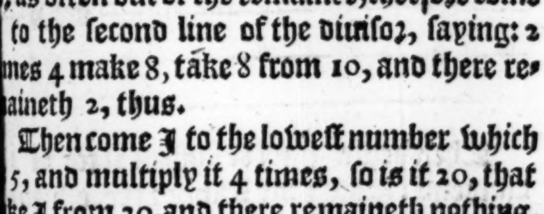


Then begin I at the highest line of the di dend therein, and that I may doe foure tim then say I four times 2 are 8, which if I ta from 9, there resteth but 1 thus.

A

And because I found the diuisor 4 times in he diuidend, I haue set, as you see, 4 in the iddle rowme which is the place of the quoti- nt: but now must I take the rest of the diui- r as often out of the remainer, therfore come to the second line of the diuisor, saying: 2 mes 4 make 8, take 8 from 10, and there re- aineth 2, thus.

Then come I to the lowest number which 5, and multiply it 4 times, so is it 20, that ke I from 20, and there remaineth nothing, that I see my quotient to be 4, which are in alue shillings, for so was the diuidend: and ereby I know that if 225 sheepe did cost 45 l. ery sheepe cost 4 s.



Scholar. This can I doe, as you shall per

ceiue

ceiue by this exãple. If a 160 soldiers do spen[d] euery moneth 68 l', what spendeth each man[?]

Example of wages.

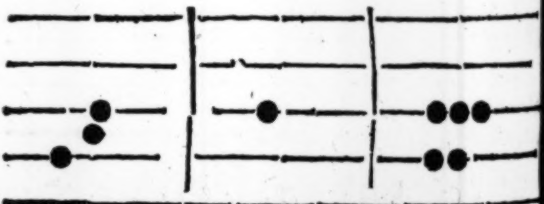
First because I cannot diuide the 68 by 160 therefore I will turne the l', into pence by mul[-]tiplication, so shall there be 16320 pence: no[w] must I Diuide this summe by the number [of] soldiers, therefore I set them in order thus.

Then begin I at the highest place of th[e] diuidend, seeking my Diuisor there, which I finde once, therefore I set 1 in the neather lin[e]

Master. Not in the neather line of the whol[e] summe, but in the neather line of that work[e] which is the third line.

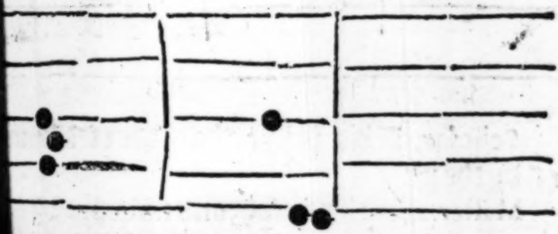
Scholar. So standeth it with reason.

Master. Then thus doe they.



The

Then seeke I againe the rest: how often I may find my diuisor: and I see that in 300 I might find 100 three times: but then the 60 will not bee so often found in 20, therefore I take 2 for my quotient: then take I 100 twice from 300, and there resteth 100, out of which with the 20 that maketh 120, I may take 60 also twice, and then stand the numbers thus.



Where I haue set the quotient 2 in the lowest line: So is euery souldiers portion 102 d, that is 8 s, 6 d.

Master. But yet because you may iustly perceiue the reason of Diuision, it shall bee good that you set your Diuisor still against those numbers from which you doe take it, as by this example I will declare.

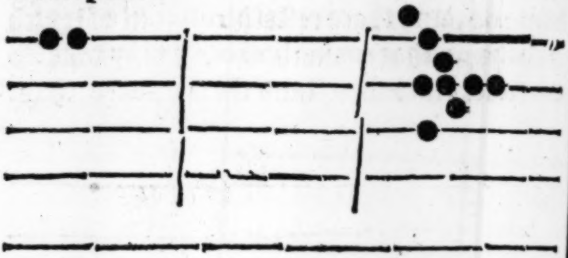
If the purchase of 200 acres of ground did cost 290 pound, what did one acre cost? Exãple of purchase.

First will I turne the pounds into pence, and will there be 69600 pence. Then in setting downe these numbers I shall doe thus.

First set the diuidend on the right hand as

ought

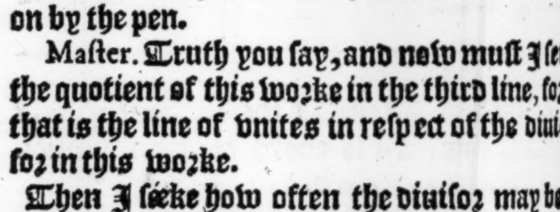
ought, and then the diuiſor on the left hand againſt thoſe numbers from which I intend to take him firſt, as heere you ſee where I haue ſet the Diuiſor two lines higher then his owne place.



Scholar. This is like the order of Diuiſion by the pen.

Maſter. Truth you ſay, and now muſt I ſet the quotient of this worke in the third line, for that is the line of vnites in reſpect of the diuiſor in this worke.

Then I ſeeke how often the diuiſor may be found in the diuidend, and that I find 3 times, then ſet I 3 in the third line for the quotient, and take away that 6000 from the diuidend, and further I ſet the Diuiſor one line lower, as you ſee heere.



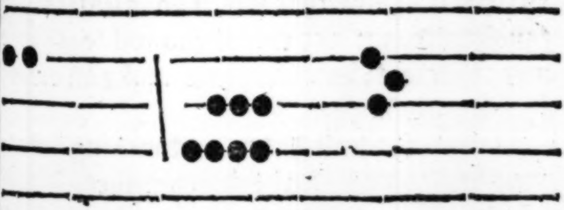
And then seeke I how often the Diuisor will be taken from the number against it, which will be foure times and 1 remaining.

Scholar. But what if it chaunce that when the Diuisor is so remoued, it cannot be once taken out of the Divident against it?

Master. Then must the Diuisor be set in another line lower.

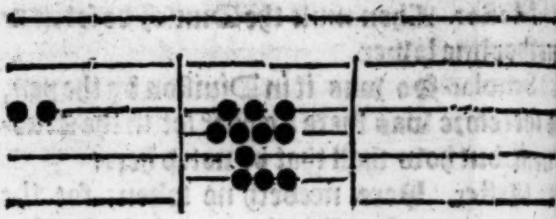
Scholar. So was it in Diuision by the pen, & therefore was there a cipher set in the Quotient: but how shall that be noted here?

Master. Here needeth no token, for the lines do represent the places, only looke, that you set your Quotient in that place which standeth for vnites in respect of the Diuisor. But now to returne to the example: I finde ye Diuisor 4 times in the Divident, & 1 remaining: for 4 times 2 makes 8, which I take from 9, and there resteth 1, as this figure following sheweth: and in the middle space for the Quotiēt, I set 4 in the second line, which is in this worke the place of vnites.



Then remoue I the Diuisor to the next

lower line, and ſeeks how often I may haue it in the Diuidend, which I may do here 8 times iuſt, and nothing remaine, as in this forme.



Where you may ſee that the whole Quotient is 348 d, that is 29 s, whereby I know that ſo much coſt the purchaſe of one acre.

Scholar. Now reſteth the proofes of Multiplication, and alſo Diuiſion.

Maſter. Their beſt proofes are each one by the other, for Multiplication is proued by Diuiſion, and Diuiſion by Multiplication, as in the worke by the pen you learned.

Scholar. If that be all, you ſhal not neede to repeate againe that which was ſufficiently taught already: and except you wil teach me any other feat, here may you make an end of this Art, I ſuppoſe.

Maſter. So will I doe as touching whole number, and as for broken number, I will not trouble your wit with it, till you haue practiſed this ſo well, that you be full perfect

so that you need not to doubt in any point that I haue taught you, and then may I boldly instruct you in the Arte of Fractions or Broken numbers: wherein I will also shew you the reasons of all that you haue now learned. But yet before I make an end, I will shew you the order of common casting, wherein are both pence, shillings, and pounds, procéeding by no grounded reason, but only by a receiued forme, and that diuersly of diuers men: for the Merchants vse one forme, and Auditors another.

Merchants vse.

But first for Merchants forme, marke this example here, in which I haue expressed this summe 198 l, 19 s 11 d. So that you may see that the lowest line serueth for pence, the next aboue for shillings, the third for pounds, and the fourth for scores of pounds.

And further you may see that the space betwéene d and s may receiue but one counter (as all other spaces likewise doe) and that one standeth in that place for 6 d.

Likewise betweene the shillings and the pounds, one counter standeth for 10 shillings

And betwene the pounds and 20l', one Counter ſtandeth for 10l'.

But beſide thoſe, you may ſee at the left ſide of ſhillings that one number ſtandeth alone, and betokeneth 5 s.

So againſt the pounds, that one Counter ſtandeth for 5 l'. And againſt the 20 poundes, the one counter ſtandeth for 5 ſcore poundes, that is, 100 pounds, ſo that euery ſide counter is fiue times ſo much as one of them againſt which he ſtandeth.

Auditors account.

NOw for the account of Auditors, take this example.

Where I haue expreſſed the ſame ſumme 198l'——19 s—11 d:

But heere you ſee the pece ſtand towards the right hand, & the other increaſing orderly toward the left hand .

Againe you may ſee, that Auditors will make two lines (yea and more) for pence, ſhillings, and all other values, if their ſumme extend thereto. Alſo you ſee, that they ſet one counter at the right end of each row, which ſo ſet there, ſtandeth for fiue of that rome: and o

on the left corner of the rowe it standeth for 10 of the same row.

But now, if you would adde, either subtract after any of both those sorts, if you marke the order of the other feate which I taught you you may easily doe the same heere without much teaching: for in addition, you must first set downe one summe, and to the same set ye other orderly, and in like manner if you haue many, but in Subtraction, you must set down first the greatest summe, and from it must you abate the other, euery denomination from his due place.

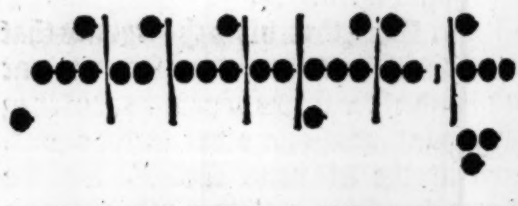
Scholar. I doe not doubt but with a little practise I shall attaine these both: but how shall I multiply and diuide after these formes?

Master. You cannot duly doe any of both by these sorts, therefore in such case you must resort to your other Artes.

Scholar. They that vse such accounts that exceed 200 in the summe they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in an other farther row, and 500 at the left hand thereof, and the thousands they set in a farther row yet, and at the left side thereof they set the 5000, and in the space ouer they set the 10000, and in a higher row 20000, which all I haue expressed in this example, which is 9869 l, 12 s, 9 d, ob, q. Ninetie seuen thousand, eight hundred, three score and nine pounds,

twelue shillings and 9 pence halfpeny farthing, for I had not told you before where, neither how you should set downe farthings, which (as you see here) must be set in a void place sideling beneath ye pence, for farthing one counter, for ob 2 counters, for ob. farthing, 3 counters & more there cannot be: for 4 farthings make 1d which must be set in his due place.

And if you desire the same summe after Auditors maner lo here it is.



But in this thing you shall take this t sufficient, and the rest you shal obserue a you may see by the working of each sort, f the diuers wits of men haue inuented diu and sundry waies, almost innumerable.

The second part of Arithmetike, touching Fractions, briefly set forth.

Scholar.

Albeit I perceiue your manifold busines doth so occupy or rather oppresse you, that you can not as yet completely end the treatise of Fractions Arithmeticall, which you haue prepared, wherin not onely sundry works of Geometrie, Musicke, and Astronomy be largely set forth, but also diuers conclusions and natural workes, touching mixtures of metals, & compositions of medicines, with other strange examples: Yet in the mean season, I cannot stay my earnest desire, but importunely craue of you some briefe preparation toward the vse of Fractions, whereby at the least I may bee able to vnderstand the common works of thē & the vulgar vse of those rules, which without them cannot well be wrought.

Master. If my leasure were as great as my will is good, you should not néed to vse any importunate crauing, for the attaining of that thing whereby I may be perswaded that

I shall any waies profit the common wealth or helpe of the honest studies of any good mẽbers in the same: wherefore, while mine attendance will permit me to walke and talke, I am well willing to helpe you as I may.

What a fraction is. Therefore first to begin with the explication of this name, Fraction, what take you it to bee?

Scholar. Mary Sir, I think a Fraction (as I haue heard it often named) to bee a broken number, that is to say, to be no whole number but a part of a number.

Master. A Fraction indeed is a broken number, and so consequently the part of another number: but that must be understood of such another number, as cannot bee diuided into any other parts then Fractions, for although I may take the third part of 60 or the fourth part of it and so of other parts diuersly, yet these parts bee not properly, nor ought not to be called Fractions, because they may bee expressed by whole numbers; for the third part of it is 20: the fourth part is 15, ỹ twelfth part is 5, and so forth of other parts, which all be whole numbers.

What a Fraction is properly. Wherefore properly a Fraction expresseth the parts or part onely of a vnite, that is to say, that the number which is the whole or entire summe of any Fraction, may not bee greater then one: and therefore it followeth that no one Fraction alone can be so greate, that

that it shall make 1. as by examples I will declare as soone as I haue taught you to know the forme how a Fraction is expressed or represented in writing.

Numeration.

BUt first to begin wt expressing of a Fraction, which is the Numeration of it: you must vnderstand that a Fraction is represented by 2 numbers, set one ouer the other and a line drawne betweene them, as thus, $\frac{1}{3}$. $\frac{3}{4}$ $\frac{2}{5}$. $\frac{10}{17}$, which 4 fractions you must pronounce thus: $\frac{1}{3}$ one third part, $\frac{3}{4}$ 3 quarters, $\frac{2}{5}$ two fift parts, $\frac{10}{17}$, ten 17 parts.

Scholar. I vnderstand the forme of their expression and pronunciation: but their meaning or valuation seemeth more obscure. Yet I thinke that by the two first fractions I vnderstand the valuation of the two latter fractions, and consequently of other.

Master. Value them then, that I may perceiue your taking of them.

Scholar. $\frac{2}{5}$ Betokeneth two fift parts, that is to say, if one be diuided into 5 parts, that fraction doth expresse two of those fiue parts: $\frac{10}{17}$ doth signifie, that if one be diuided into 17 parts

parts, I must take ten of them. And this I gather of the two first examples: for $\frac{1}{3}$ that is one third part, doth easily declare, that if any one thing be diuided into three parts, I must take out one of them: so $\frac{3}{4}$, that is three quarters, doth declare that one beeing diuided into foure quarters, I must take (for this Fraction) three of those quarters.

If there be no more difficultie in their Numeration, then I pray you goe forward to their Addition and Subtraction, and so to the other kinds of works. For I vnderstand that the same kinds of works bee in Fractions, that bee in whole numbers.

Master. There are the same kinds of works in both, albeit the order of them is diuerse, as I will anon declare: but yet more in Numeration before we leaue it. You must vnderstand; that those two numbers which expresse a Fraction, haue seuerall names. The ouermost which is aboue the line is called the Numerator, and the other beneath the line, is called the Denominator.

Scholar. And what is the reason of their diuers names? (For, in mine opinion, both be Numerators, seeing both they doe expresse the numeration of the Fraction.)

Master. You are deceiued: for one onely (which is the ouermost) doth expresse the Numeration: & the Denominator doth declare the number of parts, into which the vnite is

diuided

diuided, as in this example, when I say: Diuide a pound weight of Gold betweene 4 men, so that the first man shall haue $\frac{2}{15}$ the second $\frac{3}{15}$, the third $\frac{4}{15}$, and the fourth $\frac{6}{15}$.

Now doe you perceiue that by the Denominator (which is one in all foure fractions) it is intended that the pound waight should be diuided into so many parts, I meane 15, & by the foure seuerall Numerators is limited the diuers portion that each man should haue, that is, that when the whole is parted into 15, the first man shal haue two of those 15 parts: the second man three of them: ẏ third man 4: and the fourth man 6. And so may you see the seueral offices (as it were) of these two numbers, I meane of the Numerator and the Denominator.

And hereby you perceiue that a man can haue no more parts of any thing then it was diuided into, neither yet aptly so many: so that it were vnaptly said: You shall haue $\frac{15}{15}$ that is 15 parts of any thing, seeing it were better said you shall haue the whole thing.

Scholar. So doth it appeare reasonably, for the labour is vaine to diuide any thing and then to apply the Diuision to no vse. And much lesse reasonable were it to say $\frac{16}{15}$: for if the whole be diuided into 15 parts only, it is not possible to take 16 of them, that is to say, more then another.

Master. This is true touching the proper and

and apt vse of the name of a Fraction: yet improperly, and after a vulgar acceptation (for easinesse in worke) both those formes be called Fractions, because they be written like Fractions, although they be none in deed: for $\frac{15}{15}$, and generally all such other, where the Numerator and Denominator be equall, are not Fractions, but the whole thing with all his parts. And so $\frac{14}{12}$ is not to be called a fraction, but a mixt number, of a whole number & a fraction, for it is as much as 1 $\frac{2}{12}$, that is one whole one & $\frac{2}{12}$ parts, as shall be declared in Reduction. Therefore they do abuse ẏ names that call them fractions where the numerator is either equal or greater thē the denominator.

Scholar. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in worke write some numbers after that sort like fractions: but they needed not to call them fractions, but (as they bee) whole numbers or mixt numbers (that is, whole numbers with Fractions) expressed like fractions.

Now must you understand, that as no fraction properly can be greater then 1, so in smalnesse under one the nature of Fractions doth extend infinitely as the nature of whole numbers is to increase aboue one infinitely, so that not only one may be diuided into infinite fractions or parts, but also euery fraction may

bee

be diuided into infinite Fractions or parts, which commonly be called Fraction of fractions, and they be expressed diuersly: as for example $\frac{3}{4}$ of $\frac{2}{3}$ that is, three quarters of two third parts of one halfe part. Whereby is signified, that if one be diuided into two halfes, and the one halfe into three parts, and two of those three parts be diuided iointly into 4 quarters, this fraction of fractions doth represent three of those quarters.

Scholar. I pray you let me proue by an example in common money, whether I doe rightly vnderstand you or no. One Crowne, which I take for an vnite, doth containe 60 d, therefore the halfe of it is 30 pence: $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is fifteene pence, so then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a shilling.

Master. You perceiue this well enough: yet this note I giue you by the way, that the forme of expressing these fractions is voluntarie, and hath no other reason than the will of the Diuisor, which forme many follow: for some expresse them thus $\frac{3}{4}\frac{2}{3}\frac{1}{2}$ without any figure of distinction betwen them, which forme many follow. Some other do make lines betwene euery fraction, and adde words of distinction, after this sort $\frac{3}{4}$, of $\frac{1}{2}$, which forme is best.

Some

Some other expresse them thus in slope forme, to distinct them from severall Fractions of one whole number, for if $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ they were set in one right line thus, $\frac{3}{4}\frac{2}{3}\frac{1}{2}$: then ought it to be pronounced, three quarters, & two third parts and a halfe, which maketh almost two whole unites, lacking but one xii part. And so is it nothing agreeable with the other Fraction of Fractions: wherefore it is a great oversight in certain learned men, which do expresse them so confusedly with such severall fractions, that a man cannot know the one from the other.

Therefore some men (as Stifelius) doe expresse without a line, numbers of proportion, being applied to Addition or Substraction, because they must bee taken as two, where the line in Fractions maketh them to be taken for one: for of the Numerator and denominator is made one number.

Scholar. Then I perceive there be three severall varieties in fractions: first, when one onely fraction is set for one number, as $\frac{4}{5}$, that is, foure fifth parts. The second is, when there be set two or more severall fractions of one number: as $\frac{4}{9}\frac{2}{5}$, that is foure ninth parts, and two fift parts. The third sort is fractions of Fractions, as $\frac{4}{9}$ of $\frac{2}{5}$, that is foure ninth parts of two fifth parts.

Master. You have said well, if you under-

stand

ſtand well your owne words.

Scholar. If it ſhall pleaſe you I will by an example in the parts of an old Engliſh Angell expreſſe my meaning.

Master. Let mee heare you.

Scholar. The old Engliſh Angell did containe 7 ſhillings 6 d, that is 90 d. Now $\frac{4}{5}$ of it is 72 d. And of the ſame 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is foure nine parts, and 2 fifth parts, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that is 4 nine parts of two fifth parts, ſeeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yeeld but 16, for $\frac{1}{9}$ of 36, is but 4, and that taken 4 times maketh 16.

Master. This is plainly expreſſed, and truly, and hereby (I doubt not) but you doe perceiue, that as great a difference as is betweene 16 and 17, ſo much difference is betweene thoſe two fractions $\frac{4}{9}$ & $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that you vnderſtand theſe varieties, I will proceede to the reſt of the workes: firſt admoniſhing you, that there is another order to be followed in Fractions, then there was in whole numbers: for in whole nūbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diuiſion, and Reduction: but in Fractions (to follow the ſame aptneſſe in proceeding from the eaſieſt works to the harder) wee muſt vſe this order of the workes, Numeration, Multiplication, Diuiſion, Reduction, Addition, and Subtraction

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ction.

Scholar. That Multiplication and Diuision should go together, and Subtraction to follow Addition, naturall order doth perswade, but why Multiplication should bee first in order here next to Numeration, & Reduction in the middle, I desire to vnderstand the reason.

Master. As in the Arte of whole numbers, Order would reasonably begin with the easiest and so go forward by degrées to the hardest: euen so reason teacheth in Fractions ÿ like order. And, considering that Addition or Subtraction of Fractions can very seldome be wrought without Multiplication and Reduction: and contrariwise, Multiplication and Reduction may be wrought without this forme of Addition or Subtraction: therefore was it orderly required, that multiplication and reduction should go before Addition and Subtraction, and the same reason serueth for the placing of Multiplication before Reduction.

Scholar. Then if multiplication be the easiest, I pray you declare the forme of it first by Rule and then by example.

Master. Your example is good.

Multi-

Therefore when any two fractions be proponed to be multiplied together, the numerator of the one must bee multiplied by the numerator of the other: and the summe that amounteth therof must bee set for a new numerator: likewise the Denominator of the one must be multiplied by the denominator of the other, & that that first amounteth, shall be set for the common denominator: & this new third fractiō expresseth the product of ẏ multiplicatiō of the two first fractiōs proponed, whereof take this example, $\frac{3}{5}$ multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$.

Scholar. I perceiue then that 3. beeing the numerator of the first fraction, is multiplied by 5. being the numerator of ẏ second fraction, whereof amounteth 15. the numerator of the third fraction. And so likewise, 5 being denominator of the first fractiō, is multiplied by 12 the denominator of the second fraction, whereof amounteth 60 the new denominator, so that I perceiue how the work is done, but I do not perceiue how $\frac{15}{60}$ is greater than $\frac{3}{5}$: for I shall not vse my former manner of examination by the parts of some coine. I see that $\frac{3}{5}$ of a crowne is 36 d, & $\frac{5}{12}$, of a crowne, is 25 d, whereof ẏ one

multiplied by the other, doth make 900 d, which is 15 Crownes: but by your Multiplication there amounteth $\frac{15}{60}$, which is but 15 pence, and that is much lesse then any other of both the first fractions.

Master. That difference is betweene multiplication in whole numbers, and multiplication in broken numbers, that in whole numbers the summe that amounteth is greater then both the other whereof it came: but in fractions it is contrariwise: for the summe that amounteth is lesser then any of the other two fractions whereof it came.

Scholar. I desire much to understand the reason thereof.

Master. Although I purposed to reserve the reasons of workes Arithmeticall for the perfect booke of Arithmeticke, yet I will shew you this, because of the strangenesse of the worke.

You see in whole numbers, that of two numbers being multiplied together, is made the third number, which third number doth beare the same proportion to the number multiplied, that the multiplier doth beare to an unite. And so in fractions the third number which amounteth of multiplication, beareth the same proportion to each of the two first fractions, that the other of those two fractions doth beare to an unite.

Scholar. Sir, I understand your words thus:

thus: when 40 is multiplied by 12, there doth amount 480, which 480 doth containe 40 so many times in it, as twelue doth containe vnites, that is to say, twelue times. And so it appeareth that 480 doth containe twelue so many times also as 40 doth containe vnites, that is, 40 times. But now I see not how the third number in this example of fractions can containe any of the two former (as it happened in whole numbers) seeing it is lesser then either of them.

Master. No maruell if you cannot see that thing which is not possible to bee seene of any man, how the third number in multiplication of Fractions should bee greater then any of the two former fractions: but yet this may you see (which I said) that the third number in Fractions so multiplied, doth beare the same proportion to any of the two former fractions, that the other of those two fractions doth beare to an vnite, as in your example, $\frac{3}{5}$ being multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$. Now say I, that $\frac{15}{60}$ doth beare the same proportion to $\frac{3}{5}$ that $\frac{5}{12}$ doth beare to an vnite, as you may in your owne forme of examination by coin trie it: for in an old angel (which in times past was currât for 7 s 6 d, are 180 halfpence) which I set for the intire vnite, whose parts according to the fractions aforesaid) are these, for $\frac{15}{60}$ set 45 ob, for $\frac{3}{5}$ take 180 ob, and for $\frac{5}{12}$ put 75 ob. Now doth 45 beare the same pro-

portion to 108 that 75. doth beare to 180. for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better reserued till another time, when the knowledge of proportions in due order shall be taught. Yet in the meane season I will shew you how it commeth to passe, that in fractions the third sum must néedes bée lesser than any of the other two.

Consider this, that when a fraction is proponed, as in the former example $\frac{3}{5}$ if it be multiplied by more than 1, it wil make more than one entire number. As if I multiply $\frac{3}{5}$ by 5: that is to say, if I take it 5 times, it will make thrée entire vnities: example in a crown, $\frac{3}{5}$ of it maketh 3 s, which if I take fiue times it will amount to 15 s, that is thrée entire Crownes: so if I take the same $\frac{3}{5}$ but twice, it will yeeld 6 s, that is one entire Crowne, and $\frac{1}{5}$. Now if I take it but once, it cannot bée more than it was before, that is 5 s. And if I take it lesse than once, it cannot bée so much as it was before. Then séeing that a fraction is lesse than one, if I multiply a fraction by another fraction; it followeth that I doe take the first fraction lesse than once, and therefore the summe that amounteth, must néedes be lesse than the first fraction.

Scholar. Sir, I thanke you much for this reason. And I trust I doe perceiue the thing, as by example of this same fraction $\frac{3}{5}$ I wil expresse.

presse. If I take $\frac{3}{5}$ of a crowne once, that is to say, if I multiply $\frac{3}{5}$ by 1, it will bee as it was before, but 3 s: so if I doe multiply it by $\frac{1}{2}$, that is, if I take but halfe one time, then will it be but halfe so much: likewise if I multiply it by $\frac{1}{3}$, that is, if I take but the third part of one, it will yeeld but 12 pence, that is, the third part of the first fraction.

And so to make an end: If I take it but the twelfth part of once, that is, if I doe multiply it by $\frac{1}{12}$, it will yeeld but the twelfth part of the first fraction, which is but three pence. And it followeth, that if $\frac{1}{12}$ make 3 pence, then $\frac{5}{12}$ must needes make fiue times so much, that is, 15 pence, which was the summe that hath giuen the occasion of all this doubt.

Master. Then I perceiue you haue sufficient vnderstanding in this sort of Multiplication for this time, wherefore I will omit that I might say more of Multiplication, till wee come to reduction, and will passe to the other workes, and first to Diuision, whose place followeth Multiplication, both by naturall order, and also in easinesse of worke.

Diuision.

WHensoeuer two fractions bee proponed, that the one should bee diuided by the other, I must set downe first the Fraction that shall bee diuided, (which is called the Diuident) and then after it the other, which is ẏ Diuisor. Then shall I multiply the numerator of the Diuident by ẏ Denominator of the Diuisor, and that which amounteth, I must put for a new Numerator. Againe, I shall multiplie the Denominator of the Diuident by the Numerator of the Diuisor, and the number that amounteth thereof, I must put for the new Denominator. And this third fraction is the Quotient of the said Diuision.

Scholar. This seemeth easie in forme, as by example thus: If I would diuide $\frac{5}{8}$ by $\frac{2}{6}$, first I must multiply 5 (being the Numerator of the Diuident) by 6, which is the Denominator of the Diuisor, and thereof riseth 30: then I multiply 8 (being the Denominator of the Diuident) by 2, beeing the Numerator in the Diuisor: and so riseth 16, the which I must make in a third Fraction, thus $\frac{30}{16}$.

Master. Me séemeth you are quicker in vnderstanding now, then you were when I taught you the Art of whole number: but that

that is no maruell: for the more knowledge that any man getteth, the readier shall he finde his wit, and quicker in vnderstanding: but yet of two things I will admonish you, which you might haue obserued heere for the ease of worke, and lightnesse of vnderstanding the nature of the Quotient.

Whensoeuer you diuide one Fraction by another, either they bee both equall together, either else the one is greater then the other: if they bee equall, their Quotient shall bee such, that the Numerator and the Denominator of it shall be equall also. And if the two first Fractions be vnequall, their quotient shall declare the same by the vnequality of the Numerator and Denominator, as in these examples following shall appeare.

First of equall Fractions: $\frac{4}{5}$ and $\frac{12}{27}$ be equall together, and if the one be diuided by the other, the quotient will be $\frac{108}{108}$, as you may perceiue by that rule aforesaid.

Now in the vnequall Fractions, as $\frac{4}{9}$ & $\frac{3}{10}$, the quotient will be $\frac{40}{27}$: where the numerator is greater then the Denominator.

Scholar. I see it is so: but I see not the reason why it should be so.

Master. The reason is this: When any Fraction is diuided by another, the quotient declareth what proportion the diuident beareth to the Diuisor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$, maketh 2, which must be sounded, not two, but twice,

Note how to know the proportion betweene 2 numbers

twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

And note this, that the Numerator in the quotient repreſenteth the diuidend, and the Denominator repreſenteth the Diuiſor. And this is alwaies true, whether the greater fraction be diuided by the leſſer, or the leſſer by the greater. But this proportion will not bee exactly knowne, till you haue learned the Art of proportions: notwithſtanding ſomewhat of it I will declare in the next rule of Reduction. But now for the eaſie remembrance of ye quotion in Diuiſion: as ſoone as you haue ſet downe your two fractions the one againſt the other, then make a ſtraight line for the quotient: and as ſoone as you haue multiplied the Numerator of the diuidend, by the Denominator of the diuiſor, ſet the number that amounteth ouer the ſaid line, and then multiply the other two numbers, and ſet their totall vnder the ſame line.

Scholar. I perceiue you would not haue me truſt to memorie till I were better expert, leſt oftentimes I happen by miſſe remembrance to bee abuſed. This example I take for that declaration.

If I would diuide $\frac{2}{3}$ by $\frac{3}{4}$, I muſt ſet the numbers one againſt the other, (as heere doth appeare) and then make another line for the quotient in ſome good diſtance, where I may ſet the numbers of the quotient, as ſoone as any of them

$$\frac{2}{3} \quad \text{by} \quad \frac{3}{4}$$

them is multiplied. So then as ſoone as I haue multiplied 2 by 4, which maketh 8, I ſhall ſet that 8 ouer that line, thus: And then multiply 3 by 3, which yeeldeth 9: and that 9 muſt bee ſet vnder the ſame line, and then will the whole quotient appeare thus $\frac{8}{9}$: whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$ as 3 is to 9: but how may I perceiue that?

$$\frac{8}{}$$

Maſter. Although you ſhall better perceiue it by the Rule of Reduction, yet this example may bee declared in common coines, as in a common ſhilling of xij d', of which $\frac{2}{3}$ maketh 8 d, and $\frac{3}{4}$ doth make 9 d, and ſo you may eaſily ſee that their proportions doe agree. And if you had taken this example before when you tooke the example of $\frac{5}{8}$ and $\frac{2}{6}$, your Quotient would appeare (as this doth) more eaſie to vnderſtand; whereas that quotient being $\frac{33}{16}$, is not an eaſie proportion for you to perceiue, being yet little acquainted with proportions: whereof to giue you ſome taſte, I will enter to the rule of Reduction: in which alſo I will declare other workes, both of Multiplication, and alſo of Diuiſion, which now I muſt for a time omit, as things that doe need the helpe of Reduction.

Reduction.

Reduction.

Fiue varieties of Reduction.

THerefore will I now declare the diuersities of Reduction of Fractions, which cōmonly haue 5 varieties.

1 When there bée sundry Fractions of one intire Unite, they must be reduced to one denomination, and also into one fraction.

2 Secondarily, when there be proponed Fractions of Fractions, they must be reduced likewise into one Fraction: for otherwise they cannot be brought into one denomination.

3 Thirdly, when an improper Fraction is proponed, yͭ is to say a fraction in forme, which indeed is greater then an vnite, it must bee reduced into apt forme expressing the vnite or vnites of it, & the proper fraction distinctly. And somtimes also it shalbe néedful to conuert such a mixt number of vnites with fractions into yͤ forme of a fraction, that is, into an improper fraction: which 2 formes I estéeme but as one, because they worke on one kinde of number.

4 Fourthly, there happeneth sometimes Fractions to bee written in great numbers, which might bee written in lesser numbers: therefore is there a meane to reduce such great numbers into their smallest termes.

5 Fifthly,

5 Fifthly, when any fraction betokeneth the parts of a whole thing, which hath by common partition certaine parts, but none of like denomination with that Fraction, then may you reduce the said Fractions into another, whose denomination shall expresse the common parts of that whole thing.

Scholar. This distinction in doctrine delighteth mee much, but more with hope then present fruit: for as yet I doe not understand scarcely the varieties, and much lesse the practise and use of their workes.

Master. Reduction is an orderly alteration of numbers out of one forme into another, which is never done orderly but for some needfull use, as in every of the said 5 severall varieties I will distinctly declare.

First therefore, when two or more severall fractions of any unite be proponed, as for example $\frac{3}{16}$ and $\frac{4}{6}$, because it is hard to tell what proportion of the intire number these two fractions doe expresse, therefore was Reduction devised, to be a meane whereby these severall fractions might be brought into one denomination and fraction.

And in these fractions this is the Art for bringing them to one denomination.

Multiply first the denominators together, and the totall thereof you shall set twice down under two severall lines for two new Denominators, or rather for one common denominator:

nator. Then multiply the numerator of the first fraction, by the denominator of the second, and set the totall thereof for the numerator ouer the first line. Likewise multiply the numerator of the second fraction by the denominator of the first, and set that totall ouer the second line for the numerator of that fraction, and so are those two first fractions of seuerall denominations, brought to one denomination.

Scholar. If I vnderstand you, as I thinke I doe, my example shall declare the same. The fractions which you proponed were these, $\frac{3}{16}$, and $\frac{4}{6}$, whose denominators (being 16 and 6) I multiply together, and there amounteth 96, which I set vnder two lines thus:

Then I multiply the numerator of the first fraction by the denominator of the second, saying, 3 into 6 maketh 18, that set I ouer the first line for a new numerator, and it will be thus: $\frac{18}{96}$.

Likewise I multiply the numerator of the second fraction by the denominator of the first, saying, 4 times 16 maketh 64, that I set for the second numerator, and the fraction will appeare thus.

So that both fractions brought to one denomination, must stand thus: $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You haue done well.

Scholar. I beseech you let mee examine it after my accustomed forme, by common parts of

of coine or other meaſure.

Maſter. Goe to.

Scholar. I haue a peece of Gold which is accounted worth 8 ſhillings, and containeth 96 pence, whereof $\frac{1}{16}$, that is, the 16 part, is 6 pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{96}$. Againe $\frac{1}{6}$ of the ſame peece of gold is 16 d, ſo that $\frac{4}{6}$ maketh 64 d, that is, $\frac{64}{96}$. And ſo I find the ſums to agree with the other before.

Maſter. So haue you now the Arte to bring two ſuch fractions into one denomination. And if there be more then two, then muſt you multiply all the denominators together, and ſet the totall thereof ſo many times downe as there bee fractions, and then to get for each one a new numerator, multiplie the Numerator of the firſt, by the Denominator of the ſecond, and the totall thereof multiplie by the denominator of the third and ſo foorth, if there be more. Likewiſe multiply the Numerator of the ſecond, by the Denominator of the firſt, and the totall thereof by the Denominator of the third. And in the ſame ſort multiplie the Numerator of the third into the Denominator of the firſt, and the totall thereof into the Denominator of the ſecond, and ſo foorth, if there were moe. So theſe three fractions $\frac{2}{5}\,\frac{3}{4}\,\frac{2}{3}$ doe make by reduction theſe other 3 fractions of one denomination $\frac{24}{60}\,\frac{45}{60}\,\frac{40}{60}$. All which you may bring into one fraction by adding the numerators together, and putting that totall for the

the common Numerator, reserving still that same common denominator. And those 3 fractions make one improper fraction thus, $\frac{109}{60}$.

Scholar. All this I perceive, and also that this last Fraction is more then an Unite, and therefore you did call it an improper Fraction.

Master. There bee certaine other formes of working in this Reduction, which I will briefly touch also, to give you an occasion to exercise your wit therein.

The first variety of this reduction.

The first varietie is this: When you have made and written downe your common Denominator (as I have taught before) then to get a Numerator for the first, do thus: Divide the common denominator by the denominator of the first Fraction, and the quotient multiplied by the numerator of the same yeeldeth a new Numerator for the first new Fraction. So likewise doe with the second and the third, and with all the residue, if there be more.

Scholar. That will I prove in your last example of these 3 Fractions, $\frac{2}{5}$, $\frac{3}{4}$, $\frac{2}{3}$. When the Denominators bee multiplied, they make 60: for 5 into 4 maketh 20, and 20 by 3 yeeldeth 60, that I set downe three times, thus: 60 60 60: then to have a numerator for the first I must divide 60 by 5, (the denominator of the first) and the quotient is 12, which I must multiply by 2 (the numerator of the first) and that maketh 24, and so have I for the first fraction

fraction $\frac{24}{60}$.

Likewise for the second fraction: I divide 60 by 4, and there commeth 15: which I multiplie by 3, and so haue I 45, and the second fraction $\frac{45}{60}$. Then for the third in like sort will come $\frac{40}{60}$.

Master. Another way is this: If it happen so, that the lesser Denominator can by any multiplication make the greater, then note the multiplier, and by it multiply the Numerator ouer that lesser denominator, and for the lesser Denominator put the greater, as thus in these two fractions, $\frac{3}{12}$ and $\frac{2}{3}$, thrée being the lesser Denominator multiplied by 4, will make 12, which is the greater denominator: therefore by the same 4 I doe multiplie 2, which is the numerator ouer 3, and that maketh 8: vnder which I do put 2 being the greater denominator, which is also made by multiplication of 4 into 3, and so haue I these two fractions, $\frac{3}{12}$, $\frac{7}{18}$, thus shortly reduced without altering the one fraction. The secōd variety.

Scholar. This I vnderstand.

Master. Then mark this third way: if the denominators doe not happen so, that one by multiplication may make the other, then looke whether they both may bee parts of any other one number, as in $\frac{5}{12}$, and $\frac{8}{12}$, although the lesser taken but twice, be too great to make 18, yet they both may be parts vnto 36: therefore looke how many times twelue is in 36, and The third variety.

that

that quotient being multiplied by the Nume-rator ouer 12, the totall ſhall be put in ſtead of the Numerator ouer 12, and for 5 put 15, thus, $\frac{15}{36}$. So likewiſe looke how often is 18 in 36, and becauſe it is twice, therfore by 2 multiply 7, which is ouer 18, and it will be 14, ſet that for the Numerator, and in ſtead of 18 put 36, and then ſhal your fractions reduced ſtand thus: $\frac{15}{36}\frac{14}{36}$ in ſtead of $\frac{4}{12}$ and $\frac{7}{18}$.

And if you will prooue whether you haue wrought well or no, that may be prooued by Reduction of them againe to their former denominations, which Art ſhall bee taught in the fourth kinde of Reduction, where greater termes of fractions be reduced into ſmaller in number, but no ſmaller in proportion. And if in ſuch Reduction the ſame termes or numbers come againe that were before, then is the worke good, elſe not.

Scholar. Sir I heare your words, but I doe not underſtand many of them, which it may pleaſe you to declare.

Maſter. With a good will, when conuenient place ſerueth, but that muſt be in the ſaid fourth kinde of Reduction. In the meane ſeaſon I will declare the ſecond forme of Reduction, which teacheth how to reduce fractions of fractions into one fraction, and ſo to one denomination.

When fractions of fractions bee proponed, you ſhall multiplie the Numerators of each

into other, and set the totall for the new numerator, and then multiply all the denominators likewise, and take their totall for the new Denominator, and so are they speedily reduced.

Reduction of fractiōs into one denomination.

Scholar. If that be all, then I understand it alreadie, as by this example I will declare. These bee the fractions, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$, which I would reduce to one denomination.

Therefore beginne I with the numerators, and multiply them together, saying: 3 into 2 maketh 6, and 6 by 6 maketh 36, which multiplied by 7, yeeldeth 252, that I set ouer a line for the numerator thus:

$$\frac{252}{}$$

Then I multiply the denominator, 4 by 3 maketh 12, and that by 7 bringeth 84. which multiplied by 9, yeeldeth 756, the new denominator. And so the whole reduced fraction is this, which is too hard a fraction for me to understand yet.

$$\frac{252}{756}$$

Master. You thinke so, and no maruell, but anon you shal learne to iudge it easily: for this fraction is no more indeed than $\frac{1}{3}$. although it be in greater termes, and therfore more stranger, and more obscure.

And this sufficeth for this Reduction, saue that I will shew you by a figure of measure, the iust rate and reason of this kinde of fractions, and also the due understanding of the reduction.

The entire measure parted into 9.

Heere you see the longest measure, (which standeth for the whole and entire quantitie) first parted into Diuisions 9, whereof 7 are seuered by the second measure: and thereof againe are parted out 6, and that 6 beeing distinct into three parts, two of them are parted by the fourth measure, of which fourth measure being diuided into foure parts, the lowest measure doth containe $\frac{3}{4}$, so that the same $\frac{3}{4}$ must be named, not $\frac{3}{4}$ of the whole measure, but indeed is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholar. This example is so sensible, that I cannot choose but see it. And furthermore, I see also that the same fraction is equall to $\frac{1}{3}$ of the entire measure, as the lines which run vp and downe do expresly set forth. Also I see heere that $\frac{2}{3}$ of $\frac{6}{7}$ is equall to $\frac{4}{7}$. And further yet that $\frac{2}{7}$ of $\frac{7}{9}$ is equall to $\frac{6}{9}$ or $\frac{2}{3}$.

Master. I am glad that you see it so well, not doubting but you wil gather greater light of knowledge hereby.

But

But now it is time that we come to the third forme of reduction, which teacheth of improper fractions, that is to say, mixt numbers of unites & fractions, although they appeare like fractions, as this, $\frac{26}{5}$, which doth include 5 unites wholy, and $\frac{1}{5}$ ouer. Wherefore first you shall know them, by that the numerator is greater than the denominator.

The third forme of Reduction of improper fractions.

Scholar. Indeed sir, that appeareth reasonable, that if the numerator do expresse more parts to be taken of any unite, than the denominator doth signifie that unite to be diuided into, it must needs follow that such a fraction importeth more than the whole, that is to say, the whole with certaine parts ouer. But what reduction is there in it?

Master. There be two seuerall kindes of Reduction, concerning such fractions. Sometimes it shall be needfull to conuert these fractions into unites, and the proper fractions that will remaine. And sometimes contrariwise, it shall be meet to reduce mixt numbers, that is, unites, written with fractions, into the forme of one simple fraction, and so bee there two waies.

Scholar. What is the meane of the first way to turne improper fractions into unites with their fractions?

Master. That is thus. Your Numerator being greater than the denominator, must be diuided by the same Denominator, and the

Reduction of improper fractions into vnites, with their proper fractions.

quotient thereof expresseth the vnities: the remainer shall be put for the numerator of the fraction that resteth, & the denominator must be the same that was before.

Scholar. For example, I take $\frac{17}{5}$. And diuiding 17 by 5, the quotient will be 3, & there will remaine 2.

Master. That must you write thus, $3\frac{2}{5}$ where (you see) I haue written 3, without any line, as entire numbers ought to be written, and the 2 that remained I haue set ouer the former denominator with a line, as a proper fraction. And this number doth signifie now 3 vnites, and $\frac{2}{5}$ of one.

Scholar. Then if I would by vnites here vnderstand crownes, so it were 3 crownes, and $\frac{2}{5}$, that is 2 s.

Master. Euen so, and therefore $\frac{17}{5}$ did signifie the same. But this happeneth sometime, that when the reduction is so wrought, there remaineth nothing. And then it is not a mixt number, but a simple entire number, represented like a fraction.

Reduction of whole numbers, either alone, or ioined with fractions into fractions.

Scholar. As $\frac{15}{5}$ will make 3 iust, and $\frac{18}{3}$ will make euen 6. This I will remember. But now, what is the second forme of reduction, that you speake of, for these sorts of fractions?

Master. Whensoeuer you haue any of these two sortes of numbers, that is to say, whole numbers without fractions, or whole numbers with fraction, and you would turne them

them into the forme of a fraction, you must multiply the whole number by that denominator which you will haue to remaine still, and to the totall thereof adde the numerator, which you haue already, and all that shall you set for the new numerator, keeping still the former denominator: as if you haue $6\frac{3}{4}$ which you would conuert into an improper fraction; you must multiply 6 by 4, whereof commeth 24, and thereto adde the numerator which is 3, and so haue you 27 for the numerator, and 4 stil for the denominator.

Scholar. Then is $\frac{27}{4}$ equal to $6\frac{3}{4}$.

Master. Euen iust, and so backward (as appeareth by the former reductiõ) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of these reductions may bee the proofe of the others worke.

Scholar. This I perceiue; but now if you would turne whole numbers without fractions into any fraction, I see not how that may be done, because there is no denominator to make the multiplication by.

Master. That was well marked: but this you know, that no man intendeth to turne any whole number into a fraction, but hee hath in his minde that denominator by which the multiplication must bee made: for the proofe whereof I set downe 7, which is a whole number. And if you will haue this number conuerted into any certaine fraction, will me to do it.

Scholar. I pray you reduce 7 into a fraction.

Master. Then you care not what the fraction be, so it be some fraction.

Scholar. No, I passe not for the sort of the fraction.

Master. Then how can you thinke that you require mee to doe any thing certaine, when you leaue me to doe as I list? And seeing you stand at that stay, whether thinke you that I must first intend in minde what fraction I will make of it, before I can doe it indeed?

Scholar. Else you should do ignorantly.

Master. Then will I limit my selfe (seeing you will not) to turne it into quarters. And therefore I multiply 7 by 4 (which is the denomination of quarters) and there amounteth 28 to be set for the numerator, and the 4 must be set for the denominator, and the fraction will be thus $\frac{28}{4}$.

Scholar. Indeed I perceiue this to be reasonable, for without much triall I understand that $\frac{28}{4}$ of any thing doth make 7. And so then if I would turne 8 into fiue parts, it wil make $\frac{40}{5}$, which is all one with 8. for 8 crownes turned into fiue parts (y^t is, into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a crowne.

Master. Seeing you understand now these three kinds of Reduction, I wil declare unto you the fourth kinde, that is, when fractions

be

be written in greater termes than they neede, how they may be brought to lesser termes.

Scholar. To write any thing in greater termes than needeth, seemeth to bee a fault, and so this rule seemeth to amend that fault.

Master. It were a fault to doe any thing without need, which after must bee redressed: but in this case it is not so; neither did I say absolutely (as you do) that it needeth not to expresse those fractions in so great termes; but that the fractions doe not neede, I meane for their value to be understood: but yet it may be needfull for the ease of these works whereto they be applyed, as for example: In the first kind of Reduction this was your owne example: $\frac{3}{16}$ and $\frac{4}{6}$, which when you would reduce, you were faine to turne them first into one denomination, and so appeared they thus, $\frac{18}{96}$ & $\frac{64}{96}$, where the fractions (for their owne understanding) needed not to be turned out of smaller termes into greater, but yet the easinesse of working needed it.

Scholar. Sir, I understand now, not onely the difference of this neede (for the fractions might better bee understood as fractions severall, each in his value, when they were in lesser termes, although they could not so well be reduced) but also I understand what you meane by greater termes and lesser termes, whereof before I was in doubt: for I see you

call the numerator and Denominator, the termes of the fraction.

Master. I am glad you vnderstand it so well. Now then when you would value any fractions (because that may best be don when the terms are smallest) you shall reduce them to the smallest that you can, which thing you may doe thus: Diuide the greatest of any such two termes by the lesser, and if any thing remaine, by that remainer diuide the last diuisor: and if any thing remaine now, by that diuide the last diuisor (which was before the remainer of the first diuision) and so continue still, till nothing doe remaine in the Diuision: and then marke you last diuisor, for it is the number that will easily reduce your fraction, if you diuide both the numerator and the denominator by the same number, and put for the numerator the quotient of his diuision, & for the denominator also his quotient, that riseth by his diuision.

Scholar. I take for example $\frac{18}{96}$, and because 96. is the greatest number, I diuide it by 18. and the quotient is 5. and there resteth 6: what shall I do with this quotient?

Master. Nothing in this worke, but now seeing there remaineth somewhat, by that remainer must you diuide the last diuisor.

Scholar. If I shall diuide 18. (which was the last diuisor) by 6. that was the remainer, so is the quotient 3, and nothing resteth.

Master.

Master. As for the quotient I omit him yet: but because there doth remaine nothing, therefore is 6 (which was your last diuisor) that number by which you may reduce the fraction proponed.

Scholar. Then as you taught me, I must diuide the numerator 18. by 6, & the quotient is 3, which I must put for the numerator ouer a line, thus: And then by the said 6, must I diuide also the denominator 96, & the quotient will be 16, which I must take for the denominator, and so is the fraction $\frac{3}{16}$. And so me thinketh this Rule doth proue the work of the first reduction.

$$\frac{3}{\quad}$$

Master. That is true if the first Reduction were made of fractions in their least tearmes, and else not, without some helpe, as the second number in that place will declare.

Scholar. The second number was $\frac{4}{6}$, which was turned into $\frac{64}{96}$ by that rule. Now if I shall by this rule reduce it againe into ÿ least termes, I must diuide 96 by 64, and there remaineth 32. Wherefore I must take that 32 for the diuisor, to reduce the said fraction. Then do I diuide 64. by 32, and the quotient is 2, which I set for my numerator. Againe, I diuide 96, by 32, and the quotient will be 3, and so I haue but $\frac{2}{3}$.

Master. Muse not at the matter, for you haue done well enough: but you thinke you

haue

haue not the fraction that you looked for, that is, $\frac{4}{6}$, yet haue you one equall to it, as by the parts of a shilling you may proue.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$, and $\frac{4}{6}$ and $\frac{2}{3}$, be all three equall. And now I perceiue that because $\frac{4}{6}$ was not written in the least termes that it might bee, therefore this Reduction brought forth not it, but that other which is written in the least termes. Now vnderstand I this Rule well. But is there any other way to worke this Reduction?

Master. Yes, but first note this, that if you find no such diuisor to reduce the Fraction till you come to 1, because one doth make no diuision: therefore that fraction is already in his least terms, as by $\frac{71}{100}$ you may prooue, and so of $\frac{81}{92}$ and many other like.

But now for your better aid to finde the due proportion in least termes, with more ease for a young learner, you shall mediate or take the halfe of the numerator, and also the denominator as long as you may vpon a line, alwaies parting them with a right downe dash of your pen as you worke, which may easily be done, if the numbers be euen: as 2, 4, 6, 8, or 10, but if they be odde (though it be but one of them) then must you abbreuiate them by 3, 5, 7 or 9, &c.

And because examples doe most instruct, I haue here set downe the manner of two or

three

three, whose last number at the end of the line sheweth the least tearme or valuation of that fraction.

As for example, I would reduce $\frac{288}{576}$ into his least terme or value, wherupon I set forth $\frac{288}{576}$ with a long line drawne from it thus, $\frac{288}{576}$|—And because both the numerator, and the denominator end in euen numbers, I see this may bee abbreuiated by 2, or 4, or 6, &c. therefore on the other side of the right downe dash toward the right hand, I first take the halfe of the numerator: saying, the halfe of 2 is 1, the halfe of 8 is 4: and againe, the halfe of 8 is 4: which 144 is now a new numerator: and therefore I part it with a right down dash, as before.

Then doe I also take the halfe of 576, in saying, the halfe of 5 is 2, and the halfe of 17 is 8, and the halfe of 16 is 8, and so haue I 288 for a new denominator.

Then beginning againe: saying, the halfe of 144 is 72, and the halfe of 288 is 144: thus continuing the mediation or diuision by 2, vntill you come to the last worke, as appeareth heere in the example, where the same is reduced to $\frac{1}{2}$, which is equall to $\frac{288}{576}$.

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

Abbre-

Abbreuiated first by 5. then by 293.

$$\frac{1465}{4395} \;\Big|\; \frac{293}{879} \;\Big|\; \frac{1}{3}$$

Scholar. Sir I thank you much, this is very easie and good for a young learner.

Master. So it is, but yet notwithstanding if you can without that diuision by memory espie the greatest number that may diuide exactly both termes of your fraction proponed, then need you not to vse that diuision, as in this fraction $\frac{60}{96}$. I see that 12, is the greatest number that can diuide them both: & therefore without any worke, by memory onely, I turne that into $\frac{5}{8}$, but this ability in knowledge is got by exercise.

Yet one other way of easie Reduction in this kind there is, when your fraction hath any ciphers in the first places of both termes, then may you by casting away the ciphers, make a briefe Reduction, as thus $\frac{300}{400}$: here take away the ciphers, and it will be $\frac{3}{4}$, which is the same in value with $\frac{300}{400}$.

Scholar. And so if I haue $\frac{400}{650}$, it will be $\frac{4}{65}$.

Master. You are deceiued, for you take away more ciphers from the Numerator than you doe take from the denominator, which you may not do.

Scholar. I confesse my fault, which came of too much hast, I was more gladder of the rule

rule than wise in vsing it: but now I vnderstand it I trust.

Master. Then may I goe in hand with the fift or last kinde of reduction, which teacheth how to turne any fraction proponed into any other denomination that you list, or into any parts of common coynes, weights, or measures or such like.

The ninth kinde of Reduction

For declaration whereof, first you shall marke whether your fraction be a simple fraction, either else a fraction of sundrie parts, I meane of more termes than two. And if your fraction bee a fraction of fractions, or otherwise compound, you must reduce it to one simple fraction. And then marke well the denomination of that other fraction, into which you would turne this: for by that denominator you must multiply the numerator of your first fraction, and the totall product therof shall you diuide by the denominator of your first fraction, and that quotient shall bee the numerator of the denominator proponed: as for example, I haue this fraction $\frac{3}{5}$, which I would turne into ten parts: therefore I multiply this 10 by 3, that is the numerator of my fraction, and there ariseth 30, which I diuide by 5, and the quotient is 6, which must be the numerator to 10, and so $\frac{3}{5}$ will bee $6\,\frac{6}{10}$.

Scholar. This is easie enough to doe.

Master. Then shall you see another example of the same fraction that is not so easie:

as

as if I would turne $\frac{3}{5}$ into 8 parts, proue you that worke.

Scholar. I must multiply 8 by 3, and there amounteth 24, which I diuide by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Master. And see you nothing doubtfull in this worke?

Scholar. I see that when 24 was diuided by 5, there remained 4, which I did not passe of, because yee speake nothing of any remainer, but onely of the quotient.

Master. By likelyhood you remember what I said to you in Diuision of whole numbers, that you should not passe of the remainer there, but onely note it as a summe that could not be diuided without knowledge of fractions. Wherefore now marke this, that in all diuisions of whole numbers, when there is any remainer, you shall set it ouer a line as a numerator, and set the Diuisor for the Denominator, and that fraction doth make the Diuision complete, and is part of the quotient: as if I would diuide 48 by 5, the quotient wil be 9 $\frac{3}{5}$: so in your former worke when 24 was diuided by 5, the quotient should be 4 $\frac{4}{5}$, and so the new fraction should be thus: $\frac{4}{8}$ and $\frac{4}{5}$, of 8, that is $\frac{4}{8}$ of the entire number, & $\frac{4}{5}$ of $\frac{1}{8}$ part of any thing, which you may proue by example of some coyne.

Scholar. Then I take a crowne, whose $\frac{3}{5}$ is 3 s. Now if I would proue whether ye 3 s be $\frac{4}{8}$ and

$\frac{4}{8}$ & $\frac{4}{5}$ of $\frac{1}{8}$, I shall haue a cumbrous worke to do.

Master. Indeed for whole pence, your example is a little troublesome: yet turning the crowne into halfe pence, it is easie enough.

Scholar. That will I trie.

¶ First I see that $\frac{3}{5}$ of a crown is 3 s, which is 36 d, or 72 halfe pence. Now if I can finde that this fraction $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$ be equall vnto 3 s, then am I fully answered.

Because I cannot take $\frac{4}{8}$ of a crowne, I turne the crowne into halfe pence, as you willed me, which makes 120, which I diuide by 8, my quotient is 15, which taken foure times makes 60 ob. Now resteth me to haue $\frac{4}{5}$ of the $\frac{1}{8}$ part of a crowne: whereof $\frac{1}{8}$ part is 15 ob: y 15 being parted in 5 parts, the quotient is 3, which taken 4 times maketh 12 ob. which with my 60 before amounteth to 72, which ars then equall to $\frac{4}{5}$ my desire.

Master. I commend you for your diligence, you might haue wrought it thus either: $\frac{4}{8}$ being abbreuied as before I taught, is $\frac{1}{2}$. Now the halfe of a crowne is 2 s, 6 d. Then $\frac{4}{5}$ of $\frac{1}{8}$ is a fraction of fractions, which if you do reduce into one entire fraction, as before you haue learned, in saying, 5 times 8 is 40, for a new denominator; and once 4 is 4, for a new numerator: it maketh $\frac{4}{40}$, and abbreuiated also make $\frac{1}{10}$. Now the tenth part of a crowne is 6 s, which put to 2 s 6 d, make also 3 s your desire.

But

But now one example moꝛe foꝛ this rule, and then wee ſhall end it. If I haue $\frac{7}{15}$ of a ſoueraigne (accounting the ſoueraigne 20 ſhillings) how many ſhillings is that $\frac{7}{15}$?

Scholar. I muſt multiplie 7 by 20, and that maketh 140, which I ſhall diuide by 15, & the quotient will be 9 $\frac{5}{15}$, oꝛ elſe in leſſer termes $\frac{1}{3}$.

Maſter. That is 9 s, and one third part of a ſhilling, that is, 4 d, as by the ſame rule you may pꝛoue. And this foꝛ this time ſhall ſuffice foꝛ Reduction, ſaue that I muſt now repeat a little touching Multiplication and Diuiſion, and ſo goe foꝛward.

Multiplication.

In Multiplication it happeneth ſometime, that there bee whole numbers to bee multiplied with fractions: and may bee in two ſoꝛts: foꝛ either the whole number is ſeuerall from the fraction, and is the multiplier, oꝛ elſe the whole number is ioined with one, oꝛ both of the fractions, and ſo maketh a mixt number thereof. If it bee in the firſt ſoꝛt, then néedeth there no reduction, but only multiply the numeratoꝛ of the fraction by that whole number, and the totall thereof ſet foꝛ the new numeratoꝛ.

Reduction of whole numbers into fractions.

Scholar.

Scholar. I vnderſtand you thus. If I haue $\frac{6}{23}$ to be multiplied by 16, then muſt I multiply that 16 with 6 which is the Numerator, whereof commeth 96, and that muſt I ſet for the new Numerator, keeping ſtill 23 for the Denominator, and ſo the fraction will bee $\frac{96}{23}$ that is $4\frac{4}{23}$.

Maſter. And in this ſort of worke you may abridge the labour, thus. If it happen the Denominator to be ſuch a number, as may euenly be diuided by the ſaid whole number propoſed, then diuide it thereby, and ſet the quotient of that diuiſion for the former denominator: but reſerue ſtill the Numerator, and ſo is the Multiplication ended.

Scholar. Then I faine this example $\frac{7}{20}$ to be multiplied by 5. And becauſe 5 wil iuſtly diuide 20, therefore I take the quotiēt of that diuiſion which is 4, and ſet in ſtead of 20, and ſo the fraction will be $\frac{7}{4}$, that is $1\frac{3}{4}$.

Maſter. Which is all one with $\frac{35}{20}$, that would haue followed of the other ſort of work.

Scholar. I perceiue it very well.

Maſter. Now then for the other ſort, where the number is mixt take this way: firſt to reduce the ſaid whole number, and fraction into one fraction improper (as I ſhewed you in Reduction) and then multiply them together as if they were proper fractions.

How to multiply mixt numbers.

Scholar. $13\frac{3}{5}$ being ſet to be multiplied by $\frac{1}{3}$ firſt I muſt reduce the mixt number, as

appeareth in the margent, by multiplying 13 by 5, and that maketh 65, whereto I must adde the numerator 3, and so the fraction will
13 $\frac{3}{5}$ $\frac{68}{5}$. bee $\frac{68}{5}$, which two fractions now I shall multiply after the accustomed forme, and it will be $\frac{34}{4}$

Master. You haue done well: and so may you see that although most part of the formes of Multiplication may be wrought without Reduction, yet some cannot, as namely mixed numbers.

And yet one note more will I tell you of Multiplication, before wee leaue it: That is, whensoeuer you would multiply any Fraction by 2, which commonly is called Duplation, you may do it not onely by doubling the Numerator, but also by parting the Denominator into halfe, if it be euen.

Scholar. Then if I would double $\frac{5}{12}$, I may choose whether I will make it $\frac{10}{12}$, or els $\frac{5}{6}$. And indeed I see that all is one, but that the diuiding of the Denominator seemeth the better way to make smaller termes of the Fraction, and so they shall need the lesse reduction.

Master. It is so: and now I shall not neede to tell you that Multiplication is proued by Diuision, and Diuision likewise by Multiplication: but the like works that I shewed you in Multiplication, will I shew you in Diuision also.

Diuision

When any whole number shall be diuided by a fraction, you must multiply the said whole number with the Denominator of the fraction, and set the totall thereof for the new Numerator, and for the Denominator set the Numerator of the fraction. Diuision to diuide a whole nūber by a Fraction.

Scholar. Then 20 diuided by $\frac{3}{4}$ $\frac{20}{1}$ by $\frac{3}{4}$ will make $\frac{80}{3}$, as heere appeareth $\frac{80}{3}$.

Master. Euen so: but if you would diuide the fraction by the whole number, then multiply the Denominator by the same whole number, and set the totall for the Denominator, without changing the Numerator. To diuide Fraction by a whole number.

Scholar. Then to diuide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$. as heere appeareth $\frac{20}{23}$, by $\frac{4}{1}$ in this example. $\frac{20}{92}$.

Master. You say well. And by the same example you giue me occasion to remember another brief way to doe the same: for if you had diuided the said numerator by 4, & set the quotient for the numerator, keeping stil the old denominator, it would haue bene not only as well done, but also in a fraction of lesser terms. Another brief way.

Scholar. I gesse it to be euen so, by a like worke that you taught me in multiplication: And for proofe thereof $\frac{20}{23}$ being the diuidend, and 4 the diuisor, I diuide the Numerator 20

by 4, and the quotient is 5, which I set for 20 ouer 23, thus $\frac{5}{23}$: And I see that it is all one with $\frac{20}{92}$ as by diuiding or abreuiating both these termes by 4 & so reducing them to their least denomination, I may easily prooue: as appeareth by this example. $\frac{20}{62}$ | $\frac{5}{23}$.

Master. You conceiue it well. And if there be mixt numbers (either one or both) you must first reduce that mixt number into an improper fraction, and then work as you haue learned.

Scholar. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for Diuision, if the denominators be like, then diuide the Numerators as if it were in whole numbers, and the quotient whether it be fraction, whole number, or mixt, is a good quotient for that diuision. And generaly if one of the Numerators may iustlie diuide the other, by that quotient multiply the Denominator of the lesser Numerator, and set it that doth amount in the roome of the same denominator, and then for a Numerator to it, set the Denominator of the other fraction.

Scholar. Then if I would diuide $\frac{3}{4}$ by $\frac{12}{17}$, I see that 3 will diuide 12, and the quotient will be 4. by which I must multiply the other 4, that is the denominator vnder 3. & then it is 16, which I set for the denominator 4 and ouer

ouer it in steed of 3, I must set 17 the other Denominator, and so is it thus, $\frac{17}{16}$.

Master. And so is $\frac{17}{16}$ instead of $\frac{51}{48}$, which would haue risen by the common $\frac{3}{4}$ by $\frac{12}{17}$ worke as heere appeareth: $\frac{51}{48}$

And now for mediation (which is to diuide by 2) marke this, if the Numerator bee euen, set the halfe of it in his place without the Diuisor, and so haue you done: and if the Numerator bee not euen, then double the Denominator.

Scholar. That is, if I would mediate $\frac{6}{11}$ I may make the quotient $\frac{3}{11}$ and if I woulde mediate $\frac{7}{17}$ I must make it $\frac{7}{33}$.

Master. Now trust I that you haue sufficient knowledge in Reduction, Multiplication and Diuision: and therefore will I go in hand with Addition and Subtraction, which now will appeare easie enough.

Addition.

WHensoeuer you haue any fractions to be added, you must cōsider whether they bee of one denomination or not: & if they be of one denominatiō, then ad ỹ Numerators together, & set that ỹ amounteth, for the Numerator ouer ỹ cōmon denominator, & so haue you done; The reason is because that such differ little in Ad-

 dition

dition oꝛ Subtraction from the woꝛk of vulgar denominatiōs, where the Denominatoꝛs be of od numbers: as 3 d & 5 d, make 8 pence, where the denomination is not altered. But if the fractions be not of one denomination, oꝛ any of them be mixt of whole nūbers & fractions, then must you first reduce thē to one denomination, & after ad thē. And if they be many, then adde first 2 of them, and ſo the ſumme that doth amount of the Addition, & the third & then the fourth, &c. if you haue ſo many.

Scholar. This ſéemeth eaſie enough, now that I haue already learned to multiply and to reduce, without which two I could neuer haue wꝛought this. And therefoꝛe now I ſée good reaſon, why you did place Multiplication, and Reduction befoꝛe Addition.

M. It is well conſidered, but yet refuſe not to expꝛeſſe your vnderſtāding of it by an exāple.

Scholar. Then would I adde firſt $\frac{7}{18}$ with $\frac{5}{18}$, and becauſe the denominatoꝛs are like (& ſo néedeth no reduction) I adde 7 to 5, which maketh 12, and then is my ſum $\frac{12}{18}$, that is in ſmaller numbers, being abbꝛeuiated, $\frac{2}{3}$.

And if I haue many numbers to be added as here $\frac{3}{8}$ $\frac{4}{5}$ $\frac{9}{10}$) firſt I muſt reduce them (becauſe they haue diuers denominatoꝛs) into one denomination, & then they will be thus $\frac{150}{400}$ $\frac{320}{400}$ $\frac{360}{400}$ oꝛ in leſſe termes $\frac{15}{40}$ $\frac{32}{40}$ $\frac{36}{40}$. which by addition doe make $\frac{83}{40}$ that is $2\frac{3}{40}$.

Maſter. Now may wee goe to Subtractiō

Sub-

Subtraction.

Subtractiō hath ẏ same precepts ẏ Addition had: for if the Denominators be like then must you subtract the one numerator frō the other, & the rest is to be set ouer the common denominator, & so your Subtraction is ended: but and if you haue many fractions to be subtracted out of many, then must you reduce them to one denomination, & into two seuerall fractions, that is, all must bee subtracted into one fraction, and the residue into another fraction, and then worke as I said before.

Sch. For ẏ first example I take $\frac{5}{12}$ to be subtracted out of $\frac{7}{12}$ & the rest wil be $\frac{2}{12}$ or $\frac{1}{6}$.

For another exāple I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$. which I must reduce & it wil be thus $\frac{24}{32}$ & $\frac{28}{32}$.

Then do I subtract 24 out of 28 & there resteth 4, which I set ouer the common denominator for a Remainer thus, $\frac{4}{32}$, that is $\frac{1}{8}$.

Now for the third example, I take $\frac{3}{4}$ & $\frac{5}{6}$ to be subtracted from $\frac{7}{8}$ & $\frac{9}{10}$. And because their denominators be diuers, I do reduce them into one denomination thus: $\frac{1440}{1920}$ & $\frac{1600}{1920}$ & $\frac{1680}{1920}$ & $\frac{1728}{1920}$

Then do I adde the two first, & they make $\frac{3040}{1920}$. Also I adde the two last, & they yeelde $\frac{3408}{1920}$. Then do I subtract 3040 out of 3408, and there resteth 368, so is the remainer $\frac{368}{1920}$ that is in smaller termes, $\frac{120}{23}$. And thus haue I done with Subtraction, except you haue any more to teach me.

Master. Prooue one example more of two Fractions of diuers denominations.

Scholar. I take these two Fractions, $\frac{7}{8}$ to be subtracted from $\frac{9}{24}$, which being reduced, will stand thus $\frac{168}{192}$ and $\frac{72}{192}$. Now would I subtract 168 out of 72, but I cannot.

Master. Then may you perceiue that you mistooke the Fractions: for you can neuer subtract the greater out of the lesser, although you may adde, multiply, or diuide the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his termes lesser then $\frac{9}{24}$, yet is $\frac{9}{24}$ the lesser Fraction: for generally if you multiply the Numerators and the denominators of two fractions crosse waies, that fraction is ye greatest of whose numerator commeth the greatest summe, as in this example: 7 multiplied by 24, maketh 168: and 9 being multiplied by 8, yéeldeth but 72: therefore is the first fraction 7 the greatest of these two, so can you not subtract it out of a lesser Fraction.

But if you should subtract a Fraction out of a whole number what would you do?

Scholar. Marrie I would reduce the whole number

number into a fraction of the same Denomination that my Fraction is, and then worke by Subtraction.

Master. So may you doe, but it is easier much, if your fraction be a proper fraction, ye is to say, lesse then an Unite, to take an Unite from the whole number, and then turne it into an improper Fraction, and so worke your subtraction. As if I would subtract $\frac{3}{7}$ from 4, I may take one from 4, and turne it into $\frac{7}{7}$, frō which if I abate $\frac{3}{7}$ there will remaine $3\frac{2}{11}$. And if the first fraction be an improper Fraction, then may I take so many Unites from the whole number, that they may make an improper fraction greater then that first, and then worke by Subtraction: As if there bee proponed $\frac{10}{3}$ to be subtracted from 6, because $\frac{10}{3}$ is more then 3, and not so much as 4, I must take 4 from 6, and turne them into thirds thus, $\frac{12}{3}$ then abate $\frac{10}{3}$, & there resteth $\frac{2}{3}$; so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{3}$ which is $\frac{10}{3}$. frō 6, whole: Then set one under 6, as thus $\frac{6}{1}$: And then to reduce those 2 fractions into one Denomination as here appeareth, $\frac{10}{3}$, from $\frac{6}{1}$:
Then $\frac{10}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$ which maketh $2\frac{2}{3}$ your desire.

$\frac{18}{3}$

$\frac{10}{3}$

And thus will I make an end of the worke of common fractions for this time, not doubting, but you can apply them both unto the rule

rule of Progression, and also vnto the Golden rule, without any other teaching then you haue learned before, which might seeme tedious to repeate, saue that in some special diuersities, which be peculiar to fractions, I cānot ouerpasse, but instruct you somewhat by the way.

The Golden Rule.

THerfore as touching the Golden rule for ẏ placing of ẏ 3 nūbers proponed in the question wherby to finde the third, & for ẏ forme of their worke with other like notes, I refer you to ẏ which you haue alredy learned.

But this easie forme of working by fractions shall you note, that if your 3 numbers be fractions, for an apt work & certaine, multiply the Numerator of the first number in ẏ question, by the Denominator of the second. And all that againe multiply by the denominator of the third number, and the totall therof shall you keepe for to be the Diuisor. Then multiply the denominator of the first number by the Numerator of the second, and the whole thereof by the Numerator of the third, & the totall thereof shalbe your diuidend.

Now diuide this diuidend by the Diuisor which you found out before, and that number shall

shall be the fourth number of the question, which you seeke for: as in this example, If $\frac{3}{4}$ of a yard of veluet cost $\frac{2}{3}$ of a Soueraigne esteemed at 20 s, what shall $\frac{5}{6}$ cost?

Scholar. If it please you to let mee make the answere, I would first place these three numbers, as I learned in whole numbers thus.

$\frac{3}{4}$ N $\frac{2}{3}$
$\frac{1}{6}$

And then according to your new rule, I must multiply 3, being numerator in the first number, by 3 the Denominator of the second, and thereof commeth 9, which I multiply againe by 6, the Denominator of the third nū-ber, and so haue I 54, which I keepe for the Diuisor, then multiply I 4, the denominator of the first, by 2 the Numerator of the second and there riseth 8, which againe I multiply by 5 the Numerator of the third, and it maketh 40: then must I diuide 40 by 54. and it will be 40, that is $\frac{20}{27}$ in lesser termes, and then the figure will stand thus.

$\frac{3}{4}$ N $\frac{12}{3}$
$\frac{5}{6}$ $\frac{20}{7}$

But what that is in money, I cannot tell except I shall worke it by Reduction, as you taught me.

Master. It forceth not now, you may reduce it when you list, but it were disorderly done heere to mingle diuers works together where we do not seek the value of the thing in common money, but in an apt nūber which ye haue wel done. And therefore wil I yet shew you

you another like way of easines in worke, how you may change your three Fractions into three whole numbers, by which you shal worke as if the question were proponed in whole numbers. The first number you shall find as I taught you: now to find the Divisor of the second number, take the Numerator for the second fraction: and for the third number take that that riseth of Multiplication of the Denominator of the first, by the numerator of the third, and then worke your question.

Scholar. For example hereof, I put this question. If $\frac{11}{12}$ of 1 l' waight of siluer be worth $\frac{12}{4}$ of a Soueraigne, what is $\frac{1}{3}$ of 1 l' waight worth? For the answere, first I place the fractions in order thus

$\frac{11}{12}$ Z $\frac{12}{4}$
$\frac{1}{2}$

Then to turne these fractions into whole numbers, I multiply 11, which is the Numerator of the first by 4 (the denominator of the second) and there commeth 44: which I multiply by 2 the denominator of the third, and so amounteth 88, which I set for the Divisor in the first place. Then in the second place I set 12, which is the numerator in the second fraction, and in the third place I set the summe that amounteth of 12, being the denominator in the first number, multiplyed by one, beeing Numerator in the third number, and so the figure will stand as heere you see

88 Z 12
12

Then

Then to worke it forth, I multiply 12 by 12, and there amounteth 144, which I diuide by 88, and the quotient will be $\frac{56}{88}$, or in lesser termes, $1\frac{7}{11}$, and then the figure will stand thus.

$\frac{1}{2}$	Z	$\frac{1}{4}$	
$\frac{1}{2}$		$1\frac{7}{11}$	

Master. These two formes now you vnderstand wel enough: and as for any other at this time I will not repeate, onely this shall you marke for the proofe of this rule whether your worke be well wrought or no: Multiply the first number by the fourth, and note what amounteth: then multiply the second by the third, and marke what amounteth also. Now if those 2 numbers so amounting be equall, then is your worke well done, else you haue erred. And this shall suffice for the former rule: but in ẏ backer rule, this shall you note for ease of worke, that you multiply the Numerator of the first by the numerator of the second, and the whole thereof by the Denominator of the third, and that amounteth thereof, shall be the Diuidend. Then multiply the Denominator of the first by the Denominator of the second, and that whole by the Numerator of the third, and that that riseth thereof shall be the Diuisor. Example of this: I did lend my friend $\frac{3}{4}$ of a Portuguise seauen moneths, vpon promise that he should doe as much for mee againe, and when I should borow of him, he could lend me but $\frac{5}{12}$ of a Portuguise: now I demaund how long time must

The proofe of the golden rule.

The backe rule.

A questiō of loane.

I

I kéepe his money in iuſt recompence of my loane accounting 13 moneths in the yeare?

Scholar. The firſt number muſt be the firſt money borowed, that is $\frac{3}{4}$ of the porteguiſe: the ſecond number the 7 moneths, that is $\frac{7}{13}$ of a yeare: and the third number the money that was lent in recompence, that is 5 2 of a Porteguiſe: then I ſet the numbers thus

$\frac{3}{4}$ Z $\frac{7}{13}$
$\frac{5}{12}$

Then (as you taught mée) I multiply thrée (being Numerator in the firſt number) by 7 the Numerator of the ſecond number, and it maketh 21, which I multiply by 12, the denominator of the third, and ſo haue I 252 for ỹ diuidend: then I multiply 4 the denominator of the firſt, by 13 the denominator of the ſecond and it yéeldeth 52, which I multiply againe by 5, the Numerator of the third, and it will make 260. that is the diuiſor. Then muſt I diuide 252 by 260, ſo it will be in the ſmall fraction $\frac{63}{65}$ of yeare.

Statute of Aſsiſe of bread and Ale.

Maſter. And thus doe you ſée ſome eaſe in working better then to multiply and diuide tediouſly ſo many fractions. Another queſtion yet will I propoſe, to the intent you may ſée thereby the reaſon of the Statute of Aſſiſe of bread and Ale, which in all ſtatute bookes, in French, Latine, and Engliſh is much corrupted for want of knowledge in this art, for the right vnderſtanding whereof I propoſe this queſtion.

When

When the price of a quarter of wheat is 2 s the farthing white loafe shall waigh 68 s, then I demand, what shal such a loafe waigh when a quarter of wheat is sold for 3 s?

Scholar. The question must be wrought as it is proponed in whole numbers, and not in Fractions.

Master. You seeme to say reasonably, howbeit, in that Statute of Assise, the rate is made by the proportion of parts in a pound waight Troy, else could it not be a Statute of any long continuance, seeing the shillings doe change often, as all other monies doe: but this Statute being well understood, is a continuall rule for euer, as I will anon declare by a new table of Assise, conuerting the shillings into ounces and parts of ounces.

Therefore heere by a shilling you must understand $\frac{1}{20}$ of a pound waight, and so by pen $\frac{1}{20}$ of an ounce: wherefore although yee might worke this question proponed by whole number well inough, for that time when the Statute was made, yet to apply it to our time and to make it serue for all times generally, it is best to worke it by fractions, setting for 2 shillings $\frac{2}{20}$. and for 68 shillings $\frac{68}{20}$: and so for three shillings $\frac{3}{20}$, & then will the figure of the question stand thus:

$$\frac{2}{20}\ \mathrm{Z}\ \frac{68}{20}$$
$$\frac{3}{20}$$

In which question because all the denominators be like, you shall worke onely with the Numerators.

Schol.

Scholar. Then I shall multiply 68 by 2, whereof commeth 136, which if I divide by 3, the quotient will be 45 $\frac{1}{3}$: but how shall I make a Fraction of that to stand with the other?

Master. Haue you so soone forgotten what was taught you so lately? This is his forme.

Scholar. I remember it now, and then it signifieth 45 twenty parts, & the third deale of one twenty part.

$$\frac{45\frac{1}{3}}{20}$$

Master. So is it that maketh in shillings 45 s, 4 d: whereby you may note one greate errour in the Statute books, which haue constantly 48 s in that Assise. And by this rule if you examine the Statute, you shall finde many summs false. Wherefore for the true understanding of that Statute, and such like as I haue made mention of it, and somewhat recognised it, so doe I wish that all gentlemen and other Students of the Lawes would not neglect this art of Arithmetick, as unneedfull to their studies. Wherefore, to encourage thē thereto, and to gratifie both them and all other in generall, I will exhibite a Table of that part of the statute in two columnes, and in a third columme I will adde the correction of those errours which haue crept into it.

Heere followeth the Table.

The

The price of a quarter of wheate.		The waight of a farthing white loafe by the statute bookes.			The correction by iust Assise.		
s	d	l	s	d	l	s	d
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 1/7	2	14	4 4/7
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 2/7
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/7
5	0	1	8	2 1/2	1	7	2 2/7
5	6	1	4	8 1/4	1	4	8 8/11
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 1/13
7	0	0	19	1	0	19	5 1/7
7	6	0	18	1 1/2	0	18	1 3/5
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 1/4	0	15	1 1/3
9	6	0	14	3/4	0	14	3 15/19
10	0	0	13	7 1/2	0	13	7 1/5
10	6	0	12	1/4	0	12	11 3/7
11	0	0	12	4 1/4	0	12	4 4/11
11	6	0	11	10	0	11	9 21/23
12	0	0	11	4	0	11	4

In the cōmō bookes there is no further rate of Assise made, then into 12 s the quarter of wheat, but in an ancient copy of 200 yeares old (which I haue) there is added the rate of assise vnto 20 s the quarter, but yet was that assise also either wrong cast at the first penning, or els corrupt since that time, for lacke of iust knowledge in the Rule of Proportion, which I will adde here also to gratifie such as be desirous to vnderstand truth exactly.

The price of a quarter of wheate.		The waight of a farthing white loafe by the statute bookes.			The correction by iust Assise.		
s	d	l	s	d	l	s	d
12	6		11	0	0	10	10 14/25
13	0	0	11	0 1/2	0	10	5 7/13
13	6	6	10	1 1/2	0	10	8/9
14	0	0	9	7	0	9	8 4/7
14	6	0	9	2 1/2	0	9	4 16/29
15	0	0	9	1 1/2	0	9	0 4/5
15	6	0	9	1 1/4	0	8	9 9/31
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	2 10/11
17	0	0	8	2	0	8	0
17	6	0	7	10	0	7	10 9/35
18	0	0	7	6	0	7	6 2/7
18	6	0	7	2	0	7	4 8/37
19	0	0	7	2	0	7	1 17/19
19	6	0	5	10	0	6	11 9/13
20	0	0	5	6	0	6	9 3/5

These two tables I haue set seuerall, because no man should thinke ẏ I would either adde or take away from any law those parts which might of right seme either superfluous, either diminute; but yet I may not be so curious as to neglect manifest errours, which is not onely my part, but euery good subiects duty with sobriety to correct. And for auoiding of offence I haue rather done it in this priuate booke, then in any booke of the Statutes it selfe, trusting that all men wil take it in good part.

Scholar. I would wish so, but I dare not hope so, sith neuer good man that would reform errour, could escape the venemous tongs of enuious detractors, which because they either cannot, or list not to do any good thēselues, do delight to barke at the doings of other, but I beseech you to stay nothing for their peruerse behauiour.

Master. I consider many things that some may obiect, wherunto I am not vnprouided of iust answers, but I will not seme so hasty to make the answers before I heare their obiections; but as I trust that men are of a better nature, and more gratefull now than some haue bene in times past, as I haue done in the Statute of Assise for bread in rate of s, so will I set forth the like table in pounds and ounces, and the parts thereof, that it may be easily applied to all times: but I meane not

by this to alter any word of the statute (being so good an ordinance and of so great continuance) but onely to make it as a kinde of exposition and declaration of the said Statute, trusting that thereby the Statute may be better understood and consequently better put in execution. And here you shall note that I haue accounted the shillings after the rate of lx s. to the pound waight, because I esteeme it the most apt rate for our time. Wherefore if in the first columne you find the price of wheate directly against it, in the second columne, you may finde the waight of the farthing white loafe, in this our time: and if you double the number (as I haue done in the third columne) then haue you the waight of the halfe pennie white loafe, and so in the fourth columne is set the waight of the peny white loafe. It needeth not to tell you that, that the sight doth testify, how that euery columne is parted into 3 smaller pillers, whereof the first columne hath these 3 titles, pounds, shillings, & pence: the other three columnes haue each of them these three titles, pounds, ounces and penie waights. And as in the first columne 12 d, make a s, and 20 s make a pound, so in the other 3 columns 20 pence waight maketh an ounce, and 12 ounces doe make a pound.

Scholar.

Scholar. Sir, I doe thanke you most heartily for this, not onely in mine owne name, and in the name of all Students, but also in the name of the whole Commons, to whom the restitution of this Assise (I trust) shall bring restitution of the waight in bread, which long time hath beene abused. And if you know any like things more, wherein you would vouchsafe to declare the errours, and set forth the trueth, you cannot but obtaine great thankes of all good hearted men that loue the common wealth.

Master. I haue sundry things to declare, but I haue reserued them for a priuate booke by it selfe, yet notwithstanding because the statute of the rate of measuring of ground is so common that it toucheth all men, and yet no more common than needfull, but so much corrupt, that it is too farre out of all good rate, not onely in the English bookes of Statutes, commonly Printed, but also in the Latine bookes, & in the French also, (for I haue read each sort, and conferred them diligently) I will giue you a Table for ŷ restitution of those errours, as may suffice for this present time. And first I will propose one question to you touching the vse of that Statute, whereby you may perceiue the order how to examine the whole statute, and euery parcell thereof, and the question is this.

When the Acre of ground doth containe

foure perches in breadth, then must it containe 40 perches in length: then doe I demaund of you, how much shall the length of an Acre be, when there is in the breadth of it 13 perches: but before you shall answer to this question, I will declare vnto you another Statute, which is the ground of the former Statute. And that Statute is this. It is ordained that 3 Barly Cornes, dry and round, shall make vp the measure of an inch: 12 inches shall make a foote, and 3 foote shall make a yard (the common English bookes haue an Elne) fiue yards and a halfe shall make a perch, and 40 perches in length, & 4 in breadth, shall make an Acre. This is that Statute: whereby you may perceiue, that the intent of the statute is, that one Acre should containe 160 square perches. Now let me heare your answer to the question.

Scholar. As I perceiue by the words of ẏ Statute, a perch to be ẏ $\frac{1}{160}$ part of an Acre, so could I make those numbers all in fractions, and so worke the question, but seeing I may doe it also in whole numbers I take that forme for the most easie, therefore thus I set the question in forme. Then doe I multiply 40 by 4, and it maketh 160, which I diuide by 13, and the quotient is 12 $\frac{4}{13}$.

4 — 40
13 —

Master. Now turne that $\frac{4}{13}$ into the common parts of a perch, as they be named i[n]

th

the former Statute: howbeit it shall be best to take one of the least parts in Denomination for auoiding of much labour, as feete, whereof the perch containeth $16\frac{1}{2}$.

Scholar. Then to returne $\frac{4}{13}$ into feete, I multiply $16\frac{1}{2}$ by 4, and it maketh 66, which I must diuide by 13, and the quotient is $5\frac{1}{13}$.

Master. So I finde that if the acre hold in breadth 13 perches, it shall containe in length 12 perches, 5 foote, and $\frac{1}{13}$ of a foote, which is not fully an inch, for the inch is $\frac{1}{12}$ of a foote. But here all the Statute bookes in Latine and English (that I haue seene) doe note it to be 13 perches, 5 foote and one inch: which maketh aboue 13 perches, too many in the acre: so that I would haue thought the errour to haue crept into the printed bookes by the great negligence that Printers in our time doe vse, saue that in written copies of great antiquity, I do find the same. Yet haue I one French Copie, which hath 12 perches $\frac{1}{4}$ and one foote, and that misseth very little of the truth.

Scholar. Then I see it is true that I haue often heard say, that the truest copies of the statutes be the French copies.

Master. That is often true, but not generally, as I haue by conference tried diuersly: but in this Statute the French booke is most corrupt in all other places lightly.

But now to performe my promise, I will

 set

set forth the table for measuring of an acre of ground only by such parts as the statute doth mention, because at this time I do of purpose write it for the better understanding of that statute, and hereafter with other things I intend to set forth this same more at large.

In this Table following, I haue not done as in the other statute before compared by restitution with the faults crept into the statute, but onely haue written that true measure, which the equitie of the Statute doth pretend. For it were vile to iudge of so noble Princes and worthy counsellors, as haue authorised & set forth this statute, that they would make one Acre in any forme greater than an other, but euery one to be iust and equall with each other, which is the ground also of my worke: and hereby may all men perceiue how needefull Arithmeticke is to the students of Law. But now I think best to make an end of these matters for this present time, sith the Table hath in it none obscurity, that I should néed to declare.

The

The breadth.	The length of the Acre.			
Perches.	Perches.	Feete.	Inches.	Parts of an inch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{9}{17}$
18	8	14	8	0
19	8	6	11	$\frac{7}{19}$
20	8	0	0	0
21	7	10	2	$\frac{4}{7}$
22	7	4	6	0
23	6	15	9	$\frac{9}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{5}$
26	6	2	7	$\frac{6}{13}$
27	5	15	3	$\frac{1}{3}$

The

The breadth.	The length of the Acre.			
Perches.	Perches.	Feete.	Inches.	Parts of an inch.
28	5	11	9	$\frac{3}{7}$
29	5	8	6	$\frac{12}{29}$
30	5	5	6	0
31	5	2	7	$\frac{29}{31}$
32	5	0	0	0
33	4	14	0	0
34	4	11	7	$\frac{13}{17}$
35	4	9	5	$\frac{1}{7}$
36	4	7	4	0
37	4	5	4	$\frac{8}{37}$
38	4	3	5	$\frac{13}{19}$
39	4	1	8	$\frac{12}{39}$
40	4	0	0	0
41	3	14	10	$\frac{28}{41}$
42	3	13	4	$\frac{2}{7}$
43	3	11	10	$\frac{32}{43}$
44	3	10	6	0
45	3	9	2	0

Scholar.

Scholar. Indéede Sir, I vnderſtand the Table (as I thinke) by thoſe other which you ſet foꝛth befoꝛe. Foꝛ in the firſt Columne is ſet the perches of the bꝛeadth of any Acre, and then in the 2 Columnes following appeareth how many perches and how many foote that ſame Acre muſt haue foꝛ his length.

Maſter. You take it well: howbeit to ſpeake exactly of bꝛeadth and length, the firſt Columne doth ſometime betoken the bꝛeadth, and ſometime the length: foꝛ pꝛoperly the longeſt ſide of any ſquare doth limite his length; and the ſhoꝛter ſide doth betoken the bꝛeadth; yet it is no great abuſe in ſuch Tables, where a man cannot well change the title, to let the name remaine, although the pꝛopoꝛtions of the numbers doe change: foꝛ ſtill by the firſt Columne is expꝛeſſed the meaſure of the one ſide, and by the two other pillars in one Columne, is ſet foꝛth the meaſure of the other ſide. And this ſhall be ſufficient now foꝛ the vſe of the Golden Rule.

The Rule of Fellowſhip.

NOw ſomewhat will I touch certaine other rules, which foꝛ their ſeuerall names may ſéem diuers rules, and diſtinct from this, but indeede they are but bꝛanches of it: yet becauſe they haue

seuerall workings in appearance, but also plesant in vse, I will giue you a taste of each of them. As for the rule of Fellowship, both single and double, with time and without time, I shall néede to say little more than I haue already said in teaching the workes of whole numbers, yet an example or two will wee haue to refresh the remembrance of the same, and to declare certaine proper vses and applications of it, as this for one.

Foure men got a bootie or prise in time of warre, the prize is in value of money, 8190 l, and because the men be not of like degrée, therfore their shares may not bee equall, but the chiefest person will haue of the bootie ẏ third part, and the tenth part ouer: the second will haue a quarter, and the tenth part ouer, the third will haue the sixt part: and so there is left for the fourth man a very smal portion, but such is his lot (whether he be pleased or wroth) hee must bee content with one 20 part of the pray: Now I demaund of you, what shall euery man haue to his share?

Scholar. You must be faine to answere to your owne question, else is it not like to be answered at this time.

Master. The forme to vnderstand the solution of this question, and all such like is thus: Reduce all the denominators into one number by Multiplication, except that any of

them

them bee parts of some other of them, for all such parts you may ouerpasse, and take for them all those numbers, whose parts they bee: as in this example the shares bee these, $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{10}$ $\frac{1}{6}$ $\frac{1}{20}$ if I multiply all the denomina-tors together, beginning with 3, and so go on vnto 20, it will make 144000: but conside-ring that 3 is a part of 6, I shall omit that 3, and likewise 10, which is a part of 20, I may ouerpasse also, and then is there but three de-nominators to multiply, that is, 4, 6, and 20, which make 480, which summe I take for my worke, because all the denominators will be found in it. Then I take such parts of it, as the question importeth, that is, for the first man $\frac{1}{3}$ $\frac{1}{10}$, the $\frac{1}{3}$ 160, the $\frac{1}{10}$ is 48: which I put in one summe for the first mans share, and it maketh 208. Then for the second mans share, I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole 168. Now for the third man which must haue $\frac{1}{6}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole sum: so that if the whole pray had beene but 480 l, then were the question answered: but because the summe was of greater value, by this meanes now shall I know the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number that I found by multiplying the Denomina-tors, and in the second place the summe of the

the body. And looke what proportion is betweene the first number and the second, the same proportion shall be betweene the parts of that first number, and the parts of the second, comparing each to his like. Therfore I must put in the third place, one of the parts or shares, and then worke by the former rule of proportion or Golden Rule. And because I haue foure seuerall parts of the first number, by which I would find out foure like parts of the second number, therefore must I make 4 seuerall figures.

The reasõ of the rule.

Scholar. Now I trust I can answere to your question, as by your fauour I wil proue.

A

480 — 8190
208

B

480 — 8190

C

480 — 8190
80

D

480 — 8190
24

And to try it, I set the foure figures thus, marked with A, B, C, D, to shew their order. And then in each of them I multiply the second number by the third, and diuide their totall by the first, and so amounteth the fourth summe which I seeke for: for if I doe multiply 8190 by 208, it maketh 1703520, which being diuided by 480, maketh in the quotient 3549 for the first maine portion.

An

And so working with the other thrée figures, I finde for the second man 2866 $\frac{1}{2}$, and for the third man 1365: and then for the fourth man 409 $\frac{1}{2}$, and so euery mans share is set forth in the figure here annexed.

A		B	
480 Z 8190		480 Z 8190	
208 Z 3549		168 Z 2866 $\frac{1}{2}$	
C		D	
480 Z 8190		480 Z 8190	
80 Z 365		24 Z 409 $\frac{1}{2}$	

And thus I thinke I haue done well.

Master. If you misdoubt your working, & list to prooue it, adde all the shares together, and if they make the totall, then séemeth it well done.

The proof by addition.

Scholar. I may set them thus: and then by Addition the iust sum doth amount, ẏ is, 8190, and therefore (as you say) it séemeth to be wel wrought.

3549
2866 $\frac{1}{2}$
1365
409 $\frac{1}{2}$
8190

But I beséech you, is there any doubt in this triall, that you vse that worde, Séemeth?

The iust proofe.

Master. You may easily coniecture, that if you did assigne the first mans share to ẏ last, and so change all the rest, that one had anothers share, yet would the Addition appeare

all

all one, and therefore is not the proofe exact.

But if you will make a iust proofe for the first mans part, take $\frac{1}{3}$ & $\frac{1}{10}$ of the whole sum, and if it agree with the number in the figure, then it is well done. And so doe for the second, third, and fourth summes, and this proofe faileth not. Now will I propound certaine other questions which haue beene set forth by certaine learned men, albeit not without some ouersight, which questions I protest hartily: I doe not repeate to depraue those good men, whose labours and studies I much praise, and greatly delight in. But onely according to my profession, to seeke out trueth in all things, and to remooue all occasions of error as much as in mee lieth: and for that cause I will onely name the questions without hurting the Authors name. The first question is this.

Foure men did build an house, which cost them 3000 Crownes, their shares were such that one man should pay $\frac{1}{2}$ of the summe, and 6 Crownes ouer: the second should pay $\frac{1}{3}$ and 12 Crownes ouer: the third man must lay out $\frac{2}{3}$, abating 8 Crownes, & the fourth man should pay $\frac{1}{4}$ and 20 Crownes more: can you answer to this question?

Scholar. No in good sooth sir, and that you know best of any man, for I know no more than you haue taught me.

Master. Then I dare say you cannot do

it neither yet the best learned man that euer did propose it, for the question is impossible. For declaration whereof, I wil be bold to vse first the representation of the numbers in their aptest forme, (although I haue not yet taught that maner of worke) because it may appeare plainely that the question is not possible. For here I haue set the parts,

An impossible question.

and added them, and they make the whole sum, and $\frac{3}{4}$ and 30 more. Now, how is it possible to diuide truely either gains, either charges, so that the particulars shall be more than the totall?

$\frac{1}{2}$	-	6
$\frac{1}{3}$	-	12
$\frac{2}{3}$	-	7
$\frac{1}{4}$	-	20
$1\frac{3}{4}$	-	30

Scholar. It is against the forme of proofe by addition of parts.

Master. You say truth. And (because you shall perceiue it the better) I will trie it after the vulgar forme, as in this figure you see, where the $\frac{1}{2}$ with 6 ouer is 1506: for the totall as you heard before, 3000 the $\frac{1}{3}$ and the 12 more 1020: the $\frac{2}{3}$ would be 2000, but then abating 8, it is but 1992, and then last of all, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which all beeing added in one summe, doe make 5280, where the totall summe should be but 3000, which summe of 3000 if you diuide by $\frac{7}{4}$, so shall

1506
1022
1992
770
5110

you haue $\frac{3}{4}$ of it, that is 2250, and therto adde 30 more, then will those thrée sums make 5280: whereby you may see how this form (as wel as the other) doth declare that the particulars in that question would make more than the whole sum by $\frac{3}{4}$, and 30 more: and therefore can that question not be accepted as a possible thing: but yet do certaine learned men propound such questions, and answere to them. Therefore somewhat to say to their excuse (rather of their good meaning, then for their doing) I wil anon declare what may be said for their defence: but, in the meane season, I will propound the question as it may be wrought by good possibilitie. As if foure men build a house together, and it cost them 3000 Crownes, and then for the partition they agrée thus: that as often as the first man doth pay 6 Crownes, so often the second man shall pay 4, the third man 8, and the fourth man 3. Or else, thus: that the first man shall pay double so much as the fourth, and the second man shall paye $\frac{1}{3}$ of the first mans charge: the third man shall pay double so much as the second: (and these two waies are to one end:) but further for their agreement it is appoynted also, that the first shall giue 6 Crownes ouerplus, and the second 12, and the fourth shall giue 30: but the third man shall giue no ouerplus

2000
2250
30
5280

plus, but shall haue 8 crowns abated of his charge. Now is the question possible to be assoiled, and this is the way to do it. Marke the proportion of the seuerall charges, and set out smal numbers in that rate, by which you may reduce the worke to the Golden rule, as heere in the first forme, the numbers are alreadie named, 6, 4, 8, 3: and in the second forme, (although they be but plainly named) yet they may be the same numbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and againe 8 is double to 4. Now adde these together, and they make 21, which 21 must be set in the first number in the golden rule: for if it, with the ouerplus of each mans charge would make the totall summe of the charges: then were those seuerall sums the charges of each man besides his ouerplus: but now it is not so.

But yet, this is true, (so excellent are conclusions Arithmeticall,) that looke what proportion each of their seuerall sums doth beare to 21, the same proportion doth the iust charges of euery man (besides his ouerplus) beare to the total of the charges, the ouerplus being deducted: wherefore this may you note, that, before you doe apply the totall of his charges to the Golden rule, you must deduct the ouerplus which is 6, 12, and 20, that is in the whole 38: but then 8 must be restored for the abatement of the third man, and then remaineth to be deducted 30: Take 30 there- The Rule

foꝛe out of 3 0 0 0, and there will reſt 2 9 7 0, which I muſt ſet in the Golden Rule foꝛ the ſecond ſumm: and foꝛ the third ſumme I muſt put each of the ſmall numbers befoꝛe mentioned, which although they be not ſeueral charges, yet they repꝛeſent them in pꝛopoꝛtion. And ſo making foꝛ euery mans charge a ſeueral queſtion, the figures wil bee 4, which I marke with foure letters, A, B, C, D, thus.

A		B	
21	2970	21	2970
8	$848\frac{4}{7}$	3	$424\frac{5}{7}$

C		D	
31	2970	21	2970
8	$1131\frac{3}{7}$	3	$424\frac{2}{7}$

Where I haue ſet foꝛ bꝛiefneſſe the ſumme of euery mans charge in the fourth place, pꝛeſuppoſing that you can tell how to trie out that fourth ſumme by ſo many examples as ye haue had.

Scholar. As I truſt that I vnderſtand this foꝛme, ſo I deſire much to know what may be ſaid foꝛ them that miſtooke this queſtion.

Maſter. You ſéeme ſo deſirous to know this errour, that you haue foꝛgotten to examine, whether this woꝛke be without fault.

Scholar. Me ſeemeth this woꝛke to be wel done, becauſe the Addition of the 4 ſeuerall num-

numbers doth make the totall sum of 2970, which was to bee diuided into such foure parts.

Master. But then haue you forgotten that the first man must pay 6 Crownes more besides his share, and the second man twelue crownes more, the third man 8 crownes lesse, & the fourth man 10 crownes more, for without these your first totall of 3000 crownes wil not be made.

Scholar. Then must I adde to the first mans summe 6 more, and it will be 854 $\frac{4}{7}$: and to the second sum I must adde 12, and it will be 577 $\frac{5}{7}$: from the third sum I must abate 8, and then wil the summe bee 1123 $\frac{3}{7}$: then adding vnto the fourth sum 20. it will be 444 $\frac{16}{7}$ and these foure summes will make 3000, which is ẏ whole charge, as in this example it may appeare, where first I gather the $\frac{14}{7}$, that maketh 2, and so proceed I in the Addition to the end.

854 $\frac{4}{7}$
577 $\frac{5}{7}$
1123 $\frac{3}{7}$
444 $\frac{2}{7}$
3000

Master. Now haue you well done, and this worke in the same sums is brought of other learned men for the true solution of the question, as it was first proponed, which (as I said) was impossible: and now examine it by these seuerall summes, and see whether it doth agree with the summs in the question proponed.

The first man must pay $\frac{1}{2}$ and 6 ouer of the totall sum: how thinke you, is 854$\frac{2}{7}$ the halfe and 6 more of 3000?

Scholar. No, that it is not, for it should bée 1506: and for the second man 1012: and for the third man 1992, and for the fourth man 770, whereof not one summe agreeth to this worke. But I maruell, that so wise men could be so much ouerseene.

Master. It is commonly seene that when men will receiue things from elder writers, & will not examine the thing, they seeme rather willing to erre with their auncients for company, than to be bold to examine their workes or writings. Which scrupulositie hath ingendred infinit errors in all kinds of knowledge and in all ciuill administration, and in euery kinde of art. But these learned men did not meane any other thing by this question, than to find such numbers as should beare the same proportion together, as those nūbers in the question proponed did beare one to another: which thing you shal perceiue more plainly by another question of theirs, that is this.

A man lying vpon his death bed, bequeathed his goods, (which were worth 3600 Crownes) in this sort. Because his wife was greate with child, and he yet vncertain whether the child were male or female, he made his bequest cōditionally, that, if the wife bare a daughter, then should the wife haue half his

goods

goods, and the daughter $\frac{1}{3}$, but, if she were deliuered of a sonne, then that sonne should haue $\frac{1}{2}$ of the goods, and his wife but $\frac{1}{3}$. Now it chanced her to bring forth both a sonne and a daughter, the question is: How shall they part the goods agreeable to the testator his will?

Scholar. If some cunning Lawyers had this matter in scanning, they would determine this Testament to be quite void, and so the man to die intestate, because the Testament was made insufficient, sith this condition was not expressed in it: and also, it might haue chanced, that she should haue brought forth neither sonne nor daughter, as often hath beene seene: so is the will unsufficient in that point also.

Master. Such scanners should seeme too cunning, and yet not so cunning as cruel: for the minde of the Testator is to be taken fauorably, for the aide of the Legatories, whē there riseth such doubt. But let us trie this worke, not by force of Law, but by proportion Geometricall, seeing the Testator did mind to prouide for each sort of them.

Scholar. If the son shall haue $\frac{1}{2}$ by force of the Testament, so must the mother haue $\frac{1}{3}$. Againe, because she hath a daughter also, therfore ought she to haue $\frac{1}{2}$. & the daughter $\frac{1}{3}$. that is, both waies $\frac{1}{2}$ $\frac{1}{3}$, and $\frac{1}{3}$ $\frac{1}{2}$, which commeth to the whole goods, and $\frac{2}{3}$ more. Wherefore it seemeth also impossible.

Ma. In this matter the mind of the Testator is so to be understood, that such proportion should be betweene the portion of the wife and the son, as is between $\frac{3}{6}$ for $\frac{2}{6}$ to that is the son must haue $\frac{3}{6}$ for $\frac{2}{6}$ to his mother, so shall he haue 3 to 2, that is asmuch as his mother, and halfe asmuch more, & the mother must haue the like rate in comparison to her daughter. Then must I find out 3 numbers in such proportion, that the first may be asmuch as the second & half as much more (that is) in proportion sesquialtera, & the second to the third in y same proportion: such numbers be 6, 9, 4.

Scholar. I pray you, Sir, how shal I finde out those numbers?

Master. That will I gladly tell you.

Whatsoeuer the proportion be of any three numbers, multiply the Termes of that proportion together, & the number that amounteth, shall be the middle number of the 3: then multiply that middle number by the lesser terme, and diuide that totall by the greater, and the least number of the 3 wil amount. So if you multiply that middle number by the greater extreame, & diuide that totall by the lesser extreame, then will the greatest number of that progression amount.

Scholar. Then in this example, to find the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must diuide (as you taught me in Diuision) $\frac{1}{2}$ by $\frac{1}{3}$, & the Quotient will be $\frac{3}{2}$, that is $1\frac{1}{2}$, whereby I perceiue that

th

the proportion in this question is, as 3 to 2: Therefore (as you taught mee euen now) I multiply 3 by 2, and the sum is 6, which must bee the middle number: then, I multiply the middle number 6 by 2, which is the least tearme, and the sum is 12: that I doe diuide by 3 being of the greater Terme, and ẏ Quotient is 4: so is 4 the least number of the 3. Then I multiply 6 by 3, whereof commeth 18, & that I diuide by 2, and so haue I nine, which is the greatest number of the 3.

Master. Another way yet may you finde the third number in any progression, if you haue 2 of them: for if the middle number bee one of them which you haue, then multiply it by it selfe (as in this example 6 by 6 maketh 39) and that totall diuide by the other number which you haue, and the third number will be the Quotient.

Scholar. Then if I diuide 36 (which commeth of 6 multiplied by it selfe) by 4, the Quotient will be 9: and, if I diuide 36 by 9 the Quotient will be 4. But what if I know the first number and the third, and would haue the middle number?

Master. Multiply the two numbers together, & in their totall you must seeke the root of that number, and it shalbe the middle number: but: because as yet you haue not learned how to extract roots, therefore vse the first forme which I haue taught you, till I teach you to

extract

extract rotes. And now goe forward with the answere to the same question.

Scholar. I perceiue then that the sonne must not haue $\frac{1}{2}$ of the goods, neither the mother $\frac{1}{2}$, nor yet the daughter $\frac{1}{3}$, but yet must the goods be diuided into such proportion, ỷ the son shall haue 9 crownes for 6 to his mother, and the mother shall haue 6 crownes for euery 4 to hir daughter. Then I apply it to the golden Rule in thrée examples, thus:

where the first number is the addition of those thrée numbers, 9, 6, 4, and the third is 1 of thē seuerally: the second is the totall of the goods in ỷ testament: & then by ỷ work of the Goldē rule I find out the fourth number in euery worke, that is for the sonne, 1705 $\frac{5}{19}$: for the mother, 1136 $\frac{16}{19}$, and for the daughter, 757 $\frac{17}{19}$: the which sums added together do make the sum of the whole goods, as may be séene by this example.

19 Z 3600
9

19 Z 3620
6

19 Z 3600
4

1705 $\frac{9}{16}$
1136 $\frac{16}{19}$
757 $\frac{17}{19}$
3600

And this (me thinketh) I do perceiue, that because in this case there is a necessary remedie deuised against an vrgent inconuenience, therefore those learned mē thought they might vse the like libertie in that other question.

Scholar. Your gesse is good, but they had so good reason for them in the one, as they haue

in the other: as in another example of theirs it may better appeare, as in this.

A man left unto his three sonnes 7851 crownes to be parted in such sort, that ẏ first sonne should haue $\frac{1}{2}$, the second son $\frac{1}{3}$, and the third sonne $\frac{1}{4}$, which is not possible, for $\frac{1}{2}$ and $\frac{1}{3}$ and $\frac{1}{49}$ doth make $\frac{26}{24}$: or $\frac{23}{12}$ that is 1 $\frac{1}{12}$, so is it more than the whole: but reduce these fractions into one demonstration, ẏ least that they wil come to, and they wil be $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, and so may you part the goods into such proportion as these three Numerators beare together, ẏ is, the first to haue 6 for euery 4 to the second: and the second to haue 4 as often as the third hath 3: and so their portions will be for the first, 3623 $\frac{7}{17}$ for the second 2415 $\frac{9}{13}$: & for the third 1811 $\frac{10}{13}$. and those three shares added together, will make the totall sum of the whole goods, as you may easily see in this example.

3623	$\frac{7}{13}$
2415	$\frac{9}{13}$
1811	$\frac{10}{13}$
7851	

Another question is there proponed thus:

There are 450 Crowns to be diuided betweene three men, so that the first man must haue $\frac{1}{2}$ and $\frac{1}{3}$, the second man $\frac{1}{3}$ & $\frac{1}{4}$, the third man shall haue $\frac{2}{4}$ ond $\frac{1}{5}$.

Scholar. I maruell that any man should be so ouerseene to propound that question as a thing possible, sith $\frac{21}{23}$, $\frac{11}{34}$, $\frac{21}{45}$, doe make 1 $\frac{13}{14}$ that is almost double the whole sum.

But I perceiue it might bee thus proponed

ned, that as often as the first man did receiue 50 Crowns, so often the second man should receiue 35, and the third man 27, for $\frac{1}{3}$ & $\frac{1}{4}$, is equall to $\frac{35}{60}$, and is $\frac{1}{7}$, & equall to $\frac{35}{60}$, & $\frac{1}{4}$ & $\frac{1}{7}$ is $\frac{27}{60}$, and so working the question, the three figures will appeare in this forme: wherby the first mans portion is found to bee 200 $\frac{50}{56}$: ye second mans part is 140 $\frac{25}{56}$, ye third mans share 110 $\frac{24}{76}$: which in the whole doth make 450 crowns to be diuided between them.

112 — 450
50 — 200 $\frac{50}{56}$

112 — 450
35 — 140 $\frac{15}{56}$

112 — 450
27 — 108 $\frac{37}{56}$

Master. And thus you are (I thinke) sufficiently instructed in the rule of fellowship.

The Rule of Alligation.

NOw will I goe in hand with the Rule of Alligation: which hath his name, for that by it there are diuers parcels of sundry prices and sundry quantities alligate, bound or mixed together: wherby also it might be well called the Rule of Mixture

ture, and it hath great vse in composition of medicines, and also in mixtures of metals, and some vse it hath in mixtures of wines: but I wish it were lesse vsed therein than it is now adaies. The order of the Rule is this: When any summes are proponed to bee mixed, set them in order one ouer another, and the common number whereunto you will reduct them, set on the left hand: then marke what summes be lesser then that common number, & which be greater, and with a draught of your penne euermore linke two numbers together, so that one be lesser than the common number, & the other greater then he (for two greater or two smaller cannot well be linked together:) and the reason is this, that one greater and one smaller may be so mixed, that they will make the meane or common number very well: but two lesse can neuer make so many as the common number, being taken orderly: no more can two summes greater than the meane, euer make the meane in due order, as it shall appeare better to you hereafter. And, as it is of necessitie to linke euery smaller (once at the least) with one greater, and euery greater with one smaller: so it is at liberty to linke them oftner than once; and so may there bee to one question many solutions. When you haue so linked them, then marke how much each of the lesser numbers is smaller than the meane or common number, & that difference The reason of this rule

set

set against the greater numbers which be linked with those smaller, each with his match still on the right hand, and likewise the excesse of the greater numbers aboue the meane, you shall set before the lesser numbers which be combined with them. Then shall you (by Addition) bring all these differences into one summe, which shall be the first number in the Golden Rule: and the second number shall be the whole masse that you will haue of all those particular: the third summe shall be each difference by it selfe, and then by them shall be found the fourth number, declaring the iust portion of euery particular in that mixture: As now by these examples I will make it plaine.

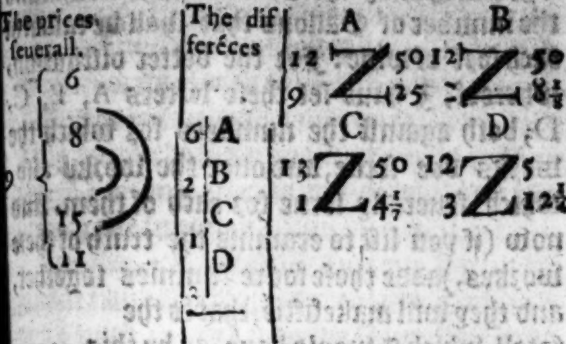
A question of mixing of wine

There are foure sorts of Wine of seuerall prices, one of 6 pence a gallon, another of 8 d, the third of a 11 d, and the fourth of 15 d the gallon. Of all these Wines would I haue a mixture made to the summe of fiftie gallons, and so that the price of eace gallon may be 9 d. Now demand I how much must be taken of euery sort of Wine?

Scholar. If it shall please you to worke the first example, that I may make the applying of it to the rule, then I trust I shall be able not onely to doe the like, but also to see the reason in the order of the worke.

Master. Marke then this forme and the placing euery kind of number in it.

The



The common price

Heere (you see) I haue set downe the seueral prices which be 6, 8, 11, 15, and haue linked together 6 with 15, and 8 with 11. The common price, 9, I haue set on the left side, and the difference betwéene it, and euery particular price, I haue set on the right hand, not against the sum (whose difference it is) but against the sum that it is linked withall: so the difference of 15 aboue 9, is 6, which I haue set, not against 15 but against 6. that is linked with 15, and the difference between 6 and 9 (that is 3) I haue set against 15. So likewise, the difference betwéene 8 and 9 is but 1: that I haue set against 11, and the difference of 1 aboue 9 (which is two) I haue set against 8. Then adde I all those foure differences, and they make 12, which I set for the first number in the Golden rule: the second number I make 50, which is the summe of gallons that I would haue, and the third sum is euery particular difference. Now, if

If you worke by the Golden rule, you shall find the number of Gallons that shall be taken of each sort of wine: For the better distinction, whereof, I haue set these letters A, B, C, D, both against the numbers for which the works doe serue, and ouer the works also, which seuerally serue for each of them. And now (if you list to examine the truth of these workes,) adde those foure summes together, and they will make fiftie, that is the

The proofe of this rule

totall which I would haue, as by this 35
example you may easily perceiue. And $8\frac{1}{3}$
(for to proue how the prices do agree) $4\frac{1}{6}$
doe this: multiply this totall summe 12
50, by the common price 9, and it 50
will make 450: then keepe that summe
by it selfe, and afterward multiply euery seuerall summe of Gallons by the price belonging to the same Gallons: and if that sum do agree with this which you haue kept first, then is your worke well done. As heere, 25
is the number of Gallons of 6 pence
price, multiple then 25 by 6, and 150
it maketh 150, which you shall set $66\frac{4}{6}$
downe, then multiply $8\frac{1}{6}$ by 8 $45\frac{5}{6}$
which is the price for the number of $187\frac{1}{2}$
Gallons, and it will make $66\frac{4}{6}$: for againe $4\frac{1}{6}$ multiplied by 11 : doth 450
make $45\frac{5}{6}$ And last of all $12\frac{1}{2}$ multiplied by 15, maketh $187\frac{1}{2}$ & these added together do

doe make 450, as in the example annexed you may see: wherefore seeing it doth agree with the former sum of 50 multiplied by 9, I may lastlie affirme this worke to be good, and wel done.

And now to proue how you can do ye like, I propound the same question, onely willing you to vse some other forme of combining or linking the summes.

Scholar. That shall I prooue with your fauour, and therefore I combine 8 with 15, and 6 with 11, and then the forme will be as followeth.

9	6	2	A
	8	6	B
	11	3	C
	15	1	D
	12		

A		B	
12	50	12	50
2	$8\frac{1}{3}$	6	25

C		D	
12	50	12	50
3	$12\frac{1}{2}$	1	$4\frac{1}{6}$

Whereby amounteth the same summe in totall of the differences, as did before: and yet now the differences be altered, as the combination is changed, whereof I vnderstand the reason by your former worke. And therefore heere appeareth no strange thing, but that now I must haue 8 $\frac{2}{6}$ gallons of sixe pence, and 25 gallons of 8 d, and 12 gallons and $\frac{3}{6}$ of 11 d, and so consequently 4 gallons and $\frac{1}{6}$ of 15 d, so ye multiplying 8 $\frac{2}{6}$ by 6, it maketh 50, & then

50
200
$137\frac{1}{2}$
$62\frac{1}{2}$
450

then 25 multiplied by 8, maketh 200: likewise 12 $\frac{1}{2}$ multiplied by 11, yeeld 137 $\frac{1}{2}$, and 4 $\frac{1}{6}$ multiplied by 15, maketh 62 $\frac{3}{6}$, which 4 summes added into one, will yeeld in the total 450, which agreeth with the multiplication of 50 (beeing the totall sum of Gallons) by 9 the common or meane price.

Master. Seeing you conceiue this work so well, I wil propound an other example vnto you of more varietie in the Alligations or combinings, as thus.

A questiõ of spices.

A Merchant beeing minded to make a bargain for spices in a mixt masse, that is to say, of Cloues, Nutmegges, Saffron, Pepper, Ginger, and Almonds, the Cloues beeing at 6 s a pound, the Nutmegges at 8 s, Saffron at 10 s, Pepper at 3 s, Ginger at 2 s, and Almonds at 1 s.

Now would he haue of each sort some, to the value of 300 l in the whole, & each pound one with another to beare in price 5 s: how much shall he haue of each sort?

Scholar. That will I trie thus.

First, I set downe those sixe seuerall prices, & at the left hand I set the common price 5 s. Then, I linke them thus, one with 10, 2 with 6, and thrée with 8, as in the example following.

Master,

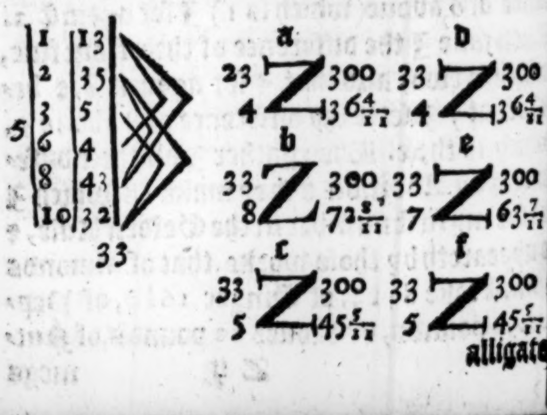
Master. I had minded to haue combined them in more varieties: but I am content to sée your owne worke first, and then more varieties in combination may follow anon.

Scholar. Then to continue as I began, I séeke the difference betwéene 1 and 5 (which is 4) and that I set against 10: then against 1 I set 5, which is the excesse of 10 aboue 5: so I gather the difference betwéene 2 and 5, which is 3, and that I set against 6, because it is combined with 2: and likewise the difference of 6 aboue (which is 1) I set against 2. Then take I the difference of thrée from fiue, which is two, and that I set against 8, & before that 3 I sette the difference of 8 aboue 5, which is thrée. Then gather I all these differences by Addition, & they make 18, which I set for my first number in the Golden Rule, & so appeareth by those workes, that of Almonds I must take 83 l $\frac{1}{3}$, of Ginger 16 l $\frac{2}{3}$, of Pepper 50 pounds, of Cloues 50 pounds, of Nut-

megs 33 pounds $\frac{1}{3}$, and of Saf-	$83\frac{1}{3}$
fron 66 pounds $\frac{2}{3}$. Then for	$33\frac{1}{3}$
triall heereof, I multiply eue-	150
rie parcell by his seuerall price,	300
as 83 $\frac{1}{3}$, which is the summe	$266\frac{2}{3}$
of Almonds, I multiplie by	$666\frac{2}{3}$
one, which is their price.	1500

Also 16 $\frac{2}{3}$ the summe of Ginger I multiply by two, which is the price of it. An so each other in his kinde, as this table annexed doth represent, and then adding them all together, I finde the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300 by the common price 5: wherefore it appeareth well wrought.

Master. Now will I make the alligation to proue your cunning somewhat better: but because you shall not thinke your selfe pressed so much, I will also note the differences, as by this example you may see, where I haue



alligate

alligate 1 with 6 and 8, and therefore haue I set against 1 both their differences, that is, 1 and 3. Likewise because 2 is combined with 8 and 10, I set before him their differences, 3 and 5. Against three I haue set only 5, which is the difference of 10, with whom 3 is combined only. Likewise, 6 is only alligate to 1, and therefore is the difference of 1 from 5: which is 4 only set against it: 8 is linked with 1 and 2, and therefore hath set 4 against him, both their differences, 4, and 3, and 10 is ioined with 2 and 3, therefore hath hee their differences, 3 and 2. And because of ease for you, in another columne I haue set the differences reduced into one number for euery seuerall sort, and haue also added them together, whereby appeareth that they make 33, and so consequently you see the workes of the Golden Rule set foorth. For the six drugges, I haue added letters, a, b, c, &c. as before. But I would not wish you to cleaue still to these elementarie aids, but accustome memorie to trust her selfe, so shall occasion of negligence be best auoided. And as for the proofe, trie it at more leisure, because the time now is short, and you sufficiently instructed in that proofe. And there resteth diuers things behinde yet, of which I would gladly giue you some taste before our departure.

Scholar. But if it may please you to let me

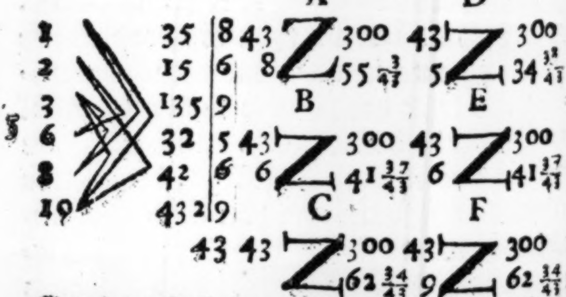
see all the variations of this question, before you go from it: for me thinketh) I could vary it two or three waies more yet.

Master. I am content to see you make two or three variations, but I would bee loth to stay to see all the variations, for it may be varied aboue 300 waies, although many of thẽ would not well serue to this purpose.

Scholar. I thought it impossible to make so many variations.

Master. Maruell not thereat: for some questions of this rule may be varied aboue 1000 waies: but I would haue you forget such fantasies til a time of more leasure. And now go forward with some variation of this questiõ.

Scholar. For the first variation, I linke the first number 1 with 8 and 10, & 2 I combine with 6 and 10, then isine I 3 with 6, 8, and 10, as in this forme.

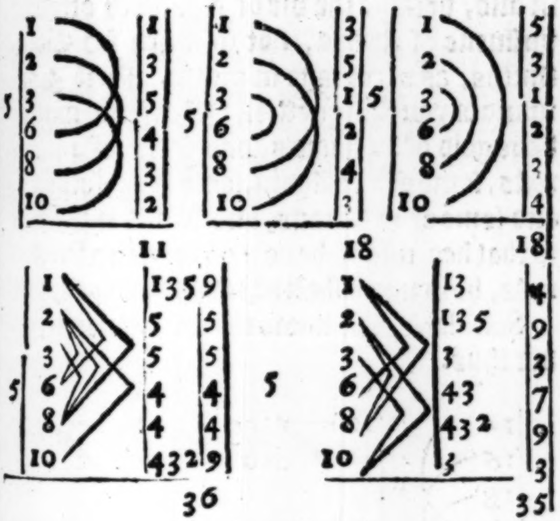


And so doth there appeare the proportion of weight for euery kinde of drugge in this mixture. Now for the triall.

Master.

Master. Nay, stay there: you shall not neede to make triall in one example so often, or if you list to do it by your selfe, I am content. But now set forth (for declaration that you conceiue the rule) two or three examples of seuerall combinations, and then will we passe to some other example, and so end this rule.

Scholar. As it pleaseth you, so will I doe. And these be the varieties: in which, as the

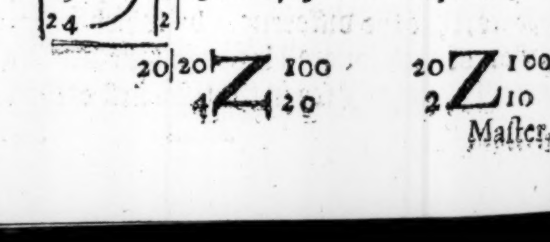


combinations are seuerall, so doth it plainely appeare, that the differences by which ye proportion of each seuerall kinde is takē, are also seuerall. And yet I see in the three first of these

fiue varieties, and in one other before, the totall ſumme of the differences to bee one, that is to ſay, 18, whereby I perceiue that the varietie of their mixture doth depend on the varietie of their differences ſeuerall, and not of the varietie of their totall ſumme.

Maſter. So is it. And ſeeing you conceiue it ſo well, I will make an end of this rule, only exhibiting vnto you one queſtion or two of the mixture of metals, that by it you may deuiſe other like, and exerciſe your ſelfe therein alſo, becauſe the vſe of it ſerueth often in buſineſſe of charge, not ſo much for Goldſmiths, as of coinage in mints. Firſt I demand of you this queſtion: If a Mint maſter haue gold of 22 karects, and ſome of 23 karects, ſome of 24: Again, ſome of 15, ſome 16, and ſome of 18 karects, and would mix them ſo that hee might haue 100 ounces of 20 karects, how much ſhall he take of euery ſort?

Scholar. To know that I anſwer in order thus.



Master. You haue wrought the question well: but how chanced you made no doubt of that new name Karect?

Scholar. Because I thought it out of time to demand such questions now, seeing you make so much haste to end: and againe in this case the proportion of the number is sufficient for my purpose in this worke: trusting, that another time you will instruct mee as well of this, as of sundry other things, which as I haue heard you talke of, so I haue a great desire to know them.

Master. Your answer is reasonable, and your request and trust (with Gods helpe) I intend to satisfie: and to goe forward with this matter, let mee see your examination of this last worke.

Scholar. First for the one part I adde together all the particular sums	10
	15
as they appeare in the worke, and	20
they make 100, as heere by their ad-	25
dition doth appeare.	20
And so it seemeth that the summes	10
are well gathered: but for the fur-	100

ther triall of them, I multiplie first

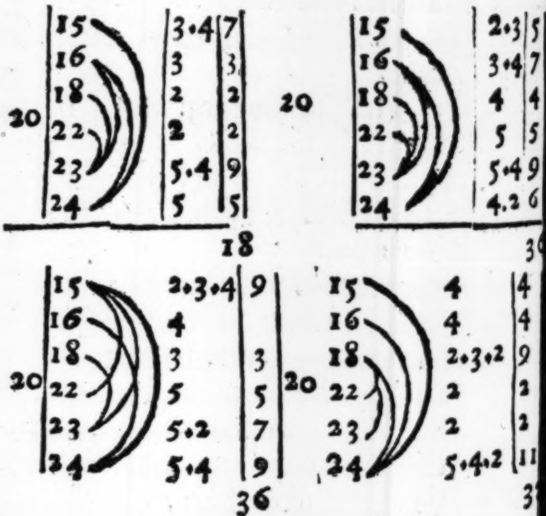
150	20, which is the common or meane
240	summe of the karects by 100, which
360	is the summe of the whole masse,
550	which I would haue, and it ma-
460	keth 2000. Then I multiply e-
240	uerie particular summe by the ka-
2000	rects that it doth containe, as 10

by 15, and that maketh 150.

Likewise I multiply 15 by 16, and it yeel-deth 240: so 20 by 18 maketh 360. And 25 by 22 yeeldeth 550: likewise 20 by 23 bringeth forth 460, and last of all 10 multiplied by 24 yeeldeth 240: which summes all ioined together make 2000 that doth agree with the like summe before, wherfore I may well say, that the worke is good. And now if (it please you) I would set forth some varieties of this question to proue my wit.

Master. Go to, let me see.

Scholar. Heere be foure varieties.



And more yet could I make, but not like to the number that you speake of in the variation of the other question.

Master. That wil I teach you at more leisure, seeing it is a thing rather of pleasure thã of any necessitie.

But now for your exercise in this rule, one other question I will propose. A Mint-master hath 6 ingots of siluer of sundry finenes, some of 4 ounces fine, and some of 5 ounces, some of 6, and other of 8, some of 11, & other of 12, and his desire is to mixe 500 pounds weight, so that in the whole masse euery poũd waight should beare 9 ounces of fine siluer: how much shall he take (say you) of euery sort of siluer?

A question of mixing of siluer.

Scholar. To finde out that, I sette the numbers thus in order.

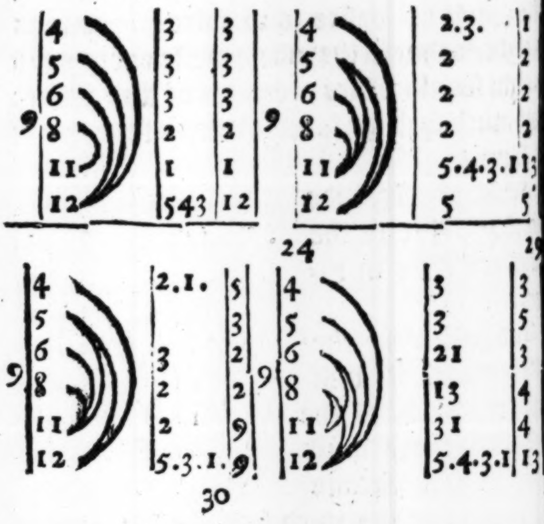
	4	2	2
	5	2	2
	6	3	3
9	8	3	3
	11	5.4	9
	12	3.1	4
			23

And gathering the differences, it will appeare, that, of the first sort there must be 43 $\frac{11}{23}$: of the second as much: of the third sort 65 $\frac{5}{23}$: and of the fourth sort as much: of the fifth sort 195 $\frac{15}{23}$: and of the sixth sort 86 $\frac{22}{23}$, which in the whole will make 500 l waight, and ounces after 9 ounces fine 4500, that is of the first sort 173 $\frac{21}{23}$: and of the second sort

217 $\frac{9}{23}$: of the third ſort 391 $\frac{7}{23}$: of the fourth ſort 521 $\frac{17}{23}$, of the fift ſort 2152 $\frac{4}{23}$, and of the ſixth ſort 1043 $\frac{11}{23}$, which all together do make 4500 ounces, agreeable to the multiplication of 9 by 500.

Maſter. This is well done of you, therefore now make three or foure varieties, and ſo an end of this rule.

Schol. Theſe 4 varieties I ſet for example.



Maſter. And by theſe it appeareth, that you can find out more, with which I will not now meddle, ſaue onely (for to ſhew you an eaſie helpe in drawing the lines of Combination) I will ſet foorth two varieties heere.

And

	4	2			4	3
	5	23			5	23
	6	23			6	23
9	8	3		9	8	23
	11	543			11	431
	12	431			12	5421
		35				39

And this shall suffice now for the rule of Alligation or mixture: for by these examples may you easily coniecture such other as doe appertaine to it, as well for the due working, as for varietie of drawing the lines of combination.

Scholar. Sir, albeit it pleased you erewhile, to put mee from my musing at the manifold varieties that may fall in these combinations, and termed them fantasies; yet my fantasie giueth me, that the consideration of this should in many other examples and cases of importance be very needfull, & the knowledge of it most profitable. Therefore ye may well thinke, that at another time conuenient I will request you to aid me héerein.

Master. Truth it is, that this consideration may fall in practise as well Politike as Philosophicall, and sundry waies in them be applied: therefore when time shall fall fit for the discussing of this consideration, you shall not want my helping hand.

The

The Rule of Falshood.

NOw will I briefly al-
so teach you somewha
of the rule of Falshoo
which beareth hi
name, not for that i
teacheth any fraud o
Falshode, but for tha
by false numbers ta-
ken at all aduentures
it teacheth how to finde those true number
that you seeke for.

The occasion of the name.

Scholar. So might any other rule be calle
the Rule of Falshood, for they work by wro
numbers, & by them find out the right num-
bers: so doth the Rule of Alligation, the Ru
of Fellowship, and the Golden Rule partly.

Master. In the golden Rule, the rule of Fe-
lowship, & the Rule of Alligation, although th
numbers that you worke by, be not the tru
numbers that you seek for, yet are they num-
bers in iust proportion, & are found by orderl
worke: whereas in this rule the numbers a
not taken in any proportion, nor found by o
derly worke, but taken at all aduentures.

And therefore I sometimes beeing mer
with my friends, and talking of such quest
ons, haue caused them that proponed su
questions, to call vnto them such children
Idiots as happened to be in the place, and

tak

take their answere, declaring that I would make them solue those questions, that seemed so doubtfull.

And indeed, I did answere to the question, and worke the triall thereof also by those answeres which they happened at all aduentures to make: which numbers seeing they be taken as manifest false, therefore is this rule called the Rule of false Positions, & for briefnesse, the Rule of Falshood: which rule, for readinesse of remembrance, I haue comprised in these few Verses following, in forme of an obscure Riddle.

Gesse at this worke as hap doth leed,
By chaunce to truth you may proceed.
And first worke by the question,
Although no truth therein be don.
Such falshood is so good a ground,
That truth by it will soone be found.
From many bate too many mo,
From too few take too few also.
With too much ioine too few againe:
To too few adde too many plain.
In crosse-wise multiply contrarie kinde,
All truth by falshood for to finde.

The sense of these Verses, and the summe of this Rule is this:

When any question is proponed appertaining to this Rule, first imagine any number that you list, which you shall name the first

fift position, and put it in ſtead of the true number, and then worke with it as the queſtion imporṫeth: and if you haue miſſed, then is the laſt number of that worke either too great or too little: that ſhall you note as hereafter ſhall be taught you, and you ſhall call it the firſt errour.

Then begin againe, and take another number, which ſhall be called the ſecond poſition, and worke by the queſtion: if you haue miſſed againe, note the exceſſe or default as it is, and call that the ſecond errour. Then multiply croſſe wiſe the firſt poſition by the ſecond errour, and againe the ſecond poſition by the firſt errour, & note their totals ſeuerally by the names of totals. Then marke whether the two errours were both alike, that is to ſay, both too much, or both too little: or whether they be vnlike, that is, the one too much, and the other too little: for, if they be like, then ſhall you ſubtract the one totall from the other (I meane the leſſer from the greater) and the remainer ſhall bee your Diuidend: ſo muſt you abate the leſſer errour out of the greater, and the reſidue ſhall bee the Diuiſor. Now diuide the Diuidend by that Diuiſor, and the Quotient will ſhew you the true number that you ſeeke for. But, and if the errours be vnlike, then muſt you adde both thoſe totals (which you noted) together, and take that whole number for the Diuidend, ſo ſhall

shall you adde both errours together, and that whole number shall bee the Diuisor, and the Quotient of that Diuision shall giue you the true number that the question seeketh for, and this is the whole rule.

Scholar. This rule seemeth so vnlike any other, that without some example I shall not easily vnderstand it.

Master. With a good will: propose halfe a score sundry questions and examples of variety, for the better vnderstanding of the worke hereof: and for the first take this example: A Mason was bound to build a wall in 40 daies, and it was couenanted so with him, that euery day that he wrought, hee should haue for his wages 2 s 1 d, and euery day that he wrought not, he should be amerced 2 s 6 d: so that when the wall was made, and the reckoning taken of the daies that he wrought, and of the other that he wrought not, the Mason had cleerely but 5 s 5 d for his worke. Now I doe demand how many daies did he worke of those 40, and how many did he not worke?

A question of Masonrie. The first example.

Scholar. I pray you expresse the order of the worke, that I may partly by imitation, and partly by comparing it with the rule, bee able againe to doe the like.

Master. This order shall you keepe in the worke of this rule: first take some number (as you list) at aduenture, as for example, I say he plaied 12 daies, and wrought 28 daies. Now

cast you the wages of euery day, and see whether it will agree with the summe of 5 s 5 d.

Scholar. The 28 daies that he wrought after 25 pence the day, yeeldeth 700 pence. Then the 12 daies that he wrought not, at 30 pence each day, doth amount to 360 pence, which if I abate out of 700 pence, there resteth 340: but you say he had not so much.

Master. He had but 65 pence, and by this supposition he should haue had 340: therefore is this summe too much by 275, which summe I must set downe after this sort, as you see heere, where first I haue made a crosse (commonlie called Saint Andrewes crosse) & at the ouer corner on ẏ left hand I haue set the first position 12: and at the other corner vnder it I haue set 275, which is the first errour, with this figure ✠, which betokeneth too much, as this line — plaine without a crosse line betokeneth too little. On the right hand of the crosse I haue left two like roomes for the second position and his errour. Therefore to prosecute the worke, I suppose he plaied 16 daies, and wrought 24.

12 ╳
275 ✠

Scholar. I was a while in doubt why you named the daies of his working, seeing they bee not set in the figure: and I doubted how you know them, or else whether that you did suppose them at all aduentures, as you did the daies that hee plaied: but now I gather

ther

ther, that seeing 40 daies is the whole time limited, then the daies that hee plaied being supposed, the rest of 40 must needs be the daies that hee wrought, and therefore 28 followed 12 of necessitie, and 24 followeth 16 also of necessitie: but yet I scarce perceiue why you set not in the figures as well 28 as 12.

Master. It forceth not which of them I take, so that in the second position I take the numbers of the same nature that is heere both of working daies, or both of idle: but now examine you this second position.

Scholar. If he plaied 16 daies, then abating 16 times 30 d, the summe will bee 480 d. And for 24 daies that hee wrought, euery day yeelding 25 d, the totall is 600 pence: so that abating 480 out of 600, there resteth 120, and as you say, it should be but 65: therefore it is too much by 55: that must be set on the right hand of the figure at the nether part, and ouer it on the same side 16, which is the second position, thus:

12 16

275 + 55 +

And as I gather by your words, it were all one if I did set 28 in stead of 12, and 24 in stead of 16.

Master. So were it. But this shall you marke, that of what nature soeuer the two positions be, of the same nature is the quotient. Therefore when the positions in this

question are 12 and 16, which both being numbers of the playing daies, the Quotient shal declare the true number of playing daies: whereas if the positions had béene 28 and 24, which are supposed to bee the working daies, then would the Quotient declare the true number of the working daies, and not of playing daies, as it will doe now. And therefore to continue the worke of this question, and to finde the true number of playing daies, I must multiplie crossewise the first position 12 by 55, that is the second errour, and the totall will be 660: then I multiply 275, and 16, and it yeeldeth 4400. Now because the errours are like, that is to say, both too much, I must subtract 660 out of 4400, and so remaineth 3740, which is the Divident. Againe, I must subtract the lesser errour 55 out of 275 that is the greater errour, and there will remaine 220, which shall bee the Divisor: then dividing 3740 by 220, the Quotient will be 17. Wherefore I say now constantly, that 17 is the true number of daies that the Mason plaied: and then it followeth, that hee wrought 23 daies, and so is the question answered.

The proof of this rule

Now for the order of triall of this worke, there needeth none other triall but onely this, to worke with this number according to the question, and if it agree, then appeareth the number to bee it that you would have. As héere

héere now séeing hee wrought 23 daies, and must haue for euery day 25 pence, the whole summe commeth to 575. Then againe séeing he plaied 17 daies, and must abate 30 pence for euery day, the whole summe of the abatement will bee 510: therefore I subtract 510 out of 575, and there will remaine 65, which maketh 5 s 5 d, the cléere wages of the Mason for his worke, according to the question.

Scholar. Now I trust I vnderstand the worke and the rule so well (and the better by this proofe) that I can be able to doe the like. And for a proofe, I take the same question all saue the last number, where I will suppose that hee had 10 s for his wages cleere. And now to ghesse at the number of the daies that hee wrought, I suppose first that he wrought 20 daies: then say I, If he wrought 20 daies, his wages must be 500 d, then did hee play other 20 daies, for which must be abated 600 d, and then he loseth 100 d. And so am I at a stay, for it is not like vnto your former worke.

Master. You should haue required of mée some question, and not haue taken a question of your owne fantasying, vntill you were more expert in this Art: for so might you as well happen on an impossible question, as on a possible: but now to goe forwards, consider that this nūber is too little by 220, séeing he should gaine by your supposition 120 d, and in this position hee loseth 100, those both make 220,

which you ſhall ſet downe for the firſt errour with this ſigne———, betokening to little, as heere in this forme following doth appeare.

And now for the reſt goe forward your ſelfe once againe.

Scholar. As my errour hath vttered my folly, ſo it hath procured me better vnderſtanding.

20

220—

Now therefore conſidering this poſition not to ſolue the queſtion, I take another, ſuppoſing that he wrought 30 daies: then for his wages he muſt be allowed 750 d, and for the 10 daies which hee wrought not he muſt abate 300 d, and ſo remaineth cleere 450 d: but it ſhould be onely 120 d, therefore is it too much by 330, which I ſet downe in the figure with the former poſition and his errour, and the figure appeareth thus:

20 30

200 330 +

Now muſt I multiply in croſſe waies 20 by 330, & it will be 6600.

Then againe I multiply 30 by 200, and it will be alſo 6600. Wherefore if I ſhall ſubtract the one out of the other, there will remaine nothing to bee the Diuident.

Maſter. In this you forget your ſelfe againe: for in as much as the ſignes in the errors be vnlike, therefore muſt you worke by Addition, adding together thoſe two totals to

make

make the Diuident, and also adding the two errors to make the Diuisor. And because you shall no more forget this part of the rule, take this briefe remembrance:

> Vnlike require Addition,
> And like desire Subtraction.

Scholar. You meane, that if the errours haue like signes, then must the Diuident and the Diuisor bee made by Subtraction, as is taught before. And if those signes bee vnlike, (as in this last example they bee) then must I by Addition gather the Diuident and the Diuisor. Therefore must I adde 6680 to 6600, and it will be 13200, which shall be the Diuident. Then againe I adde 220 to 330, and it will bee 550, which must bee the Diuisor: wherefore diuiding 13200 by 550, the Quotient will be 24, whereby I know that the Mason wrought 24 daies, and then it followeth that he plaied 16 daies.

Master. Examine your worke, whether it be agreeable to the question or no.

Scholar. For 24 daies worke the wages must be 600 d, and for 16 daies which the Mason wrought not, there must bee abated 480, and then remaineth cleere to the Mason 120 pence, as the question importeth: wherefore it is euident that 24 is the true number of daies that he wrought.

Master. Although you seeme now to vnderstand this worke, yet to acquaint your minde the better with the new trade of this rule, I thinke it good to propone to you fiue or six examples more, before I make an end of it.

Scholar. Sir I thanke you that you doe so consider my commodity and profit in knowledge, for vndoubtedly it is practise and exercise that maketh men prompt and expert in euery kinde of knowledge.

Master. You say well, so that they follow some certain precepts to gouern and rule their practise by, else may practise procure custome of errour, and a repugnance to exactnesse of knowledge, namely as long as the errour is not plainly knowen to the vulgar sort. But to returne to our worke.

There is a seruant that hath bought of veluet and damaske for his master 40 yards, the veluet at 20 s a yard, and the damaske at 12 s: and when he commeth home, his master demandeth of him how much hee hath bought of each sort: I cannot tell (saith he) exactly: but this I know, that I paid for Damaske 48 s more then I paid for Veluet: now must you ghesse how many yards there is of each sort.

Scholar. Although the ghesse seemeth difficult, yet I will prooue what I can doe: for I remember your saying, that it forceth not how fond or false the ghesse bee, so it be some-

what

what to the question, and not an answer of a contrary matter.

Therefore first I imagine that hee bought 20 yards of Damaske, for which hee should pay after the former price 240 shillings: then must hee needs haue of Veluet other 20 yards (to make vp the 40 yards) and that would cost 400 s. So that the totall of the price of the Damaske is lesse then the summe paid for Veluet 160 s, and should be more by 48: therefore the first errour is 208 too little. Then begin I againe, and suppose hee bought of Damaske 30 yards, that cost 360 s: then had hee but 10 yards of Veluet, which cost 200 s: and now the price of the Damaske is greater then the price of the Veluet by 160 shillings, and should be but 48, therfore is the second errour 112 too much, which I set in forme of figure as heere doth appeare:

Then doe I multiply in crosse waies 208 by 30, and the summe will bee 6240. Also I multiply 112 by 20, and there will amount 2240. And in as much as the signes of the errours bee vnlike, I know I must work by Addition, therfore adde I those two totals together, and they make 8480, which is the Divident: then adde I also the two errours together, 208 and 112, and they make 320, which is the Divisor. Where-

20 30

208 — 112 +

fore

fore diuiding 8480 by 320, the quotient will be 26 $\frac{1}{2}$, which is the true summe of yards of Damaske that he bought, and in Veluet 13 yards $\frac{1}{2}$, and that appeared by examination, thus: 26 $\frac{1}{2}$ yards of Damaske at 12 s the yard, maketh 318 s: then in Veluet he had but 13 yards and $\frac{1}{2}$, that cost 276 s, at 20 the yard. Now subtract 270 out of 318, and there will remaine 48, which is the number of shillings that the Damaske did cost more then the Veluet.

Master. Now shall you haue a question of another kinde.

There are thrée men that doe owe mony to mee, and I haue forgotten what the totall summe is, and what the particulars be.

Scholar. Why, then it is impossible to know the debt.

Master. Peace: ye are too hastie: there is more helpe in it then yet you sée. I haue thrée seuerall notes, whereby it appeareth that I did conferre their debts together, and found the debt of the first and the second to amount to 47 l', the debt of the first man and the third man did make 71 l', and the second man his debt with the third, did rise to 88 l'. Now can you tell what euery man did owe, and what was the whole totall?

Scholar. Nay in good faith: but as I perceiue that it must bee found by coniecture, I will I ghesse at it, supposing that the first man

di

[d]id owe 20 l, & the second man 30, and ÿ third.

Master. Nay stay there, for you are too far [g]one already: you may not suppose a severall [s]umme for every man, for it is enough to [s]uppose one summe for the first man, and let [t]he other rise as the question importeth. [T]herefore seeing you set the 1 man his debt [t]o bee 20 l', the second man cannot owe 30 l': [f]or the declaration is, that their debts added [t]ogether did make 47 l', so must the second [m]an his debt bee but 27 l'. Now this second [d]ebt with the third must make 88: therefore [s]ubtract 27 out of 88, and there will remaine [6]1, as the third man his debt. Then saith [t]he declaration, that the first and third mens [d]ebts doe make 71: but by this supposition [t]hey make 81, that is 10 too much, which I [m]ust set for the first errour. Now worke you [t]he second position.

Scholar. I suppose the first mans debt to [b]ee 24 l': then must the second mans debt (by [y]our declaration) bee but 23 l', seeing both [t]hey make but 47 l'. And the second man his [d]ebt with the third, doe make 88 l', and the se-[c]ond man oweth but 23: therefore the third [m]an must owe 65 l'. Now the third mans [d]ebt with the first should make by the decla-[r]ation 71 l', and they doe make 89 l', that is [1]8 l too much, and that is the second errour, [w]hich I set downe with the first, and their [p]osition in this forme, and then I doe multi-

ply

ply in croſſe waies 20 by 18, and it is 360. And 10 by 24 maketh 240. Alſo becauſe the ſignes of the errours be like, I muſt worke by Subtraction: therefore I ſubtract 240 out of 360, and there reſteth 120, which is the Divident: then doe I ſubtract 10 out of 18 by the ſame reaſon, and ſo is the Diviſor 8, which is found 15 times in 120: therefore I ſay, that the firſt man did owe 15 l, and then the ſecond man muſt owe 32 l: for thoſe 2 doe make 47 l, and the third man his debt is 56, for ſo much remaineth if I bate 15 out of 71, or if I take 32 out of 88.

20 24
X
10 + 18

Maſter. For the fourth example take this eaſie queſtion for the varietie in worke. Two men having ſeverall ſummes which I know not, doe thus talke together: The firſt ſaith to the ſecond, If you give me 2 s of your money, then ſhall I have three times ſo much money as you. The ſecond anſwereth, It were more reaſon that our ſummes were made equall, and ſo will it bee if you give mee 3 s of your money. Now gheſſe what each of them had.

Scholar. I imagine that the firſt had 9 s.

Maſter. Conſider evermore in your imagination, that you take a likely ſumme, as in this queſtion, take ſuch a ſumme that having 2 added unto it, may be divided into 3 parts even.

Scholar.

Scholar. Why? I remember you said be-
fore, it forceth not how fondly soever I ghes-
sed.

Master. As for the possibility of the soluti-
on, it is truth: but for easinesse in worke, the
aptest numbers are most conuenient.

Scholar. I thought no lesse, and therefore
I tooke 9 as an apt number to bee parted into
three: but I perceiue I should haue conside-
red the aptnesse of that partition after the ad-
dition of two vnto it, and then 7 had beene
more meet.

Master. That is truth, and then should the
second man his summe be 5: for although hee
haue now but the third part of 9, that is 3, yet
you must remember that he lent the first man
2, and so had he 5.

Scholar. Then to go forward: if the second
man had thrée of the first man, then should hee
haue 8, and the first man but 4: so hath hee
double to the first man: yet he said in the que-
stion they should haue equall: wherefore it ap-
peareth that he hath 4 to much.

Therefore I note that errour with his sup-
position, and ghesse againe that he hath 10 s:
whereunto I adde 2 shillings borrowed of the
second man, and then hee hath 12 shillings:
the second man hath remaining but foure,
whereunto if I adde the 2 that hee lent to the
first man, so had hee but 6 s at the beginning.
Then take 3 shillings from the first man, and
giue

giue to the second, then hath the first man but 7, and the second hath 9, which are not equall, but there are two too many, wherefore I set downe both the positions with their errours, as heere you see, and multiply acrosse, so commeth there 40 and

7 10

4 + 2 +

14: and because the signes bee like, I take 1 out of 40, and so resteth 26 to be the diuident then likewise I take 2 out of 4, and there re steth 2, by which I diuide 26, and the quo tient will bee 13, which is the summe tha the first man had. And so appeareth, that two being added thereto, the summe will be 15, hath the second man now but 5, and before h had 7: then take three from the first, and put t his 7, so haue each of them 10, and that is e quall, as the question would.

A questiō of Lambs. The fifth example.

Master. For the fifth example take th question. One man said to another, I thin you had this yeere two thousand Lambes: had I, said the other: but what with payi the tith of them, and then three seuerall losse they are much abated: for at one time I lo halfe as many as I haue now left, and at nother time the third part of so many, and t third time $\frac{1}{4}$ so many. Now ghesse you ho many are left.

Scholar. Because heere is mention ma of certaine parts, I must take a number th m

may haue all these parts, that is to say, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, which will be 24. howbeit 12 hath the same parts. Therefore first I take 12 to bee the number that doth remaine, so hath hee lost 6. 4, and 3, that is, 13, and in the whole 25, but it should be 2000.

Master. Ye are deceiued yet still: you haue forgotten the 10 part, which must bee defalked, that is, 200, so there remaineth but 1800: and now goe on againe.

Scholar. Then to finde the errour, I take 25 out of 1800, and there remaineth 1775 too few, which I set for the first errour. Then for the second position I take 24, whose halfe is 12, the third part 8, and the quarter 6, whereby riseth 50, which is too little by 1750, therefore I set downe both the positions with their errours, thus:

12	24
1775 +	1750 +

And multiply in crosse waies 1775 by 24, whereof commeth 42600. Also I multiplie 1750 by 12, and there ariseth 21000. And because the signes are like, I doe subtract the one from the other, and so remaineth the Diuident 21600: Then doe I subtract 1750 out of 1775, and there resteth 25, by which I diuide 21600, and the Quotient is 864, whereof the halfe is 432, and the third part is 288, the quarter is 216, which all being added

added together will make 1800: And if you adde thereto the tenth which was abated before, then will the whole sum be 2000. And now doth there come a question to my memorie which was demanded of me, but I was not able to answer to it: and now me thinketh I could solue it.

864
432
288
216
1800

Master. Propone your question.

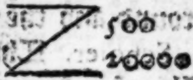
A question of sheepe & tillage. The sixth example.

Scholar. There is supposed a law made, that (for furthering of tillage) euery man that doth keepe sheepe, that for euery tenne sheepe eare and sow one acre of ground: and for his allowance in sheepe pasture, there is appointed for euery foure sheepe one acre of pasture. Now is there a rich sheepemaster which hath 7000 acres of ground, and would gladly keepe as many sheepe as hee might: by that Statute, I demand how many sheepe shall hee keepe?

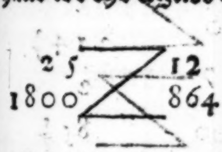
Master. Answere to the question your selfe.

Scholar. First I suppose he may keepe 500 sheepe, and for them hee shall haue in pasture after the rate of foure sheepe to an acre, 125 acres, and in arable ground 50 acres, that is 175 in all: but this errour is too little by 6825. Therefore I ghesse againe that hee may keep 1000 sheepe, that is in pasture 250 acres: and in tillage 100 acres, which maketh 350 that is too little by 6650.

some other: but to the other worke of your question: both this last question, and the next before might be wrought without the second position, by the rule of proportion, as this. When in this question ye found in the first errour, that for 500 sheepe, there must be 175 acres, then might you reduce it to the golden rule, thus. An other way of working.



If 175 acres will admit in allowance 500 sheepe, then 7000 will have 20000. And so by one position, with the helpe of the Golden rule, may you answere that question. Likewise for the question of Lambes, when you had found that 12 came of 25, you might have set the figure thus as ye see, & have sayd: If 25 doe leave but 12, what shall 1800 leave? and it would appeare to be 864.



Scholer. Sir, I thanke you for this aide, for it doeth much shorten the worke of this rule.

Master. Yet againe I will shewe you an other way to answere to this last question without this rule of False position, and that by the rule of Fellowship, for it appeareth in the proponing of the question, that 10 sheepe must have in pasture 2 acres and $\frac{1}{2}$, and for thē must there be eared but one acre: so it fol- An other way yet.

loweth, that for two acres eared, there muſt be 5 ſet to paſture. And if you put them both into one ſumme, they will make 7. Therefore looke what proportion 7 being this total, doth beare to 5 and to 2, ſuch proportions ſhall any totall in this queſtion beare to the paſture ground, and the eared ground.

Scholer. This ſerueth wonderous aptly. Therefore to proue it, I demand this by the former ſuppoſition: If a man haue 300 Acres, how much ſhall he leaue in paſture, and how much ſhall he turne to tillage? You ſay that as 7 is to 5, ſo ſhall 300 be to the acres of paſture: and as 7 is to 2, ſo is 300 to the acres of tillage, whereof for both I haue ſet examples here following, whereby appeareth that of Paſture there ſhall be $214\frac{2}{7}$ Acres, and of Tillage $85\frac{5}{7}$, which both ſummes added together, do make 300.

7	5
300	$214\frac{2}{7}$
7	2
300	$85\frac{5}{7}$

An other queſtion.

Maſter. Now take an other example: A man hath thrée ſiluer cuppes with one couer, the couer waieth 18 vnces, the ſecond cuppe wayeth euen halfe the waight of the firſt and the third. Now if the couer be put to the firſt cuppe, they wey iuſt as much as all the thrée cuppes do wey: and if the couer be ioyned with the ſecond cuppe, they wey as much as the ſecond twice, and the third: and if the couer

couer be put to the third cup, they will make twice as much as the first and the second cup. Now trie you what was the iust weight of euery cup.

Scholer. I doe set the weight of the first cup to be 9 ounces: then in as much as these two (that is to say, the couer and the first cup) doe wey the weight of the thrée cuppes, I sée that the thrée cups must wey 27 ounces, for so much is 18 and 9. Also because the first and the third do wey double so much as the second, therefore is it the third part of that weight, that is 9, and then would it followe, that the third cup also should wey 9 ounces, but then the question saith, that the couer being ioyned to the second cuppe, they wey as much as the second twice, and the third once, that should be 27, and so it doth: then béeing ioyned with the third cuppe, they should wey twice as much as the first & the second, that should be 36, and they wey but 27, so is that error 9 too little. Then begin I againe, and say, that the first cuppe doth wey 12 ounces, which I ioyne with the couer, and they make 30 ounces: then séeing the second is ' of that weight, it must néeds wey 10 ounces, and the third must wey 8 ounces, séeing the first and the third must wey 20 ounces. Now put I the couer to the second cup, and they wey 28 ounces, which should be euen so: then ioyne I the couer with the third cup, and so should

George Killness onnes this booke god giue [illegible] on it to looke

it wey twice the first, & the second, that is 44 ounces and they doe wey but 26, that is 18 too little: those errors

$$\begin{array}{ccc} 9 & & 12 \\ & \times & \\ 9- & & 18- \end{array}$$

with their positions I set downe, and multiply in crosse wayes 9 by 12, whereof commeth 108. Also 9 by 18, and that yeeldeth 162: and in as much as the signes be like, I abate the lesser out of the greater, and there doeth remaine 54. Then doe I also abate the lesser error from the greater, and so remaineth 9, by which I diuide 54, and the quotient is 6: which I take for the true weight of the first cup: which being ioyned with the couer must wey as much as the three cups, so do they wey but 24 ounces. Then seeing the second cup is the third part of that weight, for the other two cuppes (you say) must wey double his weight, the weight of the second cuppe is 8 ounces, and so the weight of the third must be 10 ounces. Now put the couer to the second cup, and it will make 26 ounces: that must be the weight of the second twice, and the third once, that is twice 8, and once 10, and so is it. Againe, put the couer to the third cup of 10 ounces, and they must wey twice as much as the first and the second, that is 28: and so is all agreeable.

A qestion of water.

Master. Then answere to this question.

There is a Cesterne with foure cockes, con-

containing 72 barrels of water: and if the greatest cocke be opened, the water wil auoid cleane in 6 houres: at the second cocke it will aske eight houres: at the third cocke it will a-uoide in no lesse then nine houres: and at the smallest it will require twelue houres: Now I demaund in what space will it auoid, all the cockes being set open?

Scholer. First I imagine that it wil auoid in two houres.

Master. Then must there auoyde by the first cocke $\frac{1}{3}$ of the water, that is 24 barrels, and by the second cocke $\frac{1}{4}$ that is 18, and by the third cocke $\frac{2}{9}$ that is 16 barrels, and by the smallest cocke $\frac{1}{6}$, that is 12 barrels, all which summes put together, doe make 70, as by their addition it doth appeare, but it should be 72, therefore the error is 2 to few.

Scholer. Then I begin againe by your fauor, because I think I vnderstand the worke, and put thrée houres for the due time: so shall there run out at the greatest cocke ', that is 36 barrels, & at the second hole $\frac{3}{8}$ that is 27, and at the third cocko $\frac{1}{3}$ that is 24, and at the smallest hole $\frac{1}{4}$, that is 18 barrels, which altogether do make 105, and should be but 72, so is it too much by 33, therefore do I set the errours in order of the figure with their positions, and worke by multiplication,

$$\begin{array}{r} 24 \\ 18 \\ 16 \\ 12 \\ \hline 70 \end{array}$$

in

in crosse, saying: 2 times 3 is 6, and two times 33 maketh 66: and because the signes are unlike, I must adde those two totals together, which make 72: also I adde the two errors, and they make 35, by which I divide 72 and the quotient riseth $2\frac{2}{35}$, whereby I see that all the cockes being set open, the water will auoide in 2 houres and $\frac{2}{35}$ of an houre.

2 3

2 — 33

Maister. This exercise maketh you to grow expert in the Rule. Therefore I will inure you somewhat more with a question or two.

A question of partners

There were two men that had béene partners, and had in account betwéene them 300 duckets: whereof the one should haue for his part 180, and the other 120: but in the parting of them, they fel at variance, so that each of them catched as many as he could: yet afterward being reconciled, they agréed that he which had gotten most part of them, should lay downe $\frac{1}{4}$ of them againe, and he that had gotten least, should lay downe $\frac{1}{3}$ of those which he had taken, and then parting them into two equall parts, each man to haue halfe thereof, and so had they their iust portions, as they ought: now I demaund of you what each of them had gotten by the skambling?

Scholler. I suppose he that had least, got

108 duckets, then the other had 192: wherefore in laying downe againe of the 192, there was put downe $\frac{1}{4}$ that is 114, and so had hée left but 48. Also of the 108: there was layde downe 36, that is' , and so hée had left 72. Then I put together 144, and 36, and it maketh 180, which I part into two parts euen, and so commeth 90 to be giuen to ech of them: which sum put to 72, maketh 162, and ioyned to 148, it maketh 238: and now I doubt how I shall go forward.

Master. You néede not to take but one of them which you list, the greater or the smaller, for all commeth to one purpose: and so may you compare it that you take to any of the other sums, remembring that you make comparison to the same in the second worke: as for example of the first part. If you compare 138 with the lesser summe due, that is 120, so is it 18 too much: and if you compare it with the greater summe, then is it 42 too little. Againe, if you compare 162 to the greater summe, the error will be 18, as it was in the other: but it will haue a contrarie signe: and if you compare it with the lesser summe, it will be 42 too much: so that the error both waies is either 18 or 42: and as for the signes it little forceth, for in them is nothing considered here, but likenesse and vnlikenesse, which in this case, doth neither further nor hinder. But now go on with the worke. Note.

Scholer. If it be so, then am I out of my greatest doubt. Then I ioyne that 90 (which I found as the halfe of the latter partition) vnto 48, which is left with the one man, and so hath he 138, which (I may say) is 18 to many, for the least should be but 120; that error doe I note, and then make a newe position, supposing the one man to haue 204, and the other to haue 96, wherefore of the 204, there must be layd downe 153, and so remaineth with him 51. Also of the 96, there must be laid downe $\frac{1}{3}$, that is 32, and so resteth with that man 64. Now of the 153 and 32 I make one summe as 185, which I must diuide into two equall parts, and so each man shall haue 92 $\frac{1}{2}$, whereunto if I adde their former portions reserued, then the one shal haue 156 $\frac{1}{2}$, and the other hath 143 $\frac{1}{2}$. Wherfore I take the lesser sum now againe, as I did before, that is 143 $\frac{1}{2}$, and finde that hee hath to many by 23 $\frac{1}{2}$, for he should haue but 120, so haue I for my two positiõs two errors, which I set downe, as here may bee seene, each error vnder his position, and then by the rule I doe multiply in crosse waies 108 by 23 $\frac{1}{2}$ and there riseth 2538, which I note then again I multiply 96 by 18, & thereof amounteth 1728.

108	96
18++	23$\frac{1}{2}$++

Now

Now because the signes are both like, that is both too many, I must worke by Subtraction, and so abating 1728 out of 2538, there will rest for the divident 810: then for the divisor I subtract 18 out of $23\frac{1}{2}$, and there remaineth $5\frac{1}{2}$, by which I divide 810, and the quotient will be $147\frac{3}{11}$, which is the iust portion of him that had the least summe. And if I do subtract it out of 300, being the totall summe, then will there remaine $152\frac{8}{11}$ as the portion that the other did get.

Master. For the proofe of this worke, you may choose whether you will examine those numbers according to the forme of the question, or else worke by other two positions for to finde the second number: and if those positions bring the same numbers ẏ did amount by the first two positions, then doth ech worke confirme other.

Scholer. By your patience, I will proue both waies, not onely to see their agreement, but also to accustom my mind to those workes: for I perceiue it is exercise that must bee the chiefe engrauer of these rules in my memory.

Master. You consider it well: then goe to.

Scholer. First I will by two other positions trie to finde the portion of him which had most.

Master. Although you may do it with any positions, yet to see the agreement of your

worke

worke the better, take the same positions that you did before, comparing them now to the greater, as you did before vnto the lesser.

Scholler. Then I suppose, that he that had most, had 192, so had the other 108. Now if I take $\frac{3}{4}$ out of 192, that will be 144, & there will rest to that man but 48. And from the second which had 108, if I take $\frac{1}{3}$, that is 36, there will remaine to him 72: then ioyning 144 with 36, it will make 180, the halfe whereof being 90. If I adde to each of those two mens portions remaining with them, the one shall haue 138, and the other 162, of which two I take the greater (that is 162) and see it to be 18 to fewe, for it should be 180, that error I note vnder this position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then take I $\frac{3}{4}$ of 204, and it will be 153, and there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is 32, and there remaineth to him 64. Now put I that 32 to 153, and it yeeldeth 185: which being parted in equall values, maketh $92\frac{1}{2}$ to be added to each mans remainder, and so the one hath $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherefore I take the greatest summe, and it is $23\frac{1}{2}$ too little, that do I note also, and set both these errors vnder their positions, as in this example following doth appeare.

And then multiplying 192 by $23\frac{1}{2}$, there doeth

doth arise 4512. 192 204.
Againe, I multiply 204 by 18, and it maketh 3672, which I do subtract out of 4512, 18— $23\frac{1}{2}$
because the signes be like, and there resteth 840 for the devident: then subtracting 18 out of $23\frac{1}{2}$, there will remaine $5\frac{1}{2}$ which I must take for the divisor. And so dividing 840 by $5\frac{1}{2}$, the quotient will be $152\frac{8}{11}$, whereby I have found an agreeable sum to that which I found by the former positions, for him that had most, which I doe subtract out of 300, that is the totall, there wil rest $147\frac{3}{11}$, which was the portion of him ẏ had the least part.

Master. So by divers positions you see, that one doeth confirme the worke of the other. Now examine those two numbers by the forme of the question, and so shall you prove your worke good also.

Scholer. If that he which gate most, had $152\frac{8}{11}$, then must he lay downe $\frac{3}{4}$ of his sum, that is $114\frac{6}{11}$, and so shall remaine with him but onely $38\frac{2}{11}$. The other which had least, that is $147\frac{3}{11}$ must put downe of his sum $\frac{1}{3}$, that is $49\frac{1}{11}$, and so doeth there remaine with him yet $98\frac{2}{11}$. Then do I adde together $114\frac{6}{11}$ and $49\frac{1}{11}$, and it will make $163\frac{7}{11}$, which I must part into two equall parts, and that wil be $81\frac{9}{11}$, to be given to each of them: so putting $81\frac{9}{11}$ unto $38\frac{2}{11}$, there doth amount 120 iust,

which

which is the true portion of him that should haue the lesser summe: and adding 81 $\frac{2}{11}$ to 98 $\frac{2}{11}$, the totall will be 180, the true portion of the other. And so is the worke by this proofe also tried to be good. And this I marke by the way, that in their scambling hee gate most (as it chanceth often) that ought to haue had least by iust partition.

Master. Let your study be to learne truth and iust art of Proportion, and to distribute and part according thereunto, as often as occasion shall be ministred. And heere would I make an end of this rule, saue that I remember one pleasant question which I cannot ouerpasse, which I wil declare somewhat largely, because you shall as well vnderstand some reason in the pleasant inuention as apt proceeding in the wittie working thereof.

An example of mixture of gold and siluer.

Hiero king of the Syracusans in Sicilia, had caused to be made a crowne of Gold of a wonderfull waight, to bee offered for his good successe in wars: in making whereof the Goldsmith fraudulently tooke out a certain portion of golde, and put in siluer for it, so that there was nothing abated of the full waight, although there was much of the value diminished. Which thing at length being vttered, (as no euil can alwaies lie hid) the king was sore moued: and being desirous to trie the truth, without breaking of the Crowne, proponed the doubt to Archimedes, vnto whose wit

wit nothing seemed unpossible, which althogh presently he could not answere unto, yet hee had good hope to deuise some policy for that inuention. and so musing thereon, as he chaunced to enter into a baine full of water to wash him, he obserued that as his bodie entred into the baine, the water did runne ouer the Tub: whereby his ready wit of such small effects coniecturing greater works, conceiued by and by a reason of solution to the Kings question, and therefore reioycing exceedingly more then if he had gotten the Crowne it selfe, forgate that he was naked, and so ran home, crying as he ran, εὕρηκα, εὕρηκα, I haue found, I haue found. And thereupon caused two masse peeces, one of golde, and another of siluer, to bee prepared of the same weight that the sayde crowne was of: and considering that golde is heauier of nature then siluer, and therefore golde of like weight with siluer must needes occupie lesse roome, by reason it is more compact and sound in substance, hee was assured, that putting the masse of golde into a vessell brimme ful of water there would not so much water run ouer, as when he should put in the siluer masse of the like weight. Wherefore he tried both, and noted not only the quantities of the water at each time, but also the difference or excesse of the one aboue ẏ other, whereby he learned what proportion in quantitie is betweene golde and siluer of equall

weight.

weight. And then putting the crowne it selfe into ẏ vessell of water brimme ful (as before) marked how much water did run out then, and comparing it with the water that ranne out when the golde was put in, noted how much it did excéede that: and likewise comparing it to the water that ran out of the siluer, marked how much it was lesse then that, and by those proportions found out ẏ iust quantitie of gold that was taken out of the crowne, and how much siluer was put in stead of it. But séeing Vitruuius which writeth this historie, doth not declare the particular worke of this triall, it shall be no inconuenience to suppose an example for declarations sake, wherein although the true and iust proportions be not expressed, yet the forme of trial shal be truly set forth. And for an example, I suppose the weight of the crowne to be 8 pound, & so of each the other two masses. And when the masse of gold was put into the water, I imagine that there ran out 2 pound of water: & when the masse of siluer was put in, I suppose there ran out 3 pound $\frac{1}{2}$. Againe, when the crowne was put in, there ran out 2 pound $\frac{1}{4}$. Now to know what quantitie of siluer was in the crowne, worke by the rule of false position, and imagine that there was 2 pound of siluer, then must there be 6 pound of golde. then say thus by the rule of proportion: if 8 pound of gold do expel 2 pound of water, what shall

shall 6 pound expel? and it will be 1 pound $\frac{1}{2}$. Againe for the siluer: if 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall 2 pound of siluer put out? it will be $\frac{7}{8}$. now adde those two weights of water together, and they will make 2 pound $\frac{3}{8}$, and it should be by the supposition 2 pound $\frac{1}{4}$, so is it too much by $\frac{1}{8}$.

Scholer. Now do I understand the worke as I thinke, therfore I pray you let me work the rest of the question. And because this first supposition did erre, I note that position, and his errour, and take a new position, esteeming ẏ siluer to be but one pound, so must there be in gold 7 pound. Then say I: if 8 pound of gold do yeeld 2 pound of water, what shall 7 pound yeeld? and it will be 1 pound $\frac{3}{4}$. Again if 8 pound of siluer expel 3 pound $\frac{1}{2}$ of water, what shall 1 pound expell? and it will be $\frac{7}{16}$. Now must I adde those two summes together, and they make 2 pound $\frac{3}{16}$, & they should make 2 pound $\frac{1}{4}$, so is it too little by $\frac{1}{16}$. Therefore I set the positions with their errours in order, as here followeth. And then I multiplie in crosse waies 2 by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: likewise 1 multiplied by $\frac{1}{8}$: maketh $\frac{1}{8}$. And because the signes be unlike, I must adde those two summes which make $\frac{1}{4}$, and that is the diuident. Againe I must adde $\frac{1}{8}$ to $\frac{1}{16}$, and it wil be

2   1

$\frac{1}{8}$ ++ $\frac{1}{16}$

be $\frac{9}{16}$, that is the diuisor. Now I shal diuide $\frac{3}{4}$ by $\frac{9}{16}$ and the quotient will be $\frac{16}{12}$, that is $1\frac{1}{3}$ whereby I know that there was put 1 ℔ and $\frac{1}{3}$ of siluer into the crowne, and so much golde taken out of it.

Master. Proue it now by examination according to the question.

Scholer. If there were 1 pound $\frac{1}{3}$ of siluer, then was there of gold 6 pound $\frac{2}{3}$. Now say I by the rule of proportion:

If eight pound of gold expell two pound of water, what shall 6 pound $\frac{2}{3}$ expell?

8	2
$6\frac{2}{3}$	$1\frac{2}{3}$

It will be one pound $\frac{2}{3}$. Again

8	$3\frac{1}{2}$
$1\frac{1}{3}$	$\frac{7}{12}$

If 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall $1\frac{1}{3}$ expell? It will be $\frac{7}{12}$.

Now must I adde together 1 pound $\frac{2}{3}$, and $\frac{7}{12}$, and they will make 2 pound $\frac{9}{36}$, that is 2 pound $\frac{1}{4}$, according to the supposition of the question, whereby I perceiue the worke to be wel done. And as I cannot but much reioice of this excellent inuention, so my desire is kindled vehemently to be perfectly instructed in euery part thereof, & namely in this point, whether the portion betweene water and gold be such, that for 8 pound of gold into a vessell full of water, there shall run out 2 pound of water: and for as much siluer, whether 3 pound $\frac{1}{2}$ of water would auoide?

Master. I perceiue your meaning, and con-

be $\frac{3}{16}$, that is the Diuisor. Now I shall diuide $\frac{1}{4}$ by $\frac{3}{16}$, and the quotient will be $\frac{16}{12}$, that is, 1 $\frac{1}{3}$: whereby I know that there was put 1 l and $\frac{1}{3}$ of siluer into the Crowne, and so much gold taken out for it.

Master. Proue it now by examination according to the question.

Scholar. If there were 1 pound $\frac{1}{3}$ of siluer, then was there of Gold 6 pound $\frac{2}{3}$. Now say I by the Rule of proportion:

If eight pound of Gold expel two pound of water, what shall 6 pound $\frac{2}{3}$ expell?

8	—	2
6 $\frac{2}{3}$	—	1 $\frac{2}{3}$

It will be 1 pound $\frac{2}{3}$. Againe, If eight pound of siluer expell three pound $\frac{1}{2}$ of water, what shall 1 $\frac{1}{3}$ expell? It will be $\frac{7}{12}$. 4 Rule.

8	—	3 $\frac{1}{2}$
1 $\frac{1}{3}$	—	$\frac{7}{12}$

Now must I adde together 1 pound $\frac{9}{36}$, and $\frac{7}{12}$, and they will make two pound $\frac{9}{3}$, that is 2 pound $\frac{1}{4}$, according to the supposition of the question, whereby I perceiue the worke to be well done. And as I cannot but much reioice of this excellent inuention, so my desire is kindled vehemently to be perfectly instructed in euery part therof, and namely in this point, whether the portion betweene water & golde, be such, that for 8 pound of gold into a vessell full of water there shall run out two pound of water, and for as much siluer, whether three pound $\frac{1}{2}$ of water would auoid?

Master. I perceiue your meaning, and

coniecture your imagination to be thus: that if you knew the exact proportion betweene gold and siluer and water both in their waight and quantities, then could you easilie finde out the mixtures of them, which thing I haue reserued for another worke that intreateth of such matters especially. And at this time you must consider that you learne Arithmeticke, which intreateth of the maner to solue doubtfull questions touching number, without regard what matter is signified by that number; els were it necessarie in Arithmeticke to teach all arts, séeing in it may be mooued questions of all arts. But seeing you are so desirous to know this thing, I will tell it you in such a sort, that you shall practise your art in finding it, & propound it in forme of a questiō. Gold beareth greater proportion to water than siluer doth, & their two proportions be in proportion together, as $\frac{48}{25}$. But to helpe you somwhat in this riddle, you shall note that the proportions of quicke-siluer vnto water, is the iust middle number proportionall in Progression Geometrical, betwéen the proportions of gold and siluer vnto water. And this proportion is $\frac{290}{21}$. Now if you wil know the iust nūbers of these two proportions, then must you find out 3 numbers in progression Geometrical, whereof the middlemost must be $\frac{190}{21}$, and the first must be vnto the last, as 25 to 48. And thus I will leaue you to finde those numbers

when you be at leasure.

Scholar. Yet Sir, I thanke you hartily for thus much, for now I see the possibilitie to finde them out. Howbeit, because this question seemeth strange, if it might please you to instruct me somewhat in the order of working for it, I should the more easily find the true working.

Master. You desire too much ease if you will study for nothing: therefore to occasion you to study the better, I will leaue this doubt wholy to your owne search. But as touching the generality of the rule, Archimides needed not to take two masses of gold and siluer equall in waight with the crowne, for the proportion might as well bee found in any other waight, yea although the masse of gold were of one waight, and the masse of siluer of another. As for example: If the crowne were of 8 pound waight as I did suppose, and I haue not so much other fine Golde, but onely one pound, and trying that by water and finding that it doth expell but $\frac{3}{4}$ of an ownce of water, yet then by it I may inferre, that 8 pound of Gold would expell six ounces of water. And likewise of siluer: whereof if I had but two pound, and finde that it doth expell three ounces of water, then might I affirme that 8 pound would expell 12 ounces, that is, one pound waight: and so is it as good as if the 3 masses were all of one waight. And thus

for this time I will make an end of this other part of Arithmetike.

Scholar. Although I cannot sufficiently thank you for this, yet your promise made me to looke for the art of extraction of rootes, whereof hitherto I haue learned nothing.

Master. I will not breake my promise, but intend (God willing) to performe it within these three or foure moneths, if I perceiue this my paines to be well taken in the meane season. And you shall not repent the tarying for it: for it shall bee increased by the tarying. And in the meane time, you shall take this Addition, not for the second part of Arithmetike which I promised, but for an augmentation of the first part, vnto which I wold haue annexed the extraction of rootes, square & cubike, namely for examples of the Statutes of Assise of wood, but that in the second part I must write of diuers other roots, and thought it best to reserue those Rules also with their examples vnto the same second part.

Scholar. Sir, although I cannot recompence your goodnes, yet I shall alwaies doe mine indeuour to occasion you not to repent your benefit on me thus imploied.

Master. That recompence is sufficient for your part.

FINIS.

THE THIRD

part of Addition to this Booke,

entreateth of briefe Rules, called Rules of Practise, of rare, pleasant, and commodious effect, *abridged into a briefer* Method then hitherto hath bene published.

With diuers other necessarie Rules, Tables, and Questions not onely profitable for Merchants, but also for Gentlemen, and all other Occupiers whatsoeuer, as by the contents of this Booke may appeare.

Set forth by IOHN MELLIS Schoole-Master.

The first Chapter of Addition entreateth of briefe Rules, called Rules of Practise, with diuers necessarie Questions, profitable not onely for Merchants, but also for all other occupiers whatsoeuer.

The working of Multiplication in Practise, is no other thing then a certaine maner of multiplying of one kind by another: whereupon is brought forth the product of the proponed number, which is accomplished by the meanes of Diuision in taking the halfe, the third, the fourth, the fift, or such other parts of the sum, which is to be multiplied.

And for the better vnderstanding of such conuersions, you shall vnderstand that in the maner and vse of these rules of Practise, you ought first to know the euen or aliquot parts of a shilling, which in this Table following doth appeare.

Item d	6	in the	$\frac{1}{2}$	of a s.
	4		$\frac{1}{3}$	
	3		$\frac{1}{4}$	
	2		$\frac{1}{6}$	
	1		$\frac{1}{12}$	

Where

Wherein as you see according to the order of these rules of Practise at 6 d the yard of any thing: You must take ½ of your number which is to be multiplied, and the product that commeth thereof shal be shillings, if any unite doe remaine it is 6 d.

For 4 d take the ⅓ of the number that is to be multiplied, and the product also produceth shillings, if any unites doe remaine, each one shall be worth in value 4 pence. The like is to be understood of the other 3, &c.

Example.	IIII
At 6 pence the yard what 379 yards?	At 2 pence the yard what 522 yards?
189 — 6 d	88 s—8 d
II	V
At 4 pence the yard what 104 yards?	At 1 peny the yard what 409?
34 s — 8 d	34 s— 1 d
III	
At 3 pence the yard what 5014 yards?	
1253—6 d	

Heere you may see in the first example, that 379 yards at 6 d the yard are worth 189 s—6 d, in taking the ½ of 379. And in the second example the 104 yards at 4 pence the yard, are

worth 34 s 8 d in taking the $\frac{1}{3}$ of 104. Likewise in the third example 5014 yards at 3 d ẏ yard, bringing forth 1253 s 6 pence, in taking the $\frac{1}{4}$ of 5014. Also in the fourth example at 2 d the yard, maketh 88 s 8 d.

And lastly in the fifth example: 409 yards at 1 d the yard, amounteth to 34 s and 1 peny, in taking the $\frac{1}{12}$ of 409: and so is to be done of all other questions the like, when the nūber of the pence is any of the euen or aliquot parts of 12 d. Item to bring the products of these shillings and all other the like into pounds is very easie in diuiding of it in your minde by 20, for it is to be vnderstood that as often as 20, is found in that product, so many pounds doth it containe: which with facility to performe, alwaies strike off the figure toward your right hand, with a right downe dash of your pen, for the 0 that appertaineth to the 20. And then begin at the left hand, in taking the $\frac{1}{2}$ of ẏ rest. And if at the last any vnite doe remaine, the same shall be ioined with the figure that is cut off, which shall represent the odde shillings cōtained in that worke.

As for example, in your third question at 3 d the yard, which amounteth to 1253 s 6 pence, the product whereof maketh 62 l 13 s 6 d, as here you may see, is easily performed by this example.

1	
125|3	1
62—13—6	

Item also for the working of one peny the

yard

yard, it is something harsh and hard to take the $\frac{1}{12}$ of some products: therefore to ease that hard worke, you shall first bring your deliuered summe into groats, by taking the $\frac{1}{4}$ part of the product, and if any vnits remaine of that $\frac{1}{4}$ part, as sometimes there may, they are pence: and must bee signified with a line from the groates with their title of pence; and because that 60 groats maketh a pound or twenty s, strike off the first figure toward your right hand, for the 0 that appertaineth to 60 (as you did euen now for the 0 that belongeth to 20:) then in taking the $\frac{1}{6}$ of that product, if there do remaine any vnits, the same shall you ioine with the figure that you cut off, esteeming thē as groats, which keepe in your minde, and by taking the $\frac{1}{3}$ part of them, you shall turne into shillings: and so haue you done: as for example by a question or two hereafter proponed shall more plainly by the worke appeare.

At 1 d the yard what 54368 yards?

1359 | 2 groates.

$\frac{1}{6}$ 226 10 s 8 d.

Here in takiking the $\frac{1}{6}$ part of 1359: in comming to the last worke, the $\frac{1}{6}$ part of 39 being taken, the remainer is 3, which ioined with the two that was cutte off, maketh 32 groats, which conuerted into shillings by taking the $\frac{1}{3}$ part, maketh as appeareth ten s 8 d. Many other waies there are, but none

more

more apt for a yong learner to vnderſtand th this: wherefore this one way well impreſſed in memory is better then 20 waies doubtfully vnderſtood.

At 1 peny the yard, what 4533 yards?

113 | 3 groats—1 d

$\frac{1}{8}$ l 18——17——9 d

At 1 peny the yard, what 64768 yards?

1619 | 2 groates.

l——299—17—4 d.

NOw followeth alſo to be vnderſtood, that if the number of pence be not an aliquot part of 12, you muſt reduce them into ſome aliquot part of 12: and after the aforeſaid maner, you ſhal make of them two or thrée products, as néede ſhall require, and adde them together into one ſumme. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken.

For pence	5. take. 3 and 2	or	4 and 1
	7. take. 4 and 3		6 and 1
	8. take. 4 and 4		6 and 2
	9. take. 6 and 3		4 4. and 1
	10 take. 6 and 4		4. 4. and 2
	11. take. 6. 4. & 1		4. 4. and 3

Here in the firſt note of this Table at 5 d, you ſhall firſt take for 3 d the $\frac{1}{4}$ of the number that

that is to be multiplied: and likewise for 2 d the $\frac{1}{6}$ of the same number, adding together both the products: But if you wil worke by 4 and 1, you must for 4 d first take the $\frac{1}{3}$ of the number that is to be multiplied: and for 1 d take the $\frac{1}{12}$ of the whole sum, or rather, which is more better, for 1 peny you may take the $\frac{1}{4}$ of the product which did come of the 4 pẽce: because that 1 d is the $\frac{1}{4}$ of 4 pence. The totall summes of these two numbers shal be the solution to the question. And in like maner is to be done of al others, as by those examples following shal appeare.

I

At 5 pence the yard what 758 yards?

3 d	189—6 d
2 d	126—4 d
	315—10 d

Otherwise.

At 5 pence the yard what 758 yards?

4 d	252—8 d
1 d	63—2 d
	315 s—10 d

II

At 7 pence the elle what 563 Elles?

4 d	187—8 d
3 d	140—9 d
	328 s—5 d

III

At 8 d the pound what 112 pounds?

4 d	37—4 d
4 d	37—4 d
	74 s—8 d

Other-

Otherwiſe.

At 8 pence ÿ pound what 112 pounds?

6 d	56—0
2 d	18—8
	74 s—8 d

IIII

At 9 pence the Elle what 356 elles?

6 d	178—0
3 d	89—0
	267 s—0 d

V

At 10 pence the peece what 795 peeces?

6 d	397—6
4 d	265—0
	662—6

VI.

At 11 d the pound what 7576 pound?

6 d	3788———0
4 d	2525———4
1 d	631———4
	6944———8 d
	347—4 s—8 d

Here in this firſt example where it is de-
maunded (at 5 d the yard) what will 75 coſt?
Firſt of 3 d I take the $\frac{1}{4}$ of 758: & thereof com-
meth 189 s 6 d: then for 2 d I take the $\frac{1}{6}$ of ÿ
ſame product, which amounteth to 126 s 4 d,
theſe two ſums added together, doe make 31
ſhillings 10 d: and ſo much are the 758 yards
worth at 5 d the yard.

Item alſo for the ſame againe: Firſt for 4 d
I take the $\frac{1}{3}$ of 758: and thereof commeth
252 s 8 d: then for 1 peny I take the $\frac{1}{4}$ of the
ſame product: that is to ſay, of 252 s 8 d, and
it yeeldeth me 63 s 2 d: which both added to-
gether

gether maketh 315 s——10 d, as before.

Item for 7 d there is taken the $\frac{1}{3}$ and the $\frac{1}{4}$ of the whole summe which is to be multiplied, and adde them together, that is to say, first for 4 pence there is taken the $\frac{1}{3}$ of 563: which comes to 187 s—8 d, as appeareth by the worke: and for 3 d there is taken the $\frac{1}{4}$ of the whole sum, which amounteth to 140 s—9 d. Both which products added together, maketh 328 s—5 d: and so much comes 563 elles to at 7 d the elle.

Item for the first 8 d there is taken for 4 d, the $\frac{1}{3}$ of the whole summe, and another $\frac{1}{3}$ for ye other 4 d, which added together, as in the example doth evidently appeare, amounteth to 74 s—8 d.

Againe, for the second worke of 112 l, there is taken first the $\frac{1}{2}$ of the whole summe for 6 d, which comes to 56 s: then for that 2 pence you have to take $\frac{1}{6}$ of the whole summe, or if you will, the $\frac{1}{3}$ of the product that came of 6 d, either of which maketh 18 s—8 d. These two summes being added togither, doe make 74 s 8 d, as in the third example appeareth.

Item, for 9 d there is taken for 6 pence the $\frac{1}{2}$ of the whole summe: & the $\frac{1}{4}$ of the whole sum for 3 d, or otherwise for the 3 d you may take the $\frac{1}{2}$ of the product that came of 6 d, because 3 pence is the $\frac{1}{2}$ of 6 pence: which added together, as plainly appeareth in the fourth example, amounteth to 267 s 0 d.

Item

Item for 10 d, first there is taken for 6 d y $\frac{1}{2}$ of the whole sum, which amounteth to 397 s —6 d. Then for 4 d there is found 265 s, both which added together, maketh 662 shillings 6 d, as appeareth in the 5 example. It may also bee wrought, as appeareth by the second note in the table, by 4 pence twice taken, and y $\frac{1}{2}$ of the product of 4 pence: or else by the $\frac{1}{6}$ of the whole sum, &c.

Item, for 11 d there is first taken the $\frac{1}{2}$ for 6 pence: then the $\frac{1}{3}$ of the whole sum for 4 d: lastly, that $\frac{1}{4}$ of y last product for 1 d. All which 3 summes added together, maketh in s 6944 8 d, and in pounds 347 4 s 8 d.

3 Rule. ITem likewise by the same reason whē you will multiply (by shillings) any number that is under 20 s, you shall haue in the product pounds, if you know the euen or aliquot parts of 20, which are here in this little Table set downe to sight.

Item s.	10	is the	$\frac{1}{2}$	of one pound.
	5		$\frac{1}{4}$	
	4		$\frac{1}{5}$	
	2		$\frac{1}{10}$	
	1		$\frac{1}{20}$	

So that for 10 s, which is the $\frac{1}{2}$ of a pound, you may take the $\frac{1}{2}$ of the number which is to be multiplied, and you shall haue in your pro-

duct

duct pounds: if an vnite doe remaine, it shall be worth ten shillings.

Likewise for 5 shillings you must take the $\frac{1}{4}$ of the number which is to be multiplied: and if there doe remaine any vnites, they shall bee fourth parts of a pound, euery vnite beeing in value 5 shillings.

For 4 s take the $\frac{1}{5}$ of the number which is to be multiplied: and if there doe remaine any vnites, they shall be fifth parts of a pound, each vnite being worth 4 shillings.

For 2 shillings you must take the $\frac{1}{10}$ of the number to be multiplied: wherfore to take the $\frac{1}{10}$ of any number, you must cut off the last figure of the same nūber (which is nearest your right hand) from all the other figures with a small right downe line or dash with a pen, & so haue you done: for all the other figures which doe remaine toward your left hand from the same figure that you doe separate, shall be the said $\frac{1}{10}$ of a pound: and that figure so separated towards your right hand, shal be so many péeces of 2 s the péece: the which figure you must double to make thereof the true number of s, as by the example shall appeare.

Finally, for 1 shilling néedeth small worke, for it is so many shillings as bee proponed in the summe, which to bring into pounds hath beene already taught in the first rule.

Example.

At 10 s the péece
What are 6543 péeces worth?

$\frac{1}{2}$ l 3267 —— 10 s

At 5 s the Elle
What comes 4373 —— Elles to?

$\frac{1}{4}$ l —— 1093 —— 5 s

At 4 s the yard
What —— 7839 yards?

$\frac{1}{5}$ l —— 1567 —— 66 s

At 2 s the pound waight
What are —— 752|7 pound waight worth?

$\frac{1}{10}$ l —— 752 —— 14 s

At 1 s the péece
What comes —— 757|3 péeces to?
$\frac{1}{3}$ l —— 378 —— 13 s.

NExtly, now followeth in order to bee vn derstoode, that if the number of shillings be not some euen or odde aliquot part of 20 you must then conuert the same number o shillings into the aliquot parts of twenty, an thereof make two or thrée products, as néede shall require: which done, adde them toge ther, and bring them into pounds. And her for thy furtherance I haue set downe a note of the order of their parts, as they are to bee taken.

s					s		
3	of	2 & 1	or		13	of	10.2.&.1.
6		4 & 2		5.& 1.	14		10.& 4
7		5 & 2			15		10.& 5
8		4 & 4		5.2.1.	16		10.5.1
9		5 & 4		4.4.1.	17		10.5.2
11		10 & 1			18		10.4.4
12		10 & 2			19		10.5.4

For 3 s according to the tenor that you see is expressed in the Table, you must first take for 2 shillings the $\frac{1}{10}$ of the number that is to be multiplied. Then for 1 shilling you must take the $\frac{1}{2}$ of the product which did come of the same $\frac{1}{10}$ part: which two sums added together produceth the effect desired.

Item, for 6 shillings according to the note set forth in the Table, first for 4 s I take the $\frac{1}{5}$ of the number that is to be multiplyed: Then for 2 s the $\frac{1}{2}$ of the product that came of 4 s, & adde them together.

Or else, as appeareth also in the Table, for 5 shillings you may take the $\frac{1}{4}$ & the $\frac{1}{5}$ part of the product that came of 5 shillings, and adde them together.

Item, for 7 s, first take for 5 s the $\frac{1}{4}$ of the product that is to be multiplied, then for 2 s, take the $\frac{1}{10}$ of the number that is to be multiplied, and adde them together, &c.

Item, for 8 s, according to reason, and the intent of the Table, for the first 4 s take the

Dd $\frac{1}{5}$ of

$\frac{1}{5}$ of the product, & the same number againe for the other 4 shillings: and adde them together.

Item, for 9 shillings: first for 5 shillings take the $\frac{1}{4}$: then for 4 shillings take the $\frac{1}{5}$: and adde them together.

Otherwise as you see by y^e^ intent of y^e^ table, worke twise for 4 shillings, as was taught euen now for 8: & then take the $\frac{1}{4}$ of y^e^ last product for the 1 shilling. But 5 & 4 is the shorter.

Item for 11 s, first dispatch 10 s: for which you must take the $\frac{1}{2}$ of the product, then lastly for 1 shilling take the $\frac{1}{10}$ part of the sum produced of the $\frac{1}{2}$ of y^e^ product, & ad thē together.

Item, for 12 shillings where I will end w^t^ y^e^ first part of my table. First take the $\frac{1}{2}$ for 10 shillings And then for 2 shillings take y^e^ $\frac{1}{5}$ of y^e^ sum that came of 10 s, take and adde them together: or else if you please, for 2 shillings you may take y^e^ $\frac{1}{10}$ of the whole giuen nūber.

To write more of the maner of taking the true parts, I omit. The desirous practitioners will (no doubt) conceiue it. Also the Table is some aide to helpe the vnperfect, whereupon by and by I will set downe three or foure of these notes in examples, and the rest I will leaue to thine owne industry and practise to labour vpon.

This is the order most commonly vsed in practise, when the number of shillings is not an aliquot part of a pound. But (louing Reader) after I haue touched the euen or aliquot parts of

of a pound that falleth out in pence and shillings, I will deliuer two new rules that shall drowne this common order quite and cleane: wherein shall be comprehended in one line or working both euen & odde parts of shillings vnder 20: without regard whether it be an aliquot or not an aliquot part: which two rules when they come in place, I commit to thy friendly iudgement in working.

Now follow the examples vpon the notes before said.

At 6 shillings the yard.

What ——3215—— yards?
4 shillings 643——
2 shillings 321——10

l.-964--10 shillings

Otherwise by Multiplication of 6.

3215——

6 shillings 1929|0
l.——964——10 shillings.

At 7 shillings the Elle
What ——4563 Elles?
5 shillings 1140——15
2 shillings 456——6
l. 1597——1 shilling.

Otherwise by multiplication of 7.

	4563
7 s	3194|1

At 8 s the péece, what 7563 péeces?

4 s	1512 . . 12
4 s	1512 . . 12
	l. 3025 . . 4 s

Otherwise by multiplication.

	7563
8 s	6050|4
	l 3025 4 s

At 13 s the péece, what 401 péeces?

10 s	200 . . 10
2 s	40 . . 2
1 s	l. 260 . . 13

Otherwise by multiplication.

	401
136	1203
	401
	521|3
	l. 260 13 s.

These and ſuch like Queſtions of compound numbers, which I haue here in this fourth rule for orders ſake ſet downe, for that it hath bene heretofore a common courſe of worke, I account but ſuperfluous. For in the 8 and ninth Chapters of this my ſimple Addition ſhall appeare, that the giuen price of any euen or odde number of ſhillings either vnder or aboue 20, ſhal be wrought at one or two workings at the moſt, how difficult ſoeuer the Queſtion be.

Item, there reſteth yet a kinde of Practiſe, how to bring pence into pounds at the firſt working: whereupon you muſt vnderſtand, that 240 pence maketh one pound, or 20 s. In conſideration whereof I cut off the laſt figure or 0, and there remaineth but 24 (of which 24) 8 d is the $\frac{1}{3}$ part thereof, 6 d is the $\frac{1}{4}$ part, 4 d the $\frac{1}{6}$ part: & 2 pence is the $\frac{1}{12}$ part thereof.

Whereupon if it were demaunded what 1486 yards or pounds of any thing commeth to, at 8 pence the yard, in pricking or cutting off ỹ firſt figure towards your right hand, for the 0 that appertaineth to 240: There is remaining of the ſaid ſumme 148, whereout I taking the $\frac{1}{3}$ part, and it commeth to 49 l', and there reſteth 1, which 1 I put to the 6, that I picke or cut off, and it maketh 16 péeces of 8 pence, which I double to make into groates,

and they make 32, whereof the $\frac{1}{3}$ part maketh 10 s, and there remaineth $\frac{2}{3}$ s, which is 8 pence, whereby it followeth, that the 1486 yards at 8 pence the yard, maketh 49 ℔ 10 s 8 d, as by the example shall appeare.

Item for 6 pence, take $\frac{1}{4}$ part of the number from the prickt figure: and if any unites doe remaine, they are so many fixe pences, whereof taking the $\frac{1}{2}$ they are shillings, if there do remaine yet one, it is in value fixe pence.

Item for 4 pence, take the $\frac{1}{6}$ part of the number from the prickt figure: If any unites remaine, they are so many groats, which to conuert into shillings, take the $\frac{1}{3}$ part: And if any thing yet remaine, they are thirds of shillings, each one in value being worth 4. pence.

Item for 3 pence, take the $\frac{1}{8}$ part from the prickt figure, if any unites remaine, they are so many pœces of 3 pence, whereof in taking the $\frac{1}{4}$ part, maketh shillings: If any thing yet remaine, they are fourth parts of shillings, each one being in value 3 pence.

Item for 2 pence, as appeareth also by the table, take the $\frac{1}{12}$ part of the number from the prickt figure: if any thing remaine, they are so many pœces of 2 pence, which by taking $\frac{3}{6}$ part, you shall turne into shillings: and if any unites remain, they are so many fixt parts of shillings, or pœces of 2 d, whether you will

I

If one cost 8 pence
What are 148|6 worth?

maketh pounds 49——10——0 pence.

If one cost 6 pence
What 786|5 worth?

maketh pounds 196——12——6 pence

At 4 pence the yard
What are 873|6 yards worth?

maketh pounds 145——12——0 d

If one cost 3 pence
What are 987|4 worth?

maketh pounds 123——8——6 pence.

At 2 pence the Elle
What comes 789|4 Elles to?

maketh pounds 65——15——8 d

But if your number of pence be not an aliquot or euen part of 24: then must you bring them into the aliquot parts of 24, and make thereof diuers products which must bee added together, as by the question hereafter following shall appeare.

Item for 5 d, first take for 3 d, then for 2 d, and adde them together according to the instruction of the second rule: Or else first take

for 4 pence, then for one penie.

Item for 7 d, first take for 4 pence: then for 3 d, and adde them together.

Item for 9 d, first take for 6 d, then for 3 d, and adde them together.

Item for 10 d, first take for 6 d, then for 4 d, and adde them together.

Item for 11 d, first take for 8 d, then for 3 d, and adde them together: as by these examples following doth appeare.

Example.

If one yard cost 5 pence,
What are 759|6 worth?

4 pence	126	12
1 penie	31	13
maketh pounds	158	5 s

Otherwise.

1 5 759|6

3 pence	94	19
2 pence	63	6
maketh pounds	158	5 s

If one cost 7 pence,
What are 98|7 worth?

4 pence	16	9	
3 pence	12	6	9
maketh pounds	28	15	9 pence.

Other

Otherwise.

1——7——98|7

6 pence	24	13	6
1 peny	4	2	3
maketh pounds	28	15	9 pence.

If one cost 9 pence,
What are 78|9 worth?

6 pence	24	13	6
3 pence	12	6	9
maketh pounds	37	0	3

Otherwise.

1——9——987

3 pence	12	0	9
maketh pounds	37	0	3.

If one cost 10 pence,
What are 98|7 worth?

6 pence	24	13	6
4 pence	16	9	0
maketh pounds	41	2	6

If one cost 11 pence,
What are 98|7 worth?

8 pence	32	18	0
3 pence	12	6	9
maketh pounds	45	4	9

But

But if you haue any shillings and pence to be multiplied together: Then are you to take for the shillings according to the instruction of the third rule. And for the pence according to the first rule before mentioned: vnlesse you can spie the aduantage thereof, and thereby helpe your selfe: as appeareth in this second example, where first I worke for 6 d: which is to be rebated out of the giuen number, and I haue 719 l' 11 s my desire.

At 19 s—6 d the yard
what 738 yards?

	738	
10 s	369	
5 s	184	10
4 s	147	12
6 d	18	9
l'	719	11 s

Otherwise by Rebating.

	73|8	
6 d	18	9 s
l'	719	11 s

The like againe is done by rebating, as by these 2 examples appeareth.

At 18 s the Elle
what 418 Elles?

2 s	41	16
l'	376	4 s

At 16 s the Elle
what 517 Elles.

4 s	103	8
l'	413	12 s

And

And now I will touch a little the euen parts of a pound, that falleth out in pence and shillings, whereof for those parts you shall take such like part of the giuen number that is to be multiplied, as the price of that giuen number beareth in proportion to a pound, which also for their better aide is here set downe.

s	d			
1	8	is the	$\frac{1}{12}$	part of a l.
2	6		$\frac{1}{8}$	
3	4		$\frac{1}{6}$	
6	8		$\frac{1}{3}$	

Item, first for 1 shilling 8 pence take the $\frac{1}{12}$ part of the giuen number, and if any thing do remaine, they are twelue parts of a pound, each one being in value 1 shilling 8 pence.

Item for 2 shillings 6 pence take the $\frac{1}{8}$ part of the number that is to be multiplyed. And if any thing do remaine, they are eight parts of a pound, each one being in value. 2 shillings 6 pence.

Item for 3 shillings 4 pence, as appeareth by the Table, you must take the $\frac{1}{6}$ part of the giuen number, and if any thing doe remaine, they are 6 parts of a pound, each one being in value 3 shillings 4 pence.

Item for 6 shillings 8 pence take the $\frac{1}{3}$ part of the number that is to bee multiplyed: And if any vnites doe remaine, they are thirds of a pound,

a pound, euery one being worth 6 ſhillings 8 pence.

Other infinite numbers there are, that may be reduced by abbreuiation into the proportionate parts of a pound: as 16 ſhillings 8 pence maketh $\frac{5}{6}$: which 16 ſhillings 8 pence is eaſily reduced into groats by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats: Then ſet 60 the groats of a pound vnder 50, cutting off the 2 ciphers, as is here performed in the margent. And then haue you brought 16 ſhillings—8 pence into the knowne parts of a pound, which maketh $\frac{5}{6}$.

16—8
3
5|0
6|0

But yet gentle Reader, for thy further inſtruction, I haue hereunto annexed in a Table, how pence and ſhillings beareth proportion to a pound, which I cõmit to thy friendly beneuolence; it will be ſome aid vnto the vngrounded practitioner: but I count him the beſt workman ỹ can preſently reduce his giuen price into the knowne and proportionate parts of a pound.

s	d	l	s	d	l
0	2	1/120	8	4	5/12
0	3	1/80	8	9	7/16
0	4	1/60	9		9/20
0	6	1/40	10		1/2
0	8	1/30	11		11/20
1	0	1/20	11	3	9/16
1	3	1/16	11	8	7/12
1	8	1/12	12		3/5
2	0	1/10	13		13/20
2	6	1/8	13	4	2/3
3	0	3/20	13	9	11/16
3	4	1/6	13		3/20
3	9	3/16	14		7/10
4	0	1/5	15		3/4
5		1/4	16	8	5/6
6		3/10	16		4/5
6	3	5/16	17	6	7/8
6	8	1/3	17		17/20
7		7/20	18	4	11/12
7	6	3/8	18	9	15/16
8		2/5	19		19/20

Here follow 4 examples vpon the 4 notes deliuered.

At 1 s 8 d the yard
What are 3884 yards worth?

maketh pounds 323—13—4 pence.

At 2 s—6 d the yard
What are 4563 yards worth?

maketh pounds 570—7—6 pence.

At 6 s—8 pence the Elle
What comes 7562 Elles to?

maketh pounds 2520—13—4 pence.

Now by custome you are able to worke by all sorts of summes being deliuered in shillings and pence, as one shilling one peny, 2 shillings two pence, 3 shillings 3 pence, & so of all other: wishing you to haue some considerations of your questions, when they are set downe, for there are many subtill abbreuiations, and great aduantages to be gotten, and easily to be perceiued.

As 3 s—8 d of 2 s and 1 s 8 d.

4 s—2 d, of 3 s—4 d, and 10 d, which 10 is $\frac{1}{4}$ of 3 s—4 pence.

5 s—8 d of 4 s—1 s 8 d.

5 s 10 d, of 5 s and 10 d, which 10 d is $\frac{1}{6}$ of 5 shilling.

And by this meane when you haue taken one product, you may oftentimes vpon the

same

same take another more briefly than vpon the sum which is to be multiplyed, &c.

NOw (gentle Reader) that you haue seene the vertue of the euen or aliquot parts of a pound in shillings alone, and also in the aliquot parts of shillings and pence: according to my promise hereafter followeth a briefer and easier method for any euen number of shillings, either vnder or aboue 20, than euer yet hath bene published: Notwithstanding Maister Humfrey Baker, whose trauell is worthy commendation, and whom for knowledge sake I reuerence, hath in some part touched this first part, though not in this method. The worke of the rule is both pleasant, ready, and briefe, as by the varietie of the examples deliuered thereupon shall appeare. And first I will set forth a question, thereby the better to expresse or teach you the order thereof: which is this.

If one yard cost 6 s, what 8574?

1 —— 6 —— 8574

maketh pounds —— 2572 —— 4 s

To the vnderstanding of this example, after you haue set downe your giuen number in forme of the rule of 3, with a line drawne vnder it, you shall presently set a pricke vnder your first figure 4, towards your right hand, draw-

drawing from the pricks as heretofore hath bene practised, a little short line, thereto set downe the shillings anon, which done, multiply the first figure 4 by 6, the value of your price, (which here you see standeth in sight aboue the line) it maketh 24: which is 1 pound 4 shillings. The 1 pound keepe to carry to the next place, and the foure shillings set downe at the end of the prescribed line towards your right hand. Thus haue you done now with sixe aboue the line, and also with 4 in the first place (for the pricke vnder foure doth represent that 4 hath done his office.) Then secondarily for a general rule take but the ½ of ẏ giuen price, which here is 3, which 3 is the number that shall now continue the rest of ẏ multiplication and end the worke, whereupon I multiply three into seuen, standing in the second place it maketh 21, and with the one I kept in minde 22; set downe 2 and keepe 2 in minde working according to the rule of Multiplication, deliuering ẏ tens in mind in their due place, which done, the product from the prick to your left hand representeth ẏ pounds and the other at the end of the line the shillings, as appeareth by the example.

If one yard cost 2 s, what 7536?

	1	2	7536
maketh pounds		753	12 s

If one yard cost 4 s, what 8792?

1	4	8792
maketh pounds	1758	8 s

If one péece cost 6 s, what 9537

1	6	9537
maketh pounds	2861	2 s

If one cost 8 s, what 7509?

1	8	7509
maketh pounds	3003	12 s

If one cost 12 what 5794

1	12	5794
maketh pounds	3476	8 s

If one cost 14 s what 3705?

1	14	30705
maketh pounds	2593	10 s

If one cost 18 s, what 5703?

1	18	5703
maketh pounds	5132	14 s

If one cost 22 s, what 953?

1	22	953
maketh pounds	1048	6 s

Let these suffice (gentle Reader) for an entrance into euen numbers. And now I will shew the like rule for any odde or vneuen part of a pound.

TO helpe you to the vnderstanding of these other questions that hereafter follow: where in my first example the giuen number is 6487. at 3 shillings the yard: I multiply 3 aboue the line into 7, it maketh 21. The one shilling is set downe, & the 1 li. I keepe. Now am I to take the $\frac{1}{2}$ of three, which because it is an odde number I cannot. Therefore I shall keepe and continue my multiplication by three still, and worke by the $\frac{1}{2}$ of the rest of the giuen figures or number, to wit, 648. And first the $\frac{1}{2}$ of 8 which is 4, multiplied into three maketh 12, thereto ioyne the 1 l in minde, it maketh 13: set downe 3, keepe one. Then againe multiply by two the $\frac{1}{2}$ of foure, it maketh sixe, and with one in minde it maketh 7. Then lastly take the $\frac{1}{2}$ of sixe, which is three, saying 3 times three is nine, which nine set downe, & so is the question answered, as appeareth by the practise, and the examples following.

At 3 s the yard what 6487?

1	3	6487
maketh pounds —	973 —	1 s

If one yard cost 5 s, what 4269?

1	5	4269

maketh pounds — 1067 — 5 s

At 7 s the elle, what 6489?

1	7	6489

maketh pounds ---2271-----3 s

If one elle cost 9 s. what 2807?

1	9	2807

maketh pounds — 1263 — 3 s

If 11 s the Pistolet, what 8263?

1	11	8263

maketh pounds — 4544 — 13 s

If one péece cost 13 s, what 4629?

1	13	4629

maketh pounds — 3008----17 s

But now note (gentle Reader) when the giuen price falleth vpon any odde nũber, as 3,5,7,9,11,13, &c. thẽ it is to be presupposed that the giuen summe to be multiplyed must be a summe made of euen numbers, as 2,4,6 8,10, &c, els cannot that question be wrought at one line or working.

Prouiding alwaies that it may beare an odde figure in the first place towards your right hand, as appeareth in these 6 examples, which all were wrought, and such like, &c.

which may beare an odde number for the price, and be done at one line or working very well.

But if the giuen price be an odde number, and the ſumme to be multiplied odde numbers alſo: then can it not be done at one working, but requireth the aid of two workings, for odde with odde will not agree, which notwithſtanding to bring to paſſe, take this for a generall rule. Firſt worke for the euen number, contained in that queſtion, or giuen price according as you haue learned, and then afterwards for the 1 odde ſhilling, take the ½ of the ſumme giuen to be multiplied omitted the firſt pricke place, as was taught for the working of one ſhilling in my firſt rule of Practiſe, and adde thoſe 2 together, and you ſhall haue your deſire.

Example.

At 3 s the yard
What are 7539 yeards worth?

2 s	753——18
1 s	376——19

maketh pounds——1130—17 s

At 7 s the elle, what 7539?

1	7	7539
6 s		2261——14
1 s		376——19

maketh pounds—— 2638—1

At 13 s the yard, what 7534?

1 —— 13 —— 75534

12 s	4520 — 8
1 s	376 — 14
maketh pounds	4897 — 2

And thus haue I abridged into these two rules, how to bring any number of shillings whatsoeuer they be into pounds, with a briefer method then euer yet hath bene published, which I commend vnto thy friendly censure and iudgment in the vse and practise thereof.

If one cost 6 s 5 d, what 1231?

1 —— 6 s —— 5 d —— 1231

6 s	369 — 6
4 d	20 — 10 — 4
1 d	5 — 2 — 7
maketh pounds	394 — 18 — 11

At 14 s 2 d, what 2825?

1 14 2 2825

14 s	1977 — 10
2 d	23 — 10 — 10
maketh pounds	2001 — 00 — 10

At 16 s 4 d, what 2531

1 16 4 2531

16 s	2024 — 16
4 d	42 — 3 — 8 d
maketh pounds	2060 — 19 — 8

At 3 s the Pistolet, what 8325?

1	3	8325	
maketh pounds		1248	15 s

At 7 s the crowne, what 6529?

1	7	6529	
maketh pounds		2285	3 s

At 9 s the peece, what 6567?

1	9	6567	
maketh pounds		2955	3 s

These three last questions may seeme something hard, yet they are easie enough if you marke them well, if I should explaine them, then are they too easie: therfore I leaue them to whet the minds of the desirous.

Item, when any one of the summes which is to be multiplied, is composed of many denominations, and the giuen number but of one figure alone: then shall you multiply all the denominations of the other sum by the same one figure, beginning first with that summe which is least in value towards your right hand, & bring the product of those pence into shillings, and the product of the shillings into pounds, as by this example appeareth.

At 3 l — 7 s — 4 d, what are 9 worth?

maketh pounds — 30 — 6 s — 0 d

But if in any of the sums that are to be mul-

multiplied there be a broken number. First work for ye whole according to the instructions that you haue learned: & then take such part of the giuen price, as that broken number beareth in proportion to the price, as in the example. After you haue wrought for 3 s and for 6 d, then are you to take the ½ of 3 s 6 d for the ½ yard, and adde that to the sum: So adding all 3 products together, which maketh 43 l 2 s 9 d, the iust price of 246 ½ Elles: and thus must you do of all other.

At 3 s 6 d the elle, what 246 ½

1	3	6	246 ½		
3 s			36	18	
6 s			6	3	
½.				1	9
maketh			43	2	9

At 16 s — 4 d the peece, what 14 ¾

1	91	4	14 ¾	
16 s		11	4	
2 d		0	4	8
¾			12	3
maketh pounds		12	00	11

If one peece cost 4 l — 3 — 6 ½ d
What are 12 peeces worth?

4 l	48	0	0
3 s	1	16	0
6 d	0	6	0
½	0	0	6
maketh pounds	50	2	6 d

The proofe.

If 12 pèeces cost 50 l 2 s 6 d
What one pèece?

	50	2	6
maketh pounds	4	3	6 $\frac{1}{2}$

Item, touching the manner how to vnderstand the order of this question, and others the like, first seeke how many times 12 is cōtained in 50, which is 4 times, and so resteth 2 pound, which 2 pound conuerted into shillings, and ioyned with the other 2 shillings, maketh 42 shillings, wherein is found 12. 3 times: resteth 6 shillings, which turned into pence, putting thereto the 6 d in the first place, it maketh 78: wherein 12 is found 6 times: resteth 6 d, which containeth 12, but $\frac{1}{2}$ a time: put that $\frac{1}{2}$ to the 6 d: and then the solution is 4 l 3 s 6 $\frac{1}{2}$, as appeareth by the practise thereof.

Item, the like is to be done of any thing that is bought or sold after fiue score to the hundreth, or the Quintall. As for example.

If

If 100 pound coſt 27 l 13 s 4 d what one pound?

27 13 4

20

s 5|53

|12

1|10

5|3

d 6|4|0

|10|0 $\frac{2}{3}$

maketh 5 s 6 $\frac{2}{3}$ d, I haue wrought this at length for the aide of the young learner, becauſe he ſhould underſtand how all the multiplication is ſet downe.

27 13 4

20

s 5|53

|12

d 6|40

|100 $\frac{2}{3}$

makes 5 s 6 $\frac{2}{3}$ d But to work it more neatly, it is by a little underſtanding ended thus.

Item, to the underſtanding of this and ſuch like queſtions, ẏ right downe line is all the guide, which is pulled downe cloſe by 20 as you ſee in the example, where 27 pound 13 ſhillings is reduced all into ſhillings, and maketh 553 ſhillings.

The 5 towards your left hand being ſepa-

rated

rated with the hanging or right downe line, is the iust number of shillings that answereth to the question. Nextly, 53 shillings is multiplyed by 12 to reduce them to pence, putting to the 4 d, it yeeldeth for the multiplication of the first figure two, 100: the one beyond the line towards the left hand, is 1 peny towards the rest of the price: then 53 also multiplied by 1 yeeldeth 53: but the 5 behinde the line towards the left hand, is also 5 pence more, towards the price, which 1 and 5 I adde together vnder the line: it maketh 6 d. So is there found now as appeareth by the Titles of shillings and pence, 5 shillings 6 pence.

Finally, I come now on this side the line towards the right hand, & vnder 12 I finde first 10, & then 3, which added together maketh 40, vnder which 40 you must put the 100, and it maketh $\frac{40}{100}$, which abbreviated commeth to $\frac{2}{5}$. So the iust price of one pound after 5 score to ye hundreth maketh 5 s 6 d. $\frac{2}{5}$ d

One example more, and so I will leaue this rule.

If

If 100 cost $10\frac{3}{4}$ d
What 9874?

100 — $10\frac{3}{4}$ — 9874.

6 d	246	17	0
4 d	164	11	4
$\frac{2}{4}$ d	20	11	5
$\frac{1}{4}$ d	10	5	$8\frac{1}{2}$

ma-keth	l 4	42	5
		20	
	s 8	45	
		12	
	d 5	$45\frac{1}{2}$	
		$\frac{91}{100}$ parts of a d	
		100	

Also the like may bee done of the usuall weights here in England (which is 112 for euery hundreth weight) in case you knowe ÿ aliquot parts of a hundreth weight, which are these, 56 l, 28 l, 14 l, and 7 l: For 56 l is the $\frac{1}{2}$ 112 pound 28 l, is the $\frac{1}{4}$ of 112 l 14 l is the $\frac{1}{8}$, and 7 l is $\frac{1}{16}$ part.

Therefore for 56 l take the $\frac{1}{2}$ of the sum of mony that 112 weight is worth.

For 28 l, take the $\frac{1}{4}$ of the summe of money that 112 l weight is worth.

For 14 l, take the $\frac{1}{8}$ of the summe that 112 l is worth.

And for 7 l, the $\frac{1}{16}$ of the summe of money that C is worth.

As for example: At 17 l 19 s the hundreth

pounds weight, that is to say, the 112 pound what shall 3 quarters and 7 pound cost?

1——17——19——3——7

2 quarterns	8	19	6
1 quarterne	4	9	9
7 pounds	1	2	$5\frac{1}{4}$
maketh pounds	14	11	$8\frac{1}{4}$

The second Chapter intreateth of the Reduction of diuers measures to others value by Rules of Practise.

NOw wil I shew a few examples of practise in reducing of measures, as Elles, Yards, Braces, Pawns of Genes &c. Much more would I haue touched, but that I fear the booke will rise to too great a volume.

In 864 elles of Antwerpe, how many yards of London?

864	864
432	216
216	648

maketh 648—— yards of London.

Item in these and such like questions of Flemmish measure to be brought into yards english: first take the $\frac{1}{2}$ of the giuen number, as

as appeareth in the first example towardes your left hand. Then take the $\frac{1}{2}$ of that product, or the $\frac{1}{2}$ of the giuen number, and adde those 2 products together they shall be yards English, as by the example you may perceiue.

The second example towards your right hand is yet briefer than ỹ first, whose worke is this: take the $\frac{1}{4}$ of the diliuered number, and that product subtract out of the giuen number, and the rest sheweth your desire. Of these two wayes vse which you thinke best.

The Proofe.

In 648 yards of London.
How many elles of Antwerpe.

$$\frac{648}{216}$$

maketh 864 Elles of Antwerpe.

Item for the vnderstanding of this work: first take the $\frac{1}{3}$ part of the yards of London, which found, adde that $\frac{1}{3}$ part and the yards together, as appeareth by the practise, & the product sheweth the elles of Antwerpe.

In

In 320 yards of London,
how many elles of Antwerpe,
maketh 426 $\frac{2}{3}$ Elles.

320 yards	Proofe.
$\frac{1}{2}$ 106	426 $\frac{2}{3}$ Elles
426 $\frac{2}{3}$	$\frac{1}{2}$ 106 $\frac{2}{3}$
	320 yards.

Other Reductions.

Item, you shal understand, that forasmuch as sixe braces of Millane make 5 Elles of Antwerpe, wherupon according to the Rules of Practise, you may reduce the one into the other by the like reasons aforesaid, in taking the $\frac{1}{6}$ part, and thē subtract the same to make Elles of Antwerpe. And againe by the contrary, taking the $\frac{1}{5}$ part with adding the giuen number, to turne the elles to Braces, as for example,

In 876 Braces, how many els of Antwerp?

876 Braces	The contrary!
(146	730 Els Flem.
Els 730 Antwerpe	146
	876 Braces.

Elles 730 Antwerpe.
182 $\frac{1}{2}$
Yards 547 $\frac{1}{2}$ English.

Thus

Thus appeareth, that 876 Braces by practise, make 730 Elles Flemmish: which elles Flemmish reduce into English yards.

So againe upon the same first question of Braces, I would know how many yardes English they make.

After the rate that 100 Braces are worth 62 ½ yards.

876 Braces.
438
109 ½
I answere 547 ½ yards.

Item, to the understanding of this worke, & such like, first take the ½ of the given braces, and after take the ¼ of that halfe, or the ½ of the given number, and adde them together, and the products are also yards English.

Item three Elles of Rochell make 5 elles at Lisbone. So likewise three elles at Lyons make 5 elles at Antwerpe.

To worke these and such like, double the Elles of Lions, and the elles of Rochell, and from their products subtract ỹ ⅙: And the rest shall bee the Elles of Antwerpe, or the Elles of Lisbone.

Example.

Example.

In 63 elles of Lyons how many elles of Antwerpe?	In 100 Elles of Rochell, how many elles of Lisbone?
63	100
63	200
126	
$\frac{1}{6}$ 21	$\frac{2}{6}$) 33$\frac{2}{3}$
Ans. 105 elles Ant.	Ans. 166 $\frac{1}{3}$ els of Lisb

Touching the proofe or returne of these & such like questions, for a generall rule, you shall first take the $\frac{1}{5}$ of the given number: & adde that $\frac{1}{5}$ and the given number together, and the $\frac{1}{2}$ of that product shall be your desire.

Example.

In 105 elles of Antwerpe, how many elles of Lyons?	In 166 $\frac{2}{3}$ elles of Lisbone, how many els of Rochell?
105	166 $\frac{2}{3}$
$\frac{1}{5}$ 21	$\frac{1}{5}$) 33 $\frac{1}{2}$
$\frac{1}{2}$ 126	200

An. 63. els of Lyons. Ans. 100 els of Roch.

The

The third Chapter teacheth of the order and worke of the Rule of three in broken number, after the trade of Merchants, digressing something from M. *Records*, which is comprehended in 3 Rules.

Now that I haue somewhat intreated of ẏ Rules of Practise, I will giue a few instructions, after my simple order, for the working of the Rule of three in broken numbers, wherein I shall neede to say the lesse, because I hope the studious learner, that hath trauelled any thing in the Ground of Artes, is not vnfurnished of knowledge capable to vnderstand me. But before I deliuer any instructions for broken numbers, I will propone a question, which shall be wrought three sundry waies, thereby to shew as it were three degrees of comparison: how farre the rule of three in broken, for more speede of worke, differeth from the whole, which I rather set downe for a view, that the studious herein may be more desirous to attaine broken: leauing any more to discourse in dialogue forme, but onely to giue instructions where neede is: and in the rest to put forth the questions with their answeres.

My first question is this.

If one yard cost 6 s 8 d, what are 789 worth at that rate?

1 —— 6 —— 8 —— 789

12 80

80 63120 d

Here the product of the summe are pence, according to the nature of the middle number.

11
137 | 1
63120 | 5260
12222 | 2220 (263
111

I answer 263 l.

1 —— $6\frac{2}{3}$ —— 789

3 20 15780

Here the product of the summe are s, according to the nature of the middle number.

1
15780 | 5260
3333 | 2220 263

1 —— $\frac{1}{3}$ —— 789

3 1 1

789

where the product is pounds, according to the title of the second number.

x
789
333 (263
I answere 263 l.

Now that you haue seene the 3 former vertues of the Rule of three, whose products haue first brought forth d, next s, and lastly pounds, I will deliuer three notes in order following, and with them a dozen questions that shall shew the works of the Rule of 3 in broken numbers or Fractions.

1 The first foure shall be sundrie questions of a Fraction comming in the second place.

2 The second foure shall bee of two Fractions comming in the second or third place.

3 The third foure of Fractions in all three places.

My first question is this.

If one yard cost mee 3 s 4 d, what are 756 worth at that price?

In setting downe the question to performe the worke, I turne 4 pence into the part of a shilling, which is $\frac{1}{3}$, and then the question standeth thus:

1 ——— $3\frac{1}{3}$ ——— 756

To the ready working of this question, and all such other like, my first note is this, which take for a generall rule: that when any one Fraction shall come either in the second or third place, that the Denominator of that fraction or fractions, must allwaies be brought vnto the number or Numerator of the first place: and thereby multiply the one into the other.

And this benefit is allwaies gotten by the vertue of bringing the Denominator of the second numbers fraction vnto the first place. For the Fraction in the middle number is now released: and the product that commeth of the multiplication, is of the nature and like Denomination of the whole number in the second place which here are shillings.

Whereupon now to worke the question, I bring three the Denominator of the fraction in the second place, vnto my first number one with a line set vnder 1 thus: and the three vnder it $\frac{1}{3}$ thus: saying, once 3 is 3 my Diuisor: that done, reduce three $\frac{1}{3}$, saying: 3 times 3 is 9, and the 1 ouer 3 make 10: my second number in the rule of 3: by which 10 I do multiply my last number 756, as appeareth by the worke thereof, and it yeeldeth 7560 s my diuidend.

Then diuiding 7560 by 3 my diuisor, it yeeldeth in quotient 2520 shillings, which maketh 126 pounds, as appeareth here most plainel[y]

bot[h]

both by the example and the worke.

At 3 s—4 d the yard, what 756 yards?

$\frac{1}{3}$——3 $\frac{1}{3}$ 756

                    10

10   7560 s

~~2~~	~~1~~	
~~7560~~	~~2520~~	126: I answere 126 l.
~~3333~~	~~2220~~	

Yet otherwise upon the same question, altering the price now into the proportion it beareth to a pound, for the 3 s 4 d is $\frac{1}{6}$ part of a pound: which example first standeth thus, as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.

$\frac{1}{6}$———$\frac{1}{6}$———756

1—$\frac{1}{6}$—756   1   1

756 pounds.

As soone as I haue caried 6 the denominator of my middle number unto my first place, as before hath been taught, I pul downe one, the numerator of 6, with a line under 6 thus, and that one in custome I pull downe in right, beeing the figure that I shall multiply my third or last number by, according to the order of the rule of three. And because one can

neither multiply nor yet diuide (though heere it is set downe in forme of multiplication, the rather for your understanding) the product of the multiplication according to the declaration of this my first rule or note is conuerted into the title of my second number, which heere are pounds. Now followeth the diuisiō performed by my diuisor 6, to make an end of ỳ question.

12
756 (126 : which maketh 126 l', as before.
666

And thus much for the varietie in working that question.

And now followeth another.

If one yard of cotten cost 8 d $\frac{1}{4}$, what 859

1 ——— 8 $\frac{1}{4}$ ——— 859

4 33 33

2577
2577

28347

2 (3 | 11 | l' s 6
288347708 (6 | 590 | 29 — 10 — 6 — $\frac{1}{4}$ d.
444412 | 220 |

Th

This queſtion was alſo wrought like the 1, and bringeth forth 29 l—10 s—6 ½ d the price of 859 yards.

If 7 pounds of any thing coſt 3 l—10 s, what comes 987 pounds to?

$7 —— 3\frac{1}{2} —— 987$

$\frac{2}{14}$ $\underline{7}$ $\frac{7}{6909}$

(7)

$6909|493\frac{7}{14}|\frac{1}{2}$ I anſwer 493 l—10 s

14

Notes vpon my ſecond rule for two Fractions comming in the ſecond and third place.

My firſt queſtion is this.

If one Ell coſt 13 s—4 d; what halfe a quarter or $\frac{1}{8}$ of an Ell?

Anſwer. Firſt bring 13 s—4 d into the parts of a pound, which is $\frac{2}{3}$, and then will the queſtion ſtand thus:

$1 — \frac{2}{3} — \frac{1}{8}$

Item for the performance of this worke, do as before was taught in the firſt Rule: firſt bring

bring 3 the denominator of the second fraction unto your first number 1, setting a line under $\frac{1}{3}$ thus: Saying, once 3 is 3: that done, bring 8 the denominator of the third fraction, setting it under 3, and multiply them together, saying, 3 times 8 maketh 24, which 24 is your diuisor: (Now haue you done with the denominator 3, and also with the denominator 8.) Therefore you shall put a line under 3 thus. And the like line also under 8, setting or pulling downe under them their owne numerators, that is 2 under 3, and also 1 under 8, as appeareth in the example, which numerators for a generall rule are euermore to bee pulled downe of custome in sight, to multiply the one by the other, according to the tenor of the rule of 3. Then I multiply the one by the other, saying: once 2 is two, which signifieth 2 l, being of the nature and like denomination of the middle number, which 2 l is to be reduced into shillings, otherwise it cannot be diuided by my first number 24.

Then diuiding 40 by 24, the quotient bringeth forth 1 $\frac{2}{3}$ s. So much is $\frac{1}{8}$ of an elle worth after that rate, Otherwise although 2 pound could not bee diuided by 24: yet it might haue beene abbreuiated to $\frac{1}{12}$ of a pound, which is worth 1 s—8 d, as before.

$1 \text{———} \frac{3}{8} \text{———} \frac{1}{2}$

3 2 1
8 20 16

——— ——— 40 (1 $\frac{2}{3}$
24 40 24

Second queſtion.

If one pound of any weight coſt 13. ſhillings 4 pence, what are $\frac{7}{8}$ of the pound worth after that rate?

Anſwere, Reduce the 13 ſhillings 4 pence into the parts of a pound: which is $\frac{2}{3}$, and then will the queſtion ſtand thus:

$$\begin{array}{c} l \\ 1 \text{———} \frac{2}{3} \text{———} \frac{7}{8} \end{array}$$

Item for the vnderſtanding of this, if you marke well the laſt example, this and the reſt lieth open, and need ſmall inſtruction. For as you did laſt, ſo now againe bring the denominator of the ſecond and third fraction vnto the firſt figure 1, multiplying the one into the other, which maketh alſo 24 your diuiſor.

Then making a line vnder 3 thus, and a line vnder 8 thus, and pulling downe their Numerators vnder each figure, that is, 2 vnder 3, and 7 vnder 8, which as I ſaid before for a generall rule I pull downe of cuſtome in ſight, to bee the two numbers that of duety ought

ought to be multiplied together, which done I bring 2 being the leſſer figure vnder 7: multiplying them together, it maketh 14: which are of the nature of the middle number: that is to wit pounds: which 14 cannot aptly bee diuided among 24, therefore are reduced into ſhillings, as is plainely to bee ſeene in the example, then 280 ſhillings parted among 24, yeeldeth for his quotient 11 s 8 d your deſire, and the iuſt price of $\frac{7}{8}$ of an ell. Otherwiſe, 14 though it could not bee diuided by 24, might by mediation or diuiſion in broken numbers haue béene diuided or abbreuiated to $\frac{7}{12}$, which in effect being reduced to his knowen parts, maketh 11 s 8 d as before. But my good will and meaning is to aid yong beginners: therefore haue I reduced the 14 l' into s, which is the eaſier way.

Now followeth the example.

			(1
1	2	7	4
3	3	8	46
8	2	7	280 | 11 $\frac{2}{3}$ s 8 d
		2	244
24		14	2
		20	
		280 s	

The third example.

If one yard cost mee 2 s—6 d, what 345 $\frac{1}{4}$ yards?

Answer. First put 6 d into the parts of a shilling, and then the question standeth thus:

1 —— 2 $\frac{1}{2}$ —— 345 $\frac{1}{4}$.

Item, to the ready understanding of this and all such like, according as before hath been declared, bring the Denominators of the second and third fractions unto the first place, multiplying them the one into the other, all which make 8 from the diuisor common. Then next reduce your second number, saying, two times 2 is 4, and 1 is 5, as was taught in the example aforesaid. Lastly, reduce your third number 345$\frac{1}{4}$ all into fourths, and they make 1381: which 1381 is to bee multiplied by 5, according to the tenor of the rule of 3: which done, maketh 6905 s: and diuided by 8 your Diuisor, yeeldeth in quotient 863$\frac{1}{8}$ s, which maketh in pounds 43 l—3 s—1 d $\frac{1}{2}$: and so much are the 345 $\frac{1}{4}$ yards worth at that price.

The same question wrought againe by two shillings 6 pence, is now conuerted into the parts of a pound, and standeth thus:

1 —— $\frac{1}{8}$ —— 345$\frac{1}{4}$

Item, after I haue brought here my second and third fraction unto my first place, and found 32 to bee my diuisor; hauing thus furnished

shed my first place with all things vnto him belonging (which is meant of bringing and multiplying the Denominators of the second and third Fractions into him) I then goe in hand to see what is to doe in my second place, where presently of custome I pull downe my Numerator 1 vnder 8: being the figure in sight that shall multiply my third number.

Then lastly I reduce $345\frac{1}{4}$ all into fourths as afore was practised, which maketh 1381, the which 1381 I am to multiply by 1 my second number, they are nothing increased, but by the Metamorphosis of my worke they are now 1381 pound, being of the nature of the middle number, as I haue often shewed you, which diuided by 32 my diuisor, yeeldeth 43 pound and $\frac{5}{32}$: which $\frac{5}{32}$ of a pound reduced into knowen numbers, make 3 shillings, 1 d, $\frac{1}{2}$, as before.

Example.

$$\begin{array}{cccccc}
 & & & \cancel{1} & & \\
 & & & \cancel{1}\cancel{0}(5 & & \\
1 \text{———} \frac{1}{2} \text{———} 345\frac{1}{4} & & & \cancel{1}\cancel{3}8\cancel{1} & & 5 \\
32 \quad\quad 1 \quad\quad \overline{} & & & \cancel{3}\cancel{2}\cancel{2}(43 & & \overline{} \\
1381 & & & \cancel{3} & & 32
\end{array}$$

Now follow 4 other questions, which are in all three places broken numbers, or whole and broken together.

Item

Item first for the finding out of your Diuisor: for you shall take this for a most certaine and generall rule; That you must multiply the numerator of the first number in the question by the denominator of the second: And also that againe by the denominator of the third: and the totall thereof shall be your diuisor.

Secondarily, for a generall rule to find out your diuidend, multiply the denominator of the first number by the Numerator of the second, and the whole thereof by the Denominator of the third. And the totall thereof shall euermore be your diuidend.

Now for an example I propone this question, thereby to make my meaning the more plaine, and to shew you as I haue done in the rest, the manner and order of the worke.

If $\frac{2}{3}$ of any waight or measure cost $\frac{5}{6}$ of a pound or 20 s, what are $\frac{3}{8}$ of the like waight or measure worth after that rate?

Example.

$\frac{2}{3}$—$\frac{5}{6}$—$\frac{3}{8}$

Item for the more plainer vnderstanding hereof, and all other the like, in broken numbers: First you shall pull downe two the numerator of the first number or fraction, with a line vnder $\frac{2}{}$ thus: that done, according as you

haue

haue learned before, bring 6 the denominator of the second fraction, and set it vnder 2, multiplying the one into the other, which maketh 12, Then lastly bring 8 the denominator of the third fraction, and set it vnder 12, multiplying that 12 by 8, which amounteth to 96: or els for more briefe, multiply 6 by 8: saying 6 times 8 makes 48, which 48 set vnder two, and multiply the one into the other, it maketh 96 as before. And this 96 is the first number in the rule of three. That shall alwaies for a most generall rule be your diuisor.

Secondly to worke for your diuidend, you shall, as hath beene sufficiently declared afore, pull down 5 the numerator of your second fraction, and set it vnder 6 with a line vnder $\frac{6}{5}$ thus. That done, as you know, you are to pull downe 3 the numerator of the third fraction, and set it vnder 8 with a line vnder $\frac{8}{3}$ thus, multiplying the one into the other according to the tenour of the rule of three, which maketh 15. Then according to my note, forget not to bring the Denominator of the first Fraction which is 3 vnder 15, and multiply them together, which maketh 45, which 45 is your deuidend. Which 45 are of the nature or denomination of the middle number, as I haue oft taught you before. And therfore are 45l', which aptly cannot bee diuided by 96. Therefore you shall reduce that 45l' into s, as you see performed in the example, which amoun-

amounteth to 900 s, which diuided by 96 your diuisor, it yéeldeth 9 s, and $\frac{36}{96}$ of a shilling, which in lesser termes is $\frac{3}{8}$, which $\frac{3}{8}$ in money maketh $4\frac{1}{2}$ d: and so much will the aforesaid $\frac{3}{8}$ cost, as by the work following shall appeare.

The example.

2	5	3	
$\frac{3}{2}$	$\frac{6}{5}$	$\frac{8}{3}$	
48		15	36
96		3	230 (9 $\frac{36}{96}$)
		45	96
		20	
		900	

Otherwise though 45 could not be diuided by 66, yet by diuisiō in broken numbers it might haue béene abbreuiated to $\frac{15}{32}$ of a l', which reduced into knowen parts will make 9 s $4\frac{1}{2}$ d, as before.

Now my second example shall be the proofes of this question.

If $\frac{3}{8}$ yards cost $\frac{15}{32}$ of a pound or 20 shillings, what shall $\frac{2}{3}$ cost?

Answer. Work as was taught you before, and you shall haue your desire.

Héere

$\frac{3}{8}$ —— $\frac{15}{32}$ —— $\frac{2}{3}$

3	15	2
32	2	
96	30	
3	8	
288	240	

Heere, as appeareth by the worke, the multiplication being ended, 240 is to bee diuided by 288, which to some perchance may seeme hard, yet notwithstanding is the worke good. Therefore abbreuiate 240 by 288, as you see heere is practised: and the end of your abbreuiation shall come to $\frac{5}{6}$ your desire $\frac{240}{288}\Big|\frac{30}{36}\Big|\frac{5}{6}$

Otherwise. $\frac{240}{288}\Big|\frac{120}{144}\Big|\frac{60}{72}\Big|\frac{30}{36}\Big|\frac{5}{6}$

Otherwise. $\frac{240}{288}\Big|\frac{40}{48}\Big|\frac{5}{6}$

The third question.

If $\frac{3}{4}$ elles cost 13 s—4 d, what 156 $\frac{1}{2}$ elles

Answer. To work this question the shortest way: reduce 13 s—4 d into the parts of a pound, which is $\frac{2}{3}$.

Then as you did afore, after you haue set downe the question, the numerator of the first fraction 3 is pulled downe vnder 4, and the

Deno-

Denominators of the other 2 Fractions mul-
tiplied into him which maketh 18 your deuisor.
Then the numerator of the second fraction
t, wois pulled downe vnder thrée of custome
now in sight, ready to multiply my third nū-
ber by: which is performed as sone as the last
number 156 $\frac{1}{2}$ is reduced into halfes.

Then lastly, I multiply that product by 4,
the denominator of my first fractiō: it yéeldeth
1504, which I diuide by 18, & my quotient is
139 l, and $\frac{1}{9}$ of a pound remaining, which is
worth 2 s 2 $\frac{2}{3}$ d: And so much will 156 $\frac{1}{2}$ els
cost, as by the worke following doth appeare.

$\frac{3}{4}$ $\frac{2}{3}$	156 $\frac{1}{2}$	x	
6	313	47	
3	2	x76 (2	
18	626	2504	139 $\frac{1}{9}$
	4	x888	
	2504	xx	

The fourth Example.

If $\frac{2}{3}$ elles cost 1 $\frac{2}{3}$ l, what commeth 29 $\frac{3}{4}$
elles to?

Item, to the workmanship of this question:
first reduce your second number to one direct
Number: in saying thrée times 1 is 3, and
2 is 5: Then bring the multiplication of the
Denominators of the second and third fracti-

 ons,

ons, which maketh 12: and multiply that 12 by fiue your first numerator, it maketh 60, which is your diuisor.

Then the reduction of the second number, which is 5 multiplied by 117 the product of ẏ last numbers reduction, make 585, which 585 yet resteth to be multiplyed by two, the denominator of the Fraction in the first place yeeldeth 1170: which diuided by your diuisor 60: yeeldeth 19 pounds 10 s, as appeareth by the worke thereof.

Thus hauing now touched the 12 questions whereof I first pretended, which wt diligence and oft practise, I trust are sufficient to ayde the desirous vnto the working of any broken numbers, I will now intreate of diuers necessary rules incident vnto trafficke, as hereafter followeth.

The fourth Chapter treateth of Losse and Gaine in the trade of Merchandise.

If one yard cost 6 s——8 d: and the same is sold againe for 8 s 6 pence: the question is, what is gained in 100 pounds laying out on such commoditie.

Answer. The rule of three direct, applyeth two manner of waies to do the same, the one is, to say: If sixe $\frac{2}{3}$ giue 8 $\frac{1}{2}$, what giueth 100? Multi-

Multiply and diuide, & looke what your quotient bringeth forth aboue your laying out, is the neare gains, and the solution to your question: If you follow the worke, your quotient will bring forth 174 l' - 10 s, which is 27 l' 10 s more then your principall, and so much is gained in the 100 pounds laying out.

Item, to worke it the other way, which I take the nearest, seeke the difference betwixt the iust price and the ouer-price, which is one shilling-10 pence. Then say by the rule of 3. If $6\frac{1}{3}$ s gaine $1\frac{5}{6}$ s, what shall 100 l' gaine? Multiply and diuide, and you shall finde 27 l' — 10 s, and so much is gained in 100 l' laying out.

Use which of these two wayes you thinke good.

The Proofe.

If a yard of cloth be deliuered for 8 s, 6 d, whereupon was gained after the rate of 27 l' 10 s in 100 pounds laying out: The question is, what the yard cost at the first hand?

Answer. Put your gaines to 100 l', all maketh 127 l' 10 s: then say, if 127 l' 10 s giue but 100 pound what giueth $8\frac{1}{2}$ s? Work and you shall finde 6 s, 8 d, the true solution to your question.

Yet another branch or proofe vpon the same first question.

If one yard cost 6 s—8 d, the question is what price the same is to be sold againe for to gaine 27 l—— 10 s, in 100 l laying out.

Answer Say by the rule of thrée, if 100 l giueth 127 l—10 s, what giueth $6\frac{2}{3}$ s. Multiply and diuide, & you shall find 8 s —— 6 d your true solution.

If one elle cost 7 s—8 d, and sold againe for 8 s — Question. What is gained in 20 l laying out in such commodities.

Answer. Séeke the difference betwixt the iust price & the ouer-price, which is 10 pence, and then apply the Rule of thrée, as before is taught, saying, If $7\frac{2}{3}$ s giue $\frac{5}{6}$ shillings, what giueth 20 l? Multiply and diuide, & you shall find 2 l. $3\frac{11}{23}$ s. and so much is gained in 20 l laying out.

The proofe also by an example of losse.

A Merchant hath bought Holland cloth at 8 s--- 6 pence the elle, which proueth not to his expectation, whereupon he is content to lose 2 l- 3 s $\frac{11}{23}$ s in 20 pounds laying out. The question is what price ought to be made of the elle abating this losse.

Answer. Doe as before in gaines hath bene taught, putting 2 l -- $3\frac{11}{23}$ s to your 20 l, all together maketh 22 l—$3\frac{11}{23}$. Then say by the rule of thrée: If 22—$3\frac{11}{23}$ s giue but 20 l, what

what shall come of 8 $\frac{1}{2}$ s? worke and you shall finde 7 s---8 d, the iust price that the ell ought to be sold for after the rate of this losse.

That it appeareth euidently, as in company the Rule is appliable as well to gaine and losse.

If 20 $\frac{1}{4}$ yards cost 26 l---10 s, how shal I sell the same againe to gaine $\frac{1}{3}$ of the principall, or to make of 3, 4: which is all one.

Answer. By the rule of 3, if 3 doe giue 4, what will 36 $\frac{1}{2}$ giue? Multiply and diuide, & you shal finde 48 l——$\frac{2}{3}$. Then say againe, if 20 $\frac{1}{4}$ yards do giue 48 $\frac{2}{3}$ pounds, as well principall as gaine, what will one yard be worth at that price? Multiply and diuide, & you shall finde 2 l 8 $\frac{16}{24}$, s.

If one Elle of cloth cost me 8 s--8 d, and afterwards I sell 10 $\frac{1}{2}$ Elles thereof for 5 l, 13 s---4 d, I would know whether I winne or lose: and how much vpon the 100 l of money.

Answer. Sée first at 8 s——8 d the elle, what 10 $\frac{1}{2}$ elles comes to, and you shal find 4 l 11 s, and I sold the same for 5 l—13 s--4 d, so that I did gaine vpon the 10 $\frac{1}{2}$ yards, 1—12—4 d Then if you would know how much is gained in the 100 l, I say by ye rule of three, if 4 l---11 s did gaine 22 s 4 d, what wil 110 l gaine?

gaine? Multiply and diuide, & you shall finde 24 pound—10 s—10 d $\frac{10}{91}$. And so much is gained in the 100 pound of money.

If 12 $\frac{1}{2}$ yards cost me 11 pound—5 s, & I sell the yard againe for 16 s, the question is, whether I doe winne or lose, and how much in or vpon the pound of money.

Answer. Looke what the 12 $\frac{1}{2}$ yards come to at 16 s the yard, & you shall finde 10 pound. But they cost 11 pound--5 s. So there is lost vpon the whole 1 pound 5 s. Then to knowe how much is lost in the pound, say by ye Rule of 3, if 11 $\frac{1}{4}$ pound do lose 1 $\frac{1}{4}$ pound, what will 1 pound lose? Multiply and diuide, and you shal find 2 s-2 d $\frac{2}{3}$, and so much is lost in the pound of money.

If I sell the C weight of any commodity for 4 pound, whereupon I do lose after 10 pound in the 100 pound, I demand how much I shal lose or gaine in the 100 pound, if in case I had sold the same for 4 pound—10 s.

Answer. Say, if 90 pound yeeld 100, how much will 4 giue? Multiply and diuide, & you shall finde 4 $\frac{4}{9}$. Then say againe, if 4 $\frac{4}{9}$ giue me 4 $\frac{1}{2}$, what will 100 come to? Multiply and diuide, and you shall find 101 pound $\frac{1}{4}$, which is more then 100 pound by one pound 5 s. And so much is gained in the 100 pound.

A Merchant hath sold Currants for the summe of 436 pound, & he hath gained therein after

after 10 pound in the 100 pound. The question is to know how much he gained in all.

Answer. Say by the rule of three, If 100 pound do gaine 10 pound, what wil 430 poūd gaine? Multiply and diuide, & you shal find 43 and so much hath he gained in all.

If one yard be worth 28 $\frac{1}{2}$ s, for how much shall 10 yards be sold to gaine after 8 l'—6 s 8 d in the 100 pound.

Ans. First adde 8 l'——6 s, 8—d to 100. Then say, if 100 l do giue 108 $\frac{1}{3}$ for principall and gaine, what will 28 $\frac{1}{2}$ s principall yeeld? Multiply and diuide, and you shall finde 30 $\frac{7}{8}$ s. Then say againe by the rule of three, If 1 yard doe giue 30 $\frac{7}{8}$ s, (which is as well the principall as the gaine) what shall 10 yards giue? Multiply and diuide, aud you shall finde 15 l—8 s—9 d. And for the same price shall ẏ 10 yards bee sold, for to gaine after the rate of 8 l——6 s——8 d vpon the 100.

A Branch or Proofe out of this Question.

A Merchant hath sold Clothes for 15 l'—— 8 s——9 d, and he hath gained in the whole, the sum of 1 l——3 s—9 d. The question is to know how much he hath gained in the 100 pound.

Answer. To know this, first rebate the gaines from the price, and there will remaine

14 l---5 s---0 d. Then say by the rule of 3 direct, if 14 l $\frac{1}{4}$ giue me 1 l---3 s---$\frac{3}{4}$ s, what will 100 l giue? Multiply and diuide, and you shall finde 8 l---6 s---8 d, the effect desired: the proofe is apparant in the question before.

Yet another Branch or Proofe of the first Question.

If 10 yards be deliuered for 15 l---8---9 whereupon was gained after the rate of 8 l 6 s---8 d vpon the 100 l, y^e question is, what the yard did cost at the first hand.

Answer. First, say by the Rule of three, if 10 with principall and gaine yeeld 19 l--8 $\frac{3}{4}$, what shall 1 yeeld? Multiplie and diuide, and you shall finde 30 $\frac{7}{8}$ s. Then say againe by the rule of three, if 108 $\frac{1}{3}$ principall and gaine giue but 100, what shall 30 $\frac{7}{8}$ s of principall and gaine yeeld? Worke, and you shal finde 28 $\frac{1}{2}$ s. And so much did the yard cost at the first penie.

If one yard cost 36 s, how much shall 12 yards be sold for to gaine after the rate of 10 l in the 100 l?

Answer. First say, if 100 giue 100 pound principall and gaine, what will 36 s giue? Multiply and diuide, and you shall finde 39 $\frac{3}{5}$ s. Then say againe by the rule of three: If 1 yard of principall and gaine yeeld 39---s what

what shall 12 yeares gaine? Multiply and divide, and you shall finde 23 l—15 $\frac{1}{7}$ s, which $\frac{1}{7}$ s in knowne nūber is 2 $\frac{2}{7}$ d. And for ẏ same price shall the 12 yards be sold, to gaine after the rate of 10 in the 100.

The Proofe.

If 12 yards be sold for 23 l—15—2 $\frac{2}{7}$ d, whereupon is gained after 10 l in the 100 l. The question is, what the yard cost at ẏ first pente?

Answer. First say, if 12 give 23 l——15 $\frac{1}{7}$ s, what one yard? Multiply and divide, and you shall finde 39 $\frac{3}{7}$ s. Then say againe by the rule of thrée, if 100 poūds give but 103 what shall 39 $\frac{3}{7}$ give? Worke, and you shall finde 36 s, the iust price of the yard at the first hand.

Item, when one Merchant selleth wares to another, & he giueth to the buyer 1 l--16 s 8 d upon the score, or 20 l: The question is, how much shall the buyer gaine upon the 100 pound after that rate.

Answer. First adde 1 l—6 s—8 d unto 20 l, and they are 21 $\frac{1}{3}$. Then say, if 20 pound give 21 $\frac{1}{3}$ what shall 100 give? Multiply and divide, and you shall find 106 $\frac{2}{3}$. So the buyer

getteth

getteth after the rate of $6\frac{2}{3}$ l' vpon the 100 pound.

Gentle Reader, other necessary questions appertaining to Losse and Gaine, you shall haue in the eight chapter of this Treatise.

The fifth Chapter entreateth of Losse and Gaine vpon time, wrought by the double Rule of three: or by the Rule of three composed, which is contained in 4 speciall selected branches or questions of diuers formes, each one of them springing from the first question, and each one of them also being a proofe to other, &c.

IF one yard cost me 2 s—8 d ready money, and after I sell the same againe for 2 s—10 d, to be paid for it at the end of 3 moneths. The question is, what I gaine vpon the 100 l in 12 moneths.

Answer. First say, if $2\frac{2}{3}$ gaine $\frac{1}{6}$, what shall 100 l' gaine? Multiply and diuide, and you shal find $6\frac{1}{4}$ l'. Then say againe by the rule of thrée, if thrée moneths gaine $6\frac{1}{4}$ pound, what shall 12 moneths gaine? Work and you shall finde 25 l', and so much shall I gaine in 12 moneths after that rate.

Item, you may also worke it at one working by the first part of the rule of 3 composed saying,

ſaying, if 2 $\frac{1}{3}$ d in thꝛée moneths do gaine $\frac{1}{6}$ of a ſhilling (which is 2 d) what will 100 l' gaine in 12 moneths: which foꝛ thy further encouragement, the woꝛke of this one example I haue here put downe, to verifie that I affirme in the firſt part of this Ground of Arts, that this rule, and ſo all others moꝛe reioyceth in Bꝛoken then in Whole.

s	moneths.	s	l'	moneths.
2 $\frac{1}{3}$	3	$\frac{1}{6}$	100	12
8				100
3		2		1200
24		7		3
6		162		3600
144		3600 | 25		
		1444		
		14		

Where the multiplication & the Diuiſion being ended, maketh 25 l' your deſire.

If a yard be deliuered foꝛ 2 s—10 d to bée paid at 3 moneths, whereupon was gained after the rate of 25 l' in the 100 foꝛ 12 moneths, the queſtion is now, what the yard coſt at the firſt hand,

Anſwer. Firſt ſay, if 12 moneths gaine 25 l', what ſhall thꝛée moneths gaine? Woꝛke and you ſhall finde 6 $\frac{1}{4}$ l'. Then ſay againe the

ſecond

ſecōd time, if $106\frac{1}{4}$ l giue but 100, what ſhall $2\frac{5}{6}$ s giue? Worke, and you ſhall finde 2 s 8 d which is the iuſt price that the yard coſt at the firſt hand.

If one yard of cloth coſt me 2 s---8 d readie money, for what terme ſhall I ſell the ſame againe for 2 s--10 d, ſo that I might gaine after the rate of 25 pound vpon the 100 pound in 12 moneths.

Anſwer. Firſt ſay, if $2\frac{2}{3}$ gaine $\frac{1}{6}$: what ſhall 100 pound gaine? Multiply and diuide, and you ſhall finde $6\frac{1}{4}$ pound. Then ſay again for the ſecond work, if 25 pound become of 12 moneths, what ſhall come of $\frac{1}{4}$? Worke, and you ſhall finde 3 moneths the iuſt terme of time that ẏ cloth ought to be deliuered at 2 s, 10 d, to gaine 25 pound vpon the 100 pound in 12 moneths.

If one yard coſt me 2 s--8 d ready money, for what price ſhall I ſell the ſame againe to be paid at the end of three moneths, ſo that I may gaine after the rate of 25 pound in ẏ 100 pound for 12 moneths.

Anſwer. Firſt ſay, if 12 gaine 25 l, what ſhall 3 moneths gaine? Multiply and diuide, & you ſhall finde $6\frac{1}{4}$ l. Then ſay for the ſecond worke, if 100 l giue $106\frac{1}{4}$, what giueth $2\frac{1}{3}$ s? Worke, and you ſhall finde 2 s—10 d, and for that price muſt the yard bee ſold to gaine after 25 pound in ẏ 100 poūd for 12 moneths.

Many

Many other of these questions I might here haue deliuered, but for feare the booke would rise to too thicke a volume, and so to make the price so much the dearer, wherby it might not be so partable to my country-men as I wish it. But these 4 I haue of purpose framed in this order, hauing relation one to another, assuring you that what questions soeuer may bee proposed within the compasse of this Rule, you shall finde by one of these 4, to make a solution. And moreouer, diuers other are yet to bee deliuered: where the Creditor giueth diuers dayes of payment, which can neuer bee well wrought, nor yet vnderstood, vnlesse you can first find by art the iust times that all those paiments, how different soeuer they be, ought to be paid at once: whereupon first I think good here to giue some instructions into such a rule, for it is the onely aide for the finishing of such questions as hereafter shal follow.

The sixth Chapter intreateth of Rules of Payment, which is a right necessarie rule, and one of the chiefest handmaids that attendeth vpon buying and selling, &c.

Example.

A Merchant doth owe a summe of money, whereof the $\frac{1}{3}$ is to be paid at 6 moneths, the $\frac{1}{2}$ at 8 moneths, and the rest at a yeare. If he would pay at one payment, the question is what time ought to be giuen him.

Answer. I haue omitted the quantitie of the summe, for you shall vnderstand, the rule is appliable, and yeeldeth a true solution to what sum soeuer shall be proponed. But now for orders sake in teaching, I doe imagine the sum to be 60 pounds, whereupon the manner of this worke is to multiply the proportionate part of the money by the time, as in company. Then 20 being the first paiment, and the $\frac{1}{3}$ of 60, which $\frac{1}{3}$ multiplied in broken numbers, by 6, his time of payment maketh $\frac{6}{3}$, which in whole numbers, as appeareth by the exãple in the margent, maketh 2 moneths: next 30 which is the $\frac{1}{2}$ multiplyed by his terme 8, yeelds 4 moneths, then ẏ rest which

$\frac{1}{3}$ by $\frac{6}{1}$	
$\frac{6}{3}$	2 moneths.
$\frac{1}{2}$ by $\frac{7}{1}$	
$\frac{8}{2}$	4 moneths.
$\frac{1}{6}$ by $\frac{12}{1}$	
$\frac{12}{6}$	2 moneths.
	8 moneths

is 10 l' muſt needes be abbreuiated into the proportion it beareth to 60 which is $\frac{1}{6}$, which multipled by his time 12 moneths, produceth $\frac{12}{6}$, maketh two moneths. All which added together, as appeareth in the margent, maketh 8 moneths, which is the iuſt time that all thoſe payments ought to be paid at once.

A Merchant hath 800 l' to pay, the $\frac{1}{6}$ thereof ready money, the $\frac{1}{4}$ at two moneths, the $\frac{1}{2}$ at foure moneths, and the reſt at a yeare. The queſtion is, if hee would pay all at one payment, what time ought to be giuen him.

Anſwer The ready money is neuer multiplyed: thẽ $\frac{1}{4}$ multiplied by 2 moneths as you did before, maketh $\frac{1}{2}$, thẽ $\frac{1}{2}$ by 4 produceth 2 moneths as appeareth here in the margent. But now for the reſt of the money, you cannot multiply it vntill you haue ſought what proportion it beareth to 800 pounds. Therefore you muſt ſubtract the ready money, the $\frac{1}{4}$ and ỹ $\frac{1}{2}$ out of the principall. The reſt will be $66\frac{2}{3}$ l', which you muſt looke what part it beareth to ỹ principall, which you ſhall finde to be $\frac{1}{12}$, the ſame you muſt alſo multiply by his time 12 moneths, and it yéeldeth 1 moneth, ſo all make $3\frac{1}{2}$ moneths, as appeareth in the margent.

	$\frac{1}{4}$ $\frac{2}{1}$	
	$\frac{2}{4}$	$\frac{1}{2}$
	$\frac{1}{2}$ $\frac{4}{1}$	
	$\frac{4}{2}$	2
$\frac{1}{12}$	$\frac{12}{1}$	
$\frac{12}{12}$		1
		$3\frac{1}{2}$

A

A Merchant is to pay 1600 l in three termes, that is to wit, 400 l at two weekes, and 600 l at foure moneths, lastly, 200 l at 5 moneths. The question is, in what time they ought to be paid at once.

Answer. Proportionate the parts, and you shall finde that 400 is $\frac{1}{3}$ part, and for 600 you shall find $\frac{1}{2}$, and likewise 200 is the $\frac{1}{6}$ part, which multiply by their times as before, and you shall haue $\frac{2}{3}$ weekes, more 8 weekes, and lastly three $\frac{1}{3}$ weekes, which together maketh 12 weekes or three moneths, your desire.

A Merchant is to pay 600 l in three terms, whereof 100 l is paid present, more 300 l at 20 daies, and the rest at 5 moneths, accounting 30 daes s to a moneth. The question is, what time ought these payments to bee payd at once.

Answer. Work, and you shall finde two moneths.

The

The seuenth chap. intreateth of buying & selling in the trade of Merchandise, wherin is taken part ready money, and diuers daies of payment giuen for the rest, and what is wonne or lost in the 100 l forbearance for 12 monethes more or lesse, according to the quantitie of money, or proportion of time, &c.

A Merchant hath bought Sattens which cost 8 shillings the yeard ready money. And hee selleth the same againe to another man for 10 s the yeard, but hee giueth 2 daies for the payment, that is to say thrée moneths for the one halfe, and fiue moneths for the other half. The question is to know how much the seller doth gaine vpon the 100 pound in 12 moneths after that rate.

Answere. Séeke first by the rules of payment, at what time those two payments ought to bee paid at once, and you shal finde foure moneths, at which time the second merchant ought to haue paid the whole entire payment. And therefore say by the first part of the Rule of thrée composed: If 8 s in 4 moneths doe gain 2 s, what will 100 l gain in 12 moneths

8—4—2—100—12			
4	12		
32	1200	2400	75
	2	232	
	2400		

Multiply and diuide, and you shall finde 75 pounds, as appeareth in the example, and so much doth the first merchant gaine vpon 100 pounds in 12 moneths.

A Merchant hath sold fiftie clothes at 9 $\frac{1}{2}$ l the peece, to be paid the one $\frac{1}{2}$ at foure months, the $\frac{1}{3}$ at fiue moneths, and the $\frac{1}{6}$ at 7 moneths, and the sellers minde is to take no more but after 8 pounds in the 100 for 12 moneths: The question is now what the first merchant gaineth in the sale of these cloathes after that rate.

Answere. First looke what the 50 cloathes come to at that price, and you shall finde 475 pounds. Then secondly, according to your direction in the Rules of payment, seeke at what time all the payments are to be performed at once. And you shall finde 4 $\frac{5}{6}$ moneths. Then thirdly say by the first part of the rule of three composed: If 100 l in 12 moneths gaine 8 l, what will 475 l gaine in 4 $\frac{5}{6}$ moneths? Work and you shall find 15 l, & $\frac{4}{9}$ of a pound, which is the neat gaines that the first merchant hath after the rate aforesaid.

A merchant hath bought Holland at 7 s 3 d the elle ready money, and hee selleth the same againe for 8 s 4 pence the elle, to be paid $\frac{1}{4}$ part in ready money, more $\frac{1}{3}$ part at 2 moneths, and the rest at 4 moneths. The question

ſtion is now to know how much the firſt mer-
chant doth gaine vpon the 100 pounds in 12
moneths after the rate.

Anſwere. According to the direction deliue-
red you in the rule of payment, the ready mo-
ney is not to bee multiplied. Then working
for the other two payments, to find out ye true
proportiō at what time they ought to bee paid
at once, you ſhall find for $\frac{1}{3}$ at 2 moneths $\frac{2}{3}$ of
a moneth. And the reſt of ye money which is $\frac{5}{12}$
multiplied by his terme 4 moneths yeeldeth 1
$\frac{2}{3}$ moneths, both which added together make 2
$\frac{1}{3}$ moneths, the iuſt time yt both the paimēts
ought to be performed at once. And therfor ſay
by the firſt part of the rule of 3 compoſed, if 7
$\frac{1}{4}$ in 2 $\frac{1}{3}$ moneths doe gaine $\frac{13}{140}$ of a l: what
ſhall 100 pounds gaine in 12 moneths after ye
rate? worke and you ſhal finde 76 pounds, 16
ſhillings 11 pence $\frac{71}{203}$ pounds. And ſo much
doth he gain vpon 100 pound in 12 moneths.

A Merchant hath bought 30 cloathes at ſix
pounds the peece for ready money. Afterward
he ſelleth 10 of them for 7 pounds the peece, for
three moneths terme. And the other 20 he ſel-
leth for 8 pounds the peece for foure moneths
terme. The queſtion is now, what he gaineth
vpon 100 pounds in 12 moneths.

Anſwere. Firſt finde the value of the 30
cloths, which amount to 180 pounds. Secon-
darily, ſeeke what the 10 peeces come to at 7 l,
and what the 20 peeces come to at 8 pounds

the one comes to 70, and the other to 160: both which together make 230, which is 50 l more than they coſt. Thirdly, as I haue taught you in the rule of payment, proportionate the firſt and ſecond prices vnto the proportion they beare vnto 230, the product of their two prices, you ſhall finde $\frac{7}{23}$ for the firſt, and $\frac{16}{23}$ for the latter. Then fourthly, multiply thoſe parts by their times, and you ſhal haue $\frac{21}{23}$ and $\frac{64}{23}$: both which together maketh 3 whole moneths, and $\frac{16}{23}$ of a moneth, which is the iuſt time that both thoſe payments are to be paid at once.

Then ſay by the firſt part of the rule of 3 compoſed: If 180 l in 3 $\frac{16}{23}$ moneths do gaine 50 l, what ſhall 100 gaine in 12 moneths? Multiply and diuide, and you ſhal finde 90 $\frac{10}{51}$ pounds. And ſo much doth he gaine vpon 100 pounds in 12 moneths.

A Merchant hath bought Cinamon which coſt him 9 s the l ready money. The queſtion is now at what price hee ought to ſell the 100 waight. To wit 112 pounds, to bee paid the $\frac{1}{4}$ at two moneths, and the reſidue at the end of three moneths, ſo that hee may gain after the rate of 10 l vpon 100 l for 12 moneths.

Anſwere. Seeke firſt by the rules of payment what terme both the payments ought to bee paid at once, where the $\frac{1}{4}$ multiplyed by his terme 2 moneths, making $\frac{2}{4}$ moneths.

Like-

Likewise the next payment which is $\frac{1}{4}$ multiplied by his terme 3 moneths, maketh 2 $\frac{1}{4}$ moneths, both which added together, maketh 2 $\frac{3}{4}$ moneths, which is the time, that both the paiments ought to be paid at once. Then say by the Rule of three, if 12 moneths do giue me 10 pound, what wil 2 $\frac{3}{4}$ moneths giue? Multiply and diuide, and you shall find 2 $\frac{7}{24}$ pound. Thē say againe by the rule of three, If one pound cost me 9 s, what will 112 pound cost? Multiply and diuide, and you shal finde 50 l— 8 s. Then say once againe: If 100 pound doe giue 102 $\frac{7}{24}$, what wil 50 $\frac{2}{5}$ l giue? Multiply and diuide, and you shall finde 51 l— 11 s— 1 $\frac{1}{5}$ d: And for that price ought I to sell 112 pound of Cinamon to be paid at 2 seueral payments aforesaid, to gaine thereby after the rate of 10 pound vpon the 100 l in 12 moneths.

Briefe Rules for our hundreth waight here at London, which is after 18 pound for the C.

Item, who that multiplieth the pence that 1 pound waight is worth by 7, and diuideth the product by 15, shal find how many poūds in money the 112 pound waight is worth.

And contrariwise, he that multiplyeth the pounds, ẏ 112 pound waight is worth by 15, and diuideth the product by 7, shall finde how

many pence in money the 112 pound weight is worth.

Example.

At 10 pence the pound weight, what is 112 pound weight worth?

Answere. Multiply ten by 7, and thereof commeth 70, the which diuide by 15, and you shall find $4\frac{2}{3}$ l. And thus the 112 l is worth 4 l — 13 s — 4 d after the rate of 10 pence the pound aforesaid.

At 6 l the 112 l waight, what is one pound worth?

Answere. Multiply 6 l by 15, and thereof commeth 90: the which diuide by 7, and you shall find 12 d $\frac{6}{7}$. So much is one pound worth when the 112 pound did cost 6 pounds.

The eight Chapter intreateth of Tares and allowances of Merchandise sold by weight, and of losses and gaines therein, &c.

At 16 poūd the 100 suttle, what shal 795 pound suttle be worth in giuing 4 pound weight vpon euery 100 for treat?

Answere. Adde 4 vnto 100, and you shal haue 104. Then say by the rule of thrée: If

104 bee worth 16 l, what are 895 l worth? Multiply and diuide, and you shall finde 237 l 13—10 $\frac{2}{13}$ d: and so much shall the 895 pound weight be worth.

Item at 3 s 4 d the pound weight, what shall 745 $\frac{1}{2}$ bee worth, in giuing 4 l weight vpon euery 100 for treat.

Answere. Sée first by the rule of 3, what the 100 pound is worth, saying: If 1 cost 3 $\frac{1}{3}$ s, what 100? Multiply and diuide, and you shall finde 16 l, $\frac{2}{3}$. Then adde 4 vnto 100, and they are 104. Then say againe by the rule of three, if 104 be sold for 16 $\frac{2}{3}$, for how much shall 754 $\frac{1}{2}$ be sold for? Multiply & diuide, and you shall find 120 l 18 s 3 $\frac{4}{3}$ d. And for so much shall the 754 $\frac{1}{2}$ be sold for at 3 s 4 pence the pound in giuing 4 vpon the 100.

Other necessary briefe rules there are for the finding of treats, or casting vp of chests of Suger, &c. which for that it is a mysterie, I omit: If any lacke instruction that way, they shall finde me ready to pleasure them.

Item if 100 l be worth 36 s, 8 d, what shall 800 l be worth in rebating 4 pound vpon euery 100 for tare and closse.

Answere. Multiply 860 by 4, and thereof commeth 3440, the which diuide by 100, and you shall haue 34 $\frac{2}{5}$ l, abate 34 $\frac{2}{5}$ from 860, &

there wil remaine 825 $\frac{1}{3}$. Then say by the rule of three. If 100 l cost 36 $\frac{2}{3}$ s, what wil 825 $\frac{1}{3}$ cost after that rate? Multiply and diuide, and you shal find 15—2—6 $\frac{1}{35}$. And so much shal the 860 cost, in rebating 4 l vpon euery 100, for tare and closse.

Item whether doth hee lose more that giueth 4 l vpon the 100: or hee that rebateth 4 l vpon the 100?

Answere. First note that he that giueth 4 l vpon 100, giueth 104 for 100. And he which rebateth 4 l vpon the 100, giueth the 100 for 96. Therefore say by the rule of three, If 104 be deliuered for 100, for how much shall the 100 be deliuered? Multiply and diuide, and you shal finde 96 $\frac{2}{13}$, and he which rebateth 4 in the 100, maketh but 96 of 100, so that hee looseth 4 in the 100, and the other which giueth 4 vpon the 100 looseth but 3 $\frac{11}{13}$ vpon the 100. Thus may you see that he which rebateth 4 in the 100, looseth more by $\frac{11}{13}$ in the 100 l, than the other which gaue 4 vpon the 100, for tare and closse.

If 100 l of any thing cost me 23 s—4 d: ye question is, how I shal sell the pound to gain after the rate of 10 l vpon the 100 l.

Answere. Say by the rule of three, if 100 giue 110 l, what shall 23 $\frac{1}{3}$ s giue? Multiply and diuide, and you shal find 1 l $\frac{17}{60}$. Then say

again

againe, If 100 pound be worth 1 $\frac{17}{63}$ l, what is one pound worth? Multiply and diuide, and you shall find 3 d $\frac{2}{25}$. And so much is the pound worth in gaining 10 l, vpon the 100.

Item a Grocer hath bought a C. weight of commodity for 6 l, 10 s. The question is now to know how many pounds thereof hee shall sell for 3 s 4 d to gaine 20 s in the C. weight.

Answere. Adde 20 s vnto 6 l—10 s, and they make 7 l—10 s. Then say, if 7 $\frac{1}{2}$ yéelde me 112 pound, what shall 1 $\frac{2}{3}$ l yéeld. Multiply and diuide, and you shal find 24 pound $\frac{8}{90}$. And so many pound onght he to sell to gaine 20 s in his C. weight.

If one pound weight cost 3 s—4 d, and I sell the same againe for 4 s, what is gained in 100 pound of money laid out in that commoditie?

Answere. You may say, If 3 $\frac{1}{3}$ s giue 4 s what will 100 l giue? But then when you haue found, you must subtract 100 l, out of the product, the rest is your neat gaine, or else to produce the neat gaine in your worke at the first. Then subtract the iust price out of the ouerprice, as I taught before in the first beginning of losse and gaine, and your conclusion shall bee all one. Multiply and diuide by which of the two waies you thinke good, and you shall finde that hee gaineth 20 l, in

in the 100 pound.

Item, if the pound weight which cost 4 s, be sold againe for 3 s—4 d, I demaund what is lost in the 100 l of money?

Answere. Say, if 4 s loose $\frac{2}{3}$ s, what shall 100 l loose? Multiply and diuide, and you shal find 16 l—13 s—4 d, and so much is lost vpon the 100 l of money.

Item, if C weight of any commoditie cost 45 l, and the buyer repenting, would loose 5 l in the 100 l of money, I demaund how the pound may be sold: his losse to be neither more nor lesse than after the rate aforesaid of 5 by the hundreth?

Answere. By the rule of 3, if 100 l loose 5 l, what shall 45 l loose? Work, & you shall finde 2 $\frac{1}{4}$ pound, which rebated from the principall 45, resteth 42 l—15 s. Lastly say, if 112 l yeeldeth but 42 l—15 s, what one pound? Multiply and diuide, and you shall finde 7 s 7 d $\frac{17}{28}$. And so much is the pound worth after that losse.

A Grocer hath bought 3 pecks of raisins, weying 175 $\frac{1}{2}$ pound, 182 $\frac{1}{4}$ l: 191 l: tare for each frayle 2 $\frac{1}{4}$ l, at 25 $\frac{1}{2}$ s the C weight. The question is, what they amount to in money.

I answere 61—3 s—4 d $\frac{23}{28}$

A Grocer hath bought 3 sacks of Almonds waighing 267 $\frac{1}{2}$ pound, tare 2 pound at 257 $\frac{1}{2}$ pound, tare 2 $\frac{1}{2}$ pound, 252 pound tare 3 poũd at 2 s—10 $\frac{1}{2}$ the pound, what amount they to in money?

I answere 110 l— 12 s — 3 $\frac{3}{4}$ d.

The ninth Chapter intreateth of lengths and breadths of Arras and other clothes, with other questions incident vnto length and breadth.

If a peece of Arras be 7 elles and $\frac{3}{4}$ long, & 5 elles, and $\frac{2}{3}$ broad, how many elles square doth the same peece containe?

Answere. Multiply the length by ẏ bredth, that is to say, 7 $\frac{3}{4}$ by 5 $\frac{2}{3}$. And thereof will come 43 $\frac{11}{12}$ elles: so many elles square doth the same peece containe.

Item more, a peece of Arras doth containe 22 elles square, & if the same were in length 3 $\frac{1}{4}$ elles, I demaund how many elles in bredth the same peece doth containe?

Answere. Diuide 22 elles by 3 $\frac{1}{4}$, & thereof commeth 6 $\frac{10}{13}$. So many elles doth the same containe in breadth.

Item more, a Merchant hath 3 $\frac{1}{4}$ elles of

Arras, at 1 $\frac{2}{3}$ elles broad, which he will chang with an other man for a peece of of Arras, tha is $\frac{7}{8}$ elles square. The question is how man elles of that squarenesse ought the first Mer chant to haue.

Answere. Multiply the first Merchant peece his length by the bredth, & you sha find it containeth 5 $\frac{5}{12}$ elles, which $\frac{5}{12}$ elle you shal diuide by $\frac{7}{8}$ and you shal find 6 $\frac{4}{1}$ ells, & so many ells of that squarnesse ough the latter merchant to giue the first.

Item a student hath bought 3 $\frac{1}{2}$ yardes o broad cloth, at 7 quarters broad, to make Gowne, and should line the same throughou with Lambe, at a foot square each skin: th question is now how many skinnes hee ough to haue.

Answere. Seeke first the number of yard square that his cloth containeth, which to do multiply 3 $\frac{1}{2}$ his length, by 1 $\frac{3}{4}$ his bredth, an you shall find 6 $\frac{1}{8}$ yardes square: then sa by the rule of three, if one yard square giu 9 foote, what shall 6 $\frac{1}{8}$? worke and you sha find 55 $\frac{1}{8}$ skinnes.

Item more, a Lawyer hath a rich peece o seeling come home, which is 24 foot and thr Inches long, and 7 foot and 2 $\frac{1}{2}$ Inches hig the Joiner is to bee paid by the yard square the question is how many yards this peec

con

ntaineth?

Answere. Multiply his length by his readth, that is, to wit $24\frac{1}{4}$ foote by $7\frac{5}{24}$ foote, nd you shall finde $174\frac{77}{96}$ foote square, which 74 you shal diuide by 9 (for so many foot make yard square) and you shall finde 19 yards he foote, and $\frac{77}{49}$ of a foote, and so many yards oth this peece hold.

Item, bought a péece of Holland cloath conining 36 elles $\frac{1}{3}$ Flemish. The question is, w many elles English it makes?

Answere. You must note, that fiue Elles lemmish do make but 3 elles English. Therefore say by the rule of thrée, if 5 elles lemish make but thrée elles English, how ny elles English wil $36\frac{1}{3}$ elles Flemmish ake? Multiply and diuide, and you shall de $21\frac{4}{5}$, and so many elles English doth $\frac{1}{3}$ elles Flemish containe. The like is to be ne of al others.

Item more, I haue bought 342 elles Flemish, of Arras worke at two elles broad emmish, and I would line the same with broad Canuas of English measure. The estioin is, how many elles English wil ue my turne?

Answere. For as much as thrée elles Eng are worth fiue elles Flemmish, therefore 3 elles English into his square, in multi-

ply-

plying 3 by himſelfe, which maketh 9. Like
wiſe multiply the Engliſh elle, which is fiu
quarters, euery way into himſelfe ſquarely
& you ſhall find 25. Then multiply 342 whic
is the length of the pœce, by two which is th
breadth, and thereof commeth 684, then ſa
by the rule of thrœ, as before: if 25 elles ſqua
of flemiſh meaſure be worth 9 elles ſquare o
Engliſh meaſure, what are 684 of flemiſ
meaſure? multiply and diuide, and you ſhal
find 246 $\frac{6}{25}$ elles Engliſh.

The ſame is alſo wrought by the backe
rule of thrœ, in ſœking the ſquares containe
in the Flemiſh elle of two elles broad (whic
are 18:) and alſo in ſœking the ſquares con
tained in the Engliſh elle (which are 25) the
ſay by the rule of 3 backward, if 18 quarter
require 342, elles, what ſhal 25 quarters giue
Multiply and diuide by the rule of three Re
uerſe, and you ſhall finde as before 246 $\frac{6}{25}$ elle
Engliſh.

Item more, at two ſhillings foure pence th
Flemiſh elle, what is the Engliſh elle wor
after the rate?

Anſwere. Say if three quarters giue 3 $\frac{1}{3}$
what giueth fiue quarters? Multiply and d
uide, and you ſhall finde 5 ſhillings 6 $\frac{4}{9}$ d.

Item more, at 8 s 4 d the Flemiſh El
ſquare, what is the Engliſh Elle worth a
t

ter that rate?

Answere. According to the reason of the last question, consider that a flemmish elle square is equall to 9 quarters of a yard English, and an English elle square is equall to 25 quarters of a yard. Therefore say by the rule of 3, if 9 quarters giue $8\frac{1}{3}$ s, what 25 quarters? Worke and find 23 s $1\frac{7}{9}$ pence. And so much is the English elle worth.

Item more, at 6 s 8 d the elle square: what shall a peece of cloath cost that is $7\frac{1}{2}$ elles long, and three $\frac{1}{4}$ elles broad?

Answere. Multiply the breadth by the length, and you shall finde $24\frac{3}{8}$ elles square. Then say by the rule of three, if 1 elle square cost $6\frac{2}{3}$ s, what $24\frac{3}{8}$? Multiply and diuide, and you shall finde 8 pounds, two s 6 pence, and so much the same peece of cloath cost.

Item more, a Mercer sold 3 peeces of silke, to wit, $24\frac{1}{4}$ $23\frac{1}{3}$, and 25 yards at $9\frac{3}{4}$ s the yard, and was glad to receiue in part of payment againe, a cloth containing $34\frac{1}{2}$ yards at $\frac{1}{3}$ shillings the yard. The question is now, what the Debitor is in the Creditors debt. Worke and you shall find he oweth the Mercer two and twenty pounds—3 s—$2\frac{1}{4}$ d.

The

The tenth Chapter intreateth of the reducing of Pawnes of Geanes into English yards.

NOte, that 100 Pawnes doe make 26 yeards, whereupon 3 Pawnes $\frac{11}{13}$ doe make one yard, and one pawne after that rate and proportion is $\frac{13}{50}$ of a yard.

In 4563 pawnes of Geanes, how many yards English?

Answere. Say by the rule of three, if 100 Pawnes doe make 26 yards, what wil 4563 Pawnes make? Multiply and diuide, and you shal finde 1186 yards $\frac{19}{50}$. So many yards do 4563 Pawnes make.

Otherwise, take some other number at your pleasure, as 10 Pawnes, which is the $\frac{1}{10}$ part of 100, then to find his proportion, take the $\frac{1}{10}$ part of 26, which is 2 $\frac{3}{5}$, and then say also by the rule of three, if 10 pawnes giue 2 $\frac{3}{5}$ yards, what wil 4563 pawnes giue? Worke, and you shal finde 1186 $\frac{19}{50}$ yeards, as before.

More, at 2 s 6 pence the Pawne of Geanes, what wil the English yard be worth after that rate?

Answere. Say by the rule of three, if $\frac{13}{50}$ of a yard cost 2 $\frac{1}{2}$ s, what one yeard? Multiply an-

and diuide, and you ſhall finde 9 s $\frac{11}{13}$ d.

More, if 346 $\frac{1}{2}$ Pawnes coſt 30 l — 13 s — 4 d ſterling, what is that the Engliſh yeard after the rate?

Anſwere. Say by the rule of thrée, If 346 $\frac{1}{2}$ Pawnes coſt 30 $\frac{2}{3}$ pounds, what are thrée $\frac{11}{13}$ Pawnes worth (for ſo many Pawnes make a yeard:) multiply and diuide, and you ſhall finde $\frac{9200}{27027}$ parts of a pound, which in knowne numbers is worth 6 s 9 pence $\frac{5272}{9009}$.

The eleuenth Chapter intreateth of Rules of Loane and Intereſt, with certaine neceſſary Queſtions and Proofes incident thereunto, &c.

Item lent my friend 326 l for 5 $\frac{1}{2}$ moneths ſimply without any Intereſt, vpon condition, to haue the like courteſie againe when I néed. But when I came to borrow, he could ſpare me but 149 l — 8 s — 4 d. The queſtion is now, how long time I ought to haue the vſe thereof, to counteruaile my friendſhip before time ſhewed him.

Anſwere. Say by the backer rule of thrée, if 326 giue 5 $\frac{1}{2}$ moneths, what time will 149 pounds $\frac{5}{12}$ giue? Multiply and diuide, and you ſhall find 12 moneths, & ſo long time ought I

to vse his money.

The proofe.

Item, lent my friend 149 pounds—8 s—4 d, for 12 moneths. The question is now, how much money hee ought to lend me againe for $5\frac{1}{2}$ moneths to recompence my friendshippe shewed him.

Answere. Say by the backer or reuerse rule of 3: if 12 moneths giue $149\frac{5}{12}$, what shall $5\frac{1}{2}$ moneths giue? Work, and you shall finde 326 pounds; and so much ought he to lend me to requite my gentlenesse or good turne.

Two other Branches yet more for proofe out of the same question.

Item, lent my friend 149 l—8 s—4 pence for 12 moneths, to haue the like friendshippe againe when I need. And comming to borrow of him, he very courteously tooke me 326 l (for that he could well then spare the same.) The question is now, how long I ought to occupy it, not vsurping friendship, but in his due time to restore it againe.

Answere. Say by the rule of three Reuerse, if $149\frac{5}{12}$ giue 12 moneths, what shall 326 pounds giue? Multiply and diuide, and you shall find, that at $5\frac{1}{2}$ moneths terme, I ought to restore it againe.

Proofe.

Proofe.

Item, lent my friend 326 l for $5\frac{1}{2}$ moneths: The queſtion is now, how many pounds hee ought to lend me for 12 moneths to recōpence the pleaſure againe?

Scholar. Worke by the rule of three Reverſe, as you haue done before, and you ſhall find 149 l—8 s—4 d.

Again, foure other ſelected queſtions of Loane and intereſt, all out of one branch, and each one alſo a neceſſary Queſtion, and a particular proofe to other.

ITem, lent my friend 430 l at intereſt for three moneths to receiue after the rate of 8 l in the 100 for 12 moneths. The queſtion is what the intereſt commeth to?

You may if you pleaſe worke it at two workings by the rule of three direct, in ſaying: If 12 moneths giue 8 l, what giueth three moneths? Multiply and diuide, and it giueth 2 pound.

Then for the ſecond worke ſay, If 100 l yeeld two l, what yeeldeth 430 l? Multiply and diuide, and you ſhall finde 8 l 12 s, and ſo much comes the loane of 430 l to for 3 mo-

neths, after the rate of 8 pounds in the 100 pounds of 12 moneths.

Otherwise wrought thus by the rule of 3 at twice also.

If 100 l giue 8 l, what giueth 430 l? multiply & diuide, and you shal find 34 pounds $\frac{2}{5}$. Then againe for the second worke say: If 12 moneths giue 34 pounds, $\frac{2}{5}$, what giueth thrée moneths? Worke & find 8 l 12 s, as before.

Otherwise yet at one working: By the first part of the rule of 5 numbers forward, in saying, if 100 pounds in 12 moneths gaine 8 l, what shall 43 pounds gaine in 3 moneths? Multiply the first by the second for your diuisor, and the other thrée the one into the other for the diuidend, and you shall find 8 pounds 12 shillings, as aforesaid.

Proofe.

Item a friend of mine receiued of me eight pounds 12 shillings for the Interest and vse of 430 pounds for thrée moneths terme: the question is now, what hee tooke in the 100 pounds for 12 moneths after that rate.

Answere. For most briefe, say by the first part or rule of 5 numbers forward: If 430 pounds in thrée moneths did pay 8 l 12 s, what doth 100 pounds in 12 moneths take after ẏ rate? Worke, & you shall finde 8 pounds, & so much he tooke vpon the 100 pounds for 12 moneths.

A third question and proofe also wroght by the Backer Rule of 5 Numbers.

Item, lent my friend 430 pound to receiue for the Interest thereof, after the rate of 8 l in the 100 for 12 moneths. The question is now, how long time my friend ought to giue the vse thereof, that it may be returned with 8 l — 12 s gaines.

You may worke it if you please by the rule of three direct at twice, in saying: If a 100 l yeeld 8 l, what yeeldeth 430 l? Multiply and diuide, and find 34 l and $\frac{2}{5}$.

Then againe for the second worke say, if 34 $\frac{2}{5}$ giue 12 moneths, what giueth 8 $\frac{3}{5}$ pound? Multiply and diuide, and you shall find three moneths, and so long time ought my friend to vse it to returne with 8 l — 12 s gaine.

Otherwise at one working by the backer rule of 5 numbers, in saying; If 100 l in 12 moneths doe gaine 8 l, how long time shall 430 l be a gaining of 8 l — 12 s? Multiply the first and the second into the last for your diuidend, and the third and fourth multiply together for your diuisor, and then diuide, and you shall finde 3 moneths, the iust time that my friend ought to vse it, to returne it with 8 l, 12 s gaine.

A Fourth deriued question out of this Branch, which is a proofe of this last, and also of the other two going before.

Item, how much money ought a merchant to deliuer after 8 l in the 100 for 12 moneth, that in three moneths he may gaine 8 l 12 s?

Answere. You may also if you please worke it by the Golden Rule of three at twice, first saying, if 3 moneths gaine 8 l, what 12 moneths gaine? you shall find 34 l. Then say againe, if 8 l be come of 100 l, what shall come of 34 l 8 s? Worke, and you shall finde the answere to the question, which is 430 l, and so much ought the merchant to deliuer.

But most briefly it is answered by the Backer Rule of 5 numbers; where I argue thus, saying: If 100 l bee 12 moneths a gaining of 8 l, then but for three moneths terme onely to take 8 l 12 s must needs bee a good round summe: to worke it, set your numbers thus, 100—12—8—3—8 $\frac{3}{5}$: multiplying the first into the second, and also by 43 the product of the fift, for your diuidend, & the third and fourth together with 5 the Denominator of your fraction for your Diuisor: then diuide and you shall finde as before, 430 l: the true solutions to your question.

Th

The twelfth Chapter intreateth of the making of Factors, which is taken in two sorts.

The first is, when the estimation of the factor, is taken upon the sending of ẏ merchant, as if the estimation of his person bee $\frac{1}{4}$, it is understood that he shall haue $\frac{1}{4}$ of ẏ gain, and the merchant the other $\frac{3}{4}$.

The other sort is when the estimation of his making is out of the sending of the Merchant, as if the order and agreement betweene them were such, that the Merchant shal put in 800 l, and the factor for his making shall haue $\frac{1}{4}$, neverthelesse he shall haue but $\frac{1}{5}$ of the gaine or profit, for the $\frac{1}{4}$ of 800 is 200 (for the estimation of his making) which with the 800 pound make 1000 l, wherof the 200 l, is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factor, under such condition, that the said Factor shall haue $\frac{1}{4}$. And after certaine time, they find in profit 124 l 6 s 8 d, I demaund how much the merchant shal haue hereof, and how much ought the factor to haue?

Answere. When the estimation of the Factor is out of the sending of the merchant it maketh,

l	s	d	
99	9	4	for the Merchant.
24	17	4	for the factor.

But if that his estimation bee at the sending of the Merchant, then it maketh but,

l	s	d	
93	5	0	for the Merchant.
31	1	8	for the Factor.

For the Merchant is then to haue $\frac{3}{4}$, and the Factor $\frac{1}{4}$.

A Merchant doth put into the hands of his Factor 800 l, and the Factor 400 l to haue the $\frac{1}{5}$ part of the profit: I demaund now for how much his person is esteemed, when the same is counted vpon the sending of the Merchant?

Answere. According to the tenor and order before prescribed in the first rule, that is, if his estimate be $\frac{1}{4}$, he shall haue the $\frac{1}{4}$ of the gaine. Therefore say by the rule of three direct, If $\frac{1}{4}$ taken put in 400 l, what is the estimate, or putting in of $\frac{1}{5}$ taking? Multiply and diuide, and you shall finde 320 l, and so much is the person of the Factor estimated.

Otherwise.

To finde the estimation of the person of the

the factor, you shall consider, that seeing it was agreed betweene them, that the factor should take the $\frac{1}{5}$, then the Merchant shall haue the residue, which are $\frac{4}{5}$: wherefore the gaine of the merchant vnto that of the factor is in such proportion as 5 vnto 4. Then if you wil know the estimation of the person of the factor; say, if 5 giue 4, what will 400 giue? Multiply and diuide, and you shall finde 320 l. And so much is the person of the Factor esteemed to bee worth.

Other conditions then these aforesaid, may also be betweene Merchants & Factors without respect, either of sending or not sending of the Merchant, where most commonly the estimation of the body of the Factor is in such proportion of the stocke which the Merchant layeth in, as the gaine of the said factor is vnto the gaine of the merchant. As thus, If a merchant doe deliuer into the hands of his factor 400 pound, and he to haue halfe the profite. The person of the said factor shall be esteemed to bee worth 400 pound, and if the factor do take but $\frac{1}{3}$ of the gaine, he should haue but $\frac{1}{2}$ so much of the gain as the merchant taketh, which must haue $\frac{2}{3}$, wherefore the person of the factor is esteemed but the $\frac{1}{2}$ of that which the merchant layeth in, that is to say, 200 pound.

And if the factor did take the $\frac{2}{3}$ of the gain, then

then the merchant shal take the residue which are $\frac{3}{5}$, wherefore the gaine of the Merchant unto the factor is then in such proportion as 3 unto 2: whereupon if you will then know the estimation of the person of the factor, say, if 3 giue 2, what shall 400 giue? Worke, & you shall finde 266 $\frac{2}{3}$ pounds. And so much is the person of the factor esteemed to be worth.

And if the merchant should deliuer unto his factor 400 pound, and the factor would lay in 80, and his person, to the end he might haue y $\frac{1}{2}$ of the gaine, I demaund how much shall his person be esteemed?

Answere. Abate 80 from 400, and there will remaine 320. And at so much shall his person be esteemed.

A merchant hath deliuered unto his factor 900 l to gouerne in the trade of Merchandise, upon condition that he shall haue the $\frac{1}{3}$ of the gaine, if any thing be gained, and also to beare the $\frac{1}{2}$ of the losse, if any thing bee lost. Now I demaund how much his person was esteemed at?

Answere. Seeing that the factor taketh the $\frac{1}{3}$ of the gaine, his person ought to be esteemed as much as $\frac{1}{2}$ of the stock, which the merchant layeth in: That is to say the $\frac{1}{2}$ of 900 pound, which is 450. The reason is, because $\frac{1}{3}$ of the gaine that the factor taketh, is the $\frac{1}{2}$ of the $\frac{2}{3}$ of the gaine that the merchant taketh, and so the

the Factor his person is esteemed to bee worth 450 pounds.

A Merchant hath delivered unto his factor 600 pound, and the factor layeth in 250 pound and his person. Now because he layeth in 150 pounds and his person, it is agreed between them, that he shal take the $\frac{2}{5}$ of the gaine. I demaund for how much his person was esteemed?

Answere. For as much as the Factor taketh $\frac{2}{5}$ of the gaine hee taketh $\frac{2}{3}$ of that which the Merchant taketh, for $\frac{2}{5}$ are the $\frac{2}{3}$ of $\frac{3}{5}$. And therefore the factors laying in, ought to bee 400 pound, which is $\frac{2}{3}$ of 600 pound that the merchant layed in. Then subtract 250, which the Factor did lay in, from 400 pound, which should haue beene his whole stocke, and there remaineth 150 pound for the estimation of his person.

More, a Merchant hath delivered unto his factor 1800 pound, upon condition that ye factor shall haue the gaine of 160 l, as though he laid in so much ready money: I demaund what portion of the gaine the said factor shall take?

Answere. See what part the 160 l (which the Factor laid in) is of 960, which is the whole stocke of their company, and you shall find $\frac{1}{6}$. And such part of the gaine shall the factor take.

But in case, that in making their couenants

nants, it were so agreed betwéene them that the Factor should haue the gaine of 160 l of the whole stocke which the Merchant layeth in, that is to say of the 800 pound, then should the Factor take $\frac{1}{5}$ of the gaine: for 160 is $\frac{1}{5}$ of 800 pound.

The thirteenth Chapter intreateth of Rules of Barter, and exchanging Merchandise, which is distinct into 7 Rules with diuers other necessary Questions incident thereunto.

The first Rule.

Two Merchants willing to change their merchandise the one with the other: The one hath 24 broad cloathes at 10 l— 10 s the peece. The other hath Mace, at 12 s the l. The question is, how many pounds of Mace he ought to giue for his clothes, to saue himselfe harmelesse and be no loser.

Answere. Seeke first by the Rule of three, what the 24 clothes cost at 10 l— 10 s the peece, and you shall find 252 l. Then to finde the quantitie of Mace, say againe by the Rule of 3, if 12 s buy one pound, what shall 252 pound buy me? Work, and you shall find 420 pound of Mace: and so many pound ought he to

to giue for his cloathes.

The proofe.

Two barter, the one hath 420 pounds of Mace at 12 s the pound, to barter or change broad clothes at 10 pounds — 10 s the peece. The question is, how many broad cloathes he ought to giue for all his Mace.

Answere. First say, if one cost 12 s, what 420? You shall find 5040 s. Then say again, if 10 $\frac{1}{2}$ pounds giue 1 cloath, what shall 5040 shillings giue? Worke, and you shall find 24 cloathes, your desire.

The second Rule.

Two change merchandise for merchandise: The one hath Pepper at 2 shillings 4 pence the pound, to sell for ready money. But in barter he will haue no lesse than three shillings the pound. And the other hath Holland at 5 s 6 pence the elle readie money. The question is now at what price hee ought to deliuer the elle in the barter to saue himselfe harmelesse.

Answere. Say by the Rule of three direct: if 2 $\frac{1}{3}$ s readie money giue 3 s in barter, what shall 5 $\frac{1}{2}$ giue in barter? you shall finde 7 $\frac{1}{14}$ s, and at that price ought the second Merchant to sell his Holland in barter.

The

The Proofe.

Two barter. The one hath Holland at 5 s 6 pence the elle to sell for ready money. And in barter hee wil haue $7 \frac{1}{74}$ s. The other hath Pepper at 2 s 4 d the pound to sell for readie money. The question is now how hee ought to sell in barter?

Answere. Say by the rule of three direct, if $5 \frac{1}{2}$ ready money giue $7 \frac{1}{74}$ s in barter, what ought $2 \frac{1}{3}$ s to take in barter? Multiply and diuide, and you shall find 3 s your desire.

The third Rule.

Two barter. The one hath cloth of Arras at 30 s the elle ready money, but in barter he will haue $35 \frac{1}{2}$ s. And the other hath white wines, which hee deliuered in barter for 16 l the tunne. The question is now what his wines cost the tunne in ready money.

Answere. Say by the rule of three direct, If $35 \frac{1}{2}$ s in barter giue but 30 readie monie, what did 16 l in barter cost? Worke and you shall find 13 l 10 s $\frac{30}{71}$. And so much cost his wines for a tunne ready money.

The proofe.

Two barter merchandise for merchandise: The one hath wines white at 13 l 10 s $\frac{30}{71}$ the

the Tunne to sell for ready money. But in barter he delivered it for 16 pounds. The other to make his match good and saue himselfe harmelesse, delivereth Arras at $35\frac{1}{2}$ s the elle. The question is now, what an elle of his Arras cost in ready money.

Answere. Say by the rule of three direct: If 16l in barter giue but 13—10—$\frac{30}{71}$ s in ready money, what shall $35\frac{1}{2}$ shillings yeeld in barter? Worke & you shall find 30 s your desire.

The fourth Rule.

Two barter. The one hath Kerseys at 14 pounds the peece ready money. But in barter he will haue 18 pounds. And yet he will haue the $\frac{1}{3}$ part of his overprice in ready money. And the other hath ginger at eight groats the pound to sell for ready money. The question is how he ought to deliuer the Ginger by the pound in barter to saue himselfe harmelesse, and make the barter equall.

Answere. Item for the working of this question, and such other the like, you must understand, if the party overselling his wares require to haue also some portion in ready money, as $\frac{1}{2}:\frac{2}{3}:\frac{3}{4}$ &c. then shall you first rebate the same demaunded part whatsoeuer it bee from the overprice, and also from the iust price. And those two numbers that shall remaine after the subtraction is made, shall be the

the two first numbers in the rule of three. And the iust price of the same merchandise shall be the third number, which by the operation of the rule of three direct shall yeeld you a true solution how and at what price you shal ouersell that your merchandise, to saue your selfe harmelesse, and make the barter equall.

Example.

Take the $\frac{1}{3}$ (of eighteene) which is the ouer price of his cloath, which $\frac{1}{3}$ of eighteene is six, which as appeareth heere in the margent, you must subtract from 18, there rest 12. And also abate it from 14, which is the iust price of the cloath, and there remaineth 8, which 8 and 12 are the two first numbers in the rule of three.

14	..	18
6		6
8	..	12

Then take eight groats or two $\frac{2}{3}$ shillings for the third number. Then say by the rule of three direct: If 8 pounds giue 12 pounds, what shall 2 $\frac{2}{3}$ s giue? Multiply and diuide, and you shall finde 4 s. And for so much shall the second Merchant sell his Ginger, or his commoditie in barter, to ballance the same equall.

The Proofe.

Two barter: The one hath fine Keyrseys at 14 pounds the peece ready money. But in

bar-

ter he will haue 18 pounds, and yet hee will haue the $\frac{1}{3}$ part of his ouerprice in ready monie. And the other hath Ginger, which he hauing cunning inough to make the barter equall, deliuered in barter for 4 ſhillings the pound. The queſtion is now, what his ginger coſt him in ready monie?

Anſwere. After you haue made the ſubtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 and 14, (as before was taught you:) then wil there remaine 8 and 12 for your two firſt nũbers in the rule of thrée. Then ſay, If 12 giue 8, what ſhall come of 4 the ouerprice of the l of Ginger? Multiply and diuide, and you ſhall finde two ſhillings 8 pence your deſire.

Two Merchants barter merchandiſe for merchandiſe. The one hath Denſhire whites at 7 l — 13 s — 4 d the peece ready money: but in barter he doth them away for 8 l — 3 s 4 d. And yet he will haue ẏ $\frac{1}{3}$ part of his ouerprice in ready money. And the other hath Cottens at 3 l the péece ready money. The queſtion is no w, at what price he ought to ſell or exchange his Cottens in barter to ſaue him ſelfe harmeleſſe, and make the barter equall?

7	13	4	8	3	4
2	14	$5\frac{1}{3}$	2	14	$5\frac{1}{3}$
4	18	$10\frac{2}{3}$	5	8	$10\frac{2}{3}$

Answere. First seeke the $\frac{1}{3}$ part of 8 l—3 s—4 d, which is, 2 l—14—5 $\frac{1}{3}$ d, which rebate from 8—3—4 d, there resteth; as appeareth by the example aboue said, 5—8—10$\frac{2}{3}$ pence, which $\frac{1}{3}$ of 8—3 d also rebated from 7——13—4 d, there resteth 4—18—10,$\frac{2}{3}$, the two first numbers in the rule of three, and the 3 l, which is the next price of the peece of Cotten, is the third number. Then say by the rule of three direct as was taught before: If 4—18 10 $\frac{2}{3}$ d did giue 5 l—8 s—$\frac{2}{3}$ d, what shall 3 l giue? Multiply and diuide, and you shal finde 3—6—0 $\frac{72}{89}$ pence, the iust price that he ought to deliuer his Cottens in barter.

The fifth Rule.

Two merchants will change merchandise for merchandise. The one hath Kerseys at 40 s the peece to sell for ready money. And in barter he wil sell them for 56 s—8 d, and he wil gain after 10 l vpon the 100 l. And yet hee will haue the $\frac{1}{2}$ of his ouerprice in ready mony. The other hath Flaxe at 3 d the pound ready money. The question is now, how he shall sell the pound of his Flaxe in barter?

Answere. Seé first at 10 l vpon the 100 l, what the 56$\frac{2}{3}$ s commeth to, in saying (by the rule of three direct) if a 100 l, giue 110 l, what 56$\frac{2}{3}$ s? Multiply and diuide, and you shall find 3 l—2 s—4 d, of which the $\frac{1}{2}$ that hee demandeth

deth in ready money, is 1 l——11 s——2 d the sum 31 s——2 d abated from 40 s, and also from 56 s—8 d: there wil remaine 8 shillings—10 pence, and 25 s—6 d, for the two first numbers in the rule of thrée, and 3 pence the price of the pound of flaxe for the third nūber. Then multiply and diuide, and you shall find 8 $\frac{12}{13}$ pence. And for so much shal he sell the pound of flaxe in barter.

The sixt Rule.

Two are willing to exchange merchandise. The one hath Norwich Grograins at 25 s the peece ready money: and in barter he wil haue 30 s, & he wil haue the $\frac{1}{4}$ part of his ouer price in ready mony. The other hath Norwich stockings at 40 s the dozen to sell for ready mony. But in as much as the first Merchants Grograines are no better, he would deliuer them so to ballance the barter, that he may gaine after 10 pounds in the 100 pounds. The question is now, how he shall sell his hose the dozen in barter according to his request?

Answere. Say, if 100 giue 110, what shall 40 s giue, which is the iust price of the dozen of stockings? Multiply and diuide, and you shall finde 44 s. Then take the $\frac{1}{4}$ of 30 s, which is 7 s—6 d. And subtract it from 25 s, and also from 30 s, and there will remaine

17 s——6 d——& 22 s——6 d, for the two firſt numbers in the rule of three, and 44 ſhillings, which is the iuſt price (with his gaine in the dozen of ſtockings) for the third number. Then multiply and diuide, and you ſhall finde 56 s 6 $\frac{6}{7}$: and for ſo much he is to ſell his dozen of ſtockings in barter.

The ſeuenth Rule.

Two Merchants will change their Merchandiſe one with the other. The one hath 720 Elles of Cambricke at 5 s the Elle to ſell for ready money, but in barter he requireth 6 s —8 d. And yet notwithſtanding hee loſeth by it after 10 pounds vpon the 100 l, whereupon he requireth one halfe of his ouerprice in ready money, and the other Merchant hauing ſkill inough to make the barter equall, deliuereth Engliſh Saffrons at 30 s the pound. The queſtion is now, what his Saffrons coſt the pound in ready monie?

Anſwere. You muſt firſt ſeeke what is loſt vpon the 100 l, which to doe you may ſay (if you pleaſe.) If 100 l looſe 10, what ſhall 6 $\frac{2}{3}$ looſe? Work, & you ſhall find $\frac{2}{3}$ s (or 8 d) which muſt be rebated from 6 s—8 d. So reſteth 6 s ſtil. Or you may ſay, if 100 l giue me but 90 pounds, what ſhall 6 s — 8 d giue? Worke this way either, and you ſhall finde alſo as before, directly in your quotient 6 s your deſire

fire. Then are you next to cast vp what the 720 Elles of Cambricke commeth to at 6 s 8 d the Elle, and you shall finde 240 pounds; ẏ ½ whereof the Cambricke Merchant will haue in ready money (which is 120 l:) nextly you must cast what the Cambricke commeth to after his losse in the 100 l, which as you found is but 6 s an Elle, and you shall finde 216 poūds. Now must you subtract his ready money (which is a 120 l) out of 240 l, and also out of 216 l, and there will remaine 120 pounds, & 96 pounds for your two first numbers in the rule of three, and 30 shillings is the ouerprice of your Saffron for the third number. Then multiply and diuide, and you shall finde 24 s And so much did his Saffron cost in readie monie.

Two Merchants barter. The one hath 50 Clothes to put away for ready monie at a 11 pounds the cloth, and in barter putteth them away for 12 pounds, taking Holland cloth at 20 d the Flemmish Elle, which was worth no more but 18 d. The question is now, what Holland payeth for the Cloth? and what hee winneth or looseth by the bargaine?

Answer. 50 Clothes at 11 l the Cloth commeth to 550 l, and put away at 12 l the peece maketh 600 l. Then to finde what Holland payeth for the Cloth, say by ẏ rule of three direct, If 20 d buy 1 Elle, what 600 l? Worke

and you shall finde 7200 Elles. Now to find the estate of his gaine or losse, you must seeke what his 7200 Elles commeth to at 18 d the Elle. Worke by the rule of Proportion direct, and you shall finde 540 pounds, which is not so much as his Clothes were worth in ready money by 10 pounds: and so much lost the first Merchant by his exchange.

A Venetian hath in London, 100 peeces of silke, to put away for ready mony at 3 poũds the peece. But in barter he deliuered them for 4 pounds the peece, taking wools of a Felmonger 7 pounds 10 s the C. weight, which was worth no more but 6 l the C. ready money. The question is now, what wools paieth for the silkes, and which of them winneth or loseth by the barter?

Answere. 100 peeces of silke at 3 pound, is 300 l, and 4 l is 400 l. Then to finde what wooles payeth for the silke, say by the rule of three direct. If $7\frac{1}{2}$ l buy me 1 C. waight, what 400 pounds? Worke, and find $53\frac{1}{3}$ C. weight of wool. Now, to finde the estate of their gaine or losse, cast vp his wool at 6 pounds the C (for so much they were worth ready money) and you shal find 320 pound, which is 20 pounds more than the silkes were to bee sold for ready money, whereupon the Venetian gained 20 pounds by the barter.

A

A Merchant hath $53\frac{1}{3}$ C. waight of wool at 6 pounds the C. to sel for ready mony, but in barter he wil haue 7 pounds — 10 s: and an other doth barter with him for Silkes, which are worth 3 pounds a peece ready money. The question is now, how he ought to deliuer his silkes the peece in barter, and how many payeth for the wooles?

Answere. Say, by the Rule of Proportion, (or by the rule of three direct) If 6 pound for C. weight ready money yeeld me 7 l — 10 s, what wil 3 pound yeeld, which is the iust price of a peece of silke in barter, to make the Trucke equal? Worke, and find 3 pounds — 15 s, the price of a peece of silk in barter: then say, if three pound 15 s require 1 peece of silke, how many peeces of silke are bought with 400 pound, which is the value of the $53\frac{1}{3}$ C. weight of wool at 7 pound — 10 s? Worke by the rule of three direct, & you shal finde 106 peeces of silk, and $\frac{2}{3}$ of a peece, and so many peeces of silk paieth for the wool, and neither partie hath aduantage of other.

Two men wil change merchandise the one with the other. The one of them hath Beere at 6 s 8 d the barrel to sell for ready money. But in Barter hee wil sel the barrel for 8 s, and yet he wil gaine moreouer after 10 l vpon the 100 l. And the other hath white Spanish wool at 20 s the Roue, to sell for readie monie. The question is now, how hee shall

deliuer the Roue of wool in barter to saue him selfe harmelesse.

Answere. Say, if $6\frac{2}{3}$ s, which is the iust price of the barrell of Beere, be sold in barter for 8 shillings: for how much shal 20 shillings (which is the iust price of the Roue of wooll) be sold in barter? Worke by the rule of three direct, and you shall find 24 s. Then for because the first Merchant will gaine after 10 l. vpon the 100 l. hee maketh his 100 l——— 110 l. And therefore say by the rule of three, if the second Merchant of 110 pound doe make but 100 pound, how much shal hee make of 24 s? Multiply and diuide, and you shal finde 21 s——9 d $\frac{9}{11}$ of a penie. And for so much shall he sel the Roue of wool to be deliuered in barter, to the ende the first Merchant may gaine 10 in the 100.

Two Merchants wil change their commodities the one with the other. The one of them hath white paper at 4 s the reame, to sel for ready money. And in barter he wil do it alway for 5 s, and yet hee will gaine moreouer after the rate of 10 l vpon the 100 l. And the other hath Mace at 14 s—6 d ye pound waight to sel in barter. Now I demaund what the pound did cost in ready money?

Answere. Say, if 5 s, (which is the ouer price of the paper in barter) become of 4 s the iust price, of how much shal come 14—s $\frac{1}{2}$, which is the surprice of the pound of Mace in barter?

barter? Multiply & diuide, and you shal finde 11 s $\frac{2}{5}$. Then for because the first Merchant of Paper wil gaine after 10 vpon the 100. Say, if 100 doe giue a 110, what shal 11 $\frac{2}{5}$ shillings giue? Worke, and you shal finde 12 s—9 d $\frac{12}{25}$: and so much did the pound of Mace cost in ready money.

The Fourteenth Chapter intreateth of *Exchanging of money from* one place to another.

Exchange is no other thing than to take or receiue money in one City to render or pay the value thereof in another Citie: or else to giue money in one place, and receiue the value thereof in another, at tearme of certaine dayes, moneths, or faires, according to the diuersitie of the place.

But this practise chiefly consisteth in the knowledge of the money or Coines in diuers places, of which, for thy benefite, (after a few examples giuen to the introduction of this worke) I wil set downe certaine notes of the diuersitie of the common and vsual coynes in most places of Christendome for trafique.

And first I wil begin at Antwerpe, where they vse to make their accounts by Deniers de gros,

gros, that is to say by pence Flemmish, wher-of 12 doe make 1 s Flemmish, and 20 s doe make one pound de gros.

Item, a Merchant delivered at Antwerpe 400 pounds Flemmish, to receive in London 20 s sterling, for every 23 s—4 d Flemmish. The question is now, how much sterling mony is to be received at London for the said 400 pounds of Flemmish?

Answere. Say by the rule of three, if 23 $\frac{1}{3}$ Flemmish give 20 s sterling, what 400 pouds Flemmish? Worke, and you shall finde 342 pounds 17 s 1 $\frac{5}{7}$ pence, and so much sterling shal I receive in London for the said 400 pounds Flemmish.

Otherwise also wrought by Rules of Practise in taking the $\frac{1}{7}$ of the Flemmish money delivered, and abating the same from the principall, the rest is English money sterling, as before.

400 .. 0 .. 0

---

57 .. 2 .. 10 $\frac{2}{7}$

---

342 .. 17 .. 1 . $\frac{5}{7}$ sterling.

A Merchant at London delivered 200 pouđ sterling for Antwerpe at 23 s—5 d Flemmish the pound sterling. The question is, how much he must receive at Antwerpe?

Answere.

Answere. Say by the Rule of three, if 1 l sterling, giue 23 s—5 d Flemmish, what 200 pounds sterling? Worke, and you shall finde 234 l—3 s—4 d. So many pounds Flemmish shal hee receiue at Antwerpe for the said 200 pounds sterling.

Otherwise by practise.

1 ··	23 ·· 5 ·· 200		
	3 s 4 d	33 ·· 6 ·· 8	
	1 d	·· 16 ·· 8	

maketh 234 l ·· 3 ·· 4 sterling.

In London 200 pound sterling is deliuered by Exchange, for Antwerpe at 23 s—9 pence Flemmish the pound sterling. The questiō is, at what rate the Flemmish money ought to bee returned to gaine 4 pound vpon the 100 l sterling at London?

Answere. First say by the Rule of three direct: If 1 pound sterling giue $23\frac{3}{4}$ Flemmish, what 200 pounds sterling? Multiply & diuide, and you shal finde 237 pounds—10 s. The which to returne to gaine 8 pounds sterling in London, say by the backer rule, if 200 pounds sterling require the exchange 23 s—9 d Flemmish, what the exchange to make 208 l sterl? Worke by the rule, and finde 22 s— 10 d $\frac{1}{26}$ d Flemmish, the effect in the question required.

If

If I take up money at Antwerpe after 19 s—4 d Flemmish, to pay for the same at London 20 shillings sterling, and when the day of payment is come, I am forced to returne the same money againe in London, to pay my bill of exchange: so that for 20 shillings which I take up here at London, I must pay 19 s—6 d at Antwerpe. I demaund whether I do win or lose? and how much in or upon the C pounds of money?

Answere. Say by the Rule of three: If 16. $\frac{1}{3}$ giue 19 $\frac{1}{3}$, what will 100 l' giue? Multiply and diuide, and you shall find 99 l—2 s $\frac{106}{117}$, which being abated from 10 l', there wil remaine 17 shillings $\frac{11}{117}$, and so much I doe lose upon the 100 pound of money.

If I take up at London 20 shillings sterl. to pay at Antwerpe 22 s — 4 d, and when the day of payment is come, my Factor is constrained to take up money againe at Antwerpe, wherewith to pay the aforesaid summe; and there hee doth receiue 23 s — 4 d Flemmish, for the which I must pay 20 s at London: the questiō is now, whether I do win or lose? and how much upon the C pound of money after that rate?

Answere. Say by the Rule of Proportion. If 22 ^{1} s giue 23 $\frac{1}{3}$ s, what will 200 pounds giue?

Mul-

Multiply and diuide, and you shall finde 104 pounds 9 s $\frac{17}{67}$, from the which abate 100 l, & there will remaine 4 pounds 9 s $\frac{17}{67}$, and so much is there gained vpon the 100 pounds of monie.

In Antwerpe is deliuered 200 pounds Flemish by exchange for London at 20 s sterling for euery 23 s—4 d Flemmish. The question is, at what rate the same is to be returned to gaine 10 pounds vpon the 100 pounds Flemmish in Antwerpe.

Answere. First, say by the rule of thrée, if 23 $\frac{1}{3}$ Flemish giue 20 s, what shall 200 poũds giue? Worke, and you shall finde 171 pounds ——8 s——6 $\frac{6}{7}$ d. Then say againe by the rule of thrée direct, if 171 l—8 s—6 $\frac{6}{7}$ d sterling, giue me 110 l Flemmish, what shall 20 s sterling giue? Worke and you shall find 24 s 6 d Flemmish. And at the same rate ought the same to be returned at Antwerpe to gaine 10 l vpon 100 Flemmish.

A Merchant of Antwerpe deliuereth 234 l—3 s—4 d Flemmish, to receiue at London 200 pounds sterling. The questiõ is now, how the exchange goeth after this rate?

Answere. Say by the rule of thrée direct, If 200 giue 20, what doth one giue? Multiply and diuide, and you shall finde 23 shillings ——5—d. And for so much goeth the Exchange.

Item, the Exchange from London into France

France, is not like as it is in Flaunders, but is deliuered by the French Crowne, which is worth 50 soulx Turnois the peece.

Wherupon also you must note, that in France they make their accounts by Francks, Soulx, & Deniers Turnois, whereof 12 Deniers maketh one Soulx Turnois, & 20 Soulx maketh 1 l Turnois, which they call a Liure or Franke. But the Merchants to make their accounts do vse French crowns: which is currant among them for 51 soulx Turnois. But by exchange it is otherwise, for it is deliuered but for 50 soulx Turnois the Crowne, or as the taker vp of the money can agree with the deliuerer. And note that this △ character representeth the Crowne by exchange, and is euer 50 soulx Turnois for French money.

A Merchant deliuereth at London 240 l sterling after 5 shillings — 6 pence ÿ crowne, to receiue at Paris 50 soulx Turnois for euery Crowne. I demand how much Turnois or French money payeth the bils for the said 240 pounds sterling?

Answere. Say by the rule of three, if $\frac{1}{2}$ s sterling giue me 50 s Turnois, what shall 240 l sterling giue? Reduce the pounds into shillings, then multiply and diuide, and you shal finde 2181 Liures — 16 Soulx, 4 — Deniers

niers, and $\frac{4}{11}$ Turnois, and so much payeth the bils at Paris for the 240 pounds sterling.

A Merchant delivereth at Roan, or elswhere in France 1430 pound or Francks, the which franck or pound is 20 soulx, or a pound Tournois, to receive in London 6 s 4 d sterling for every △ of 50 soulx Turnois. The question is, how much sterling money I ought to receive at London for my 1430 pound Turnois.

Answere. Say, if two $\frac{1}{2}$ pounds give me $\frac{1}{3}$ s, what wil 1430 give me? Worke, and you shal finde $3622\frac{2}{3}$ sterling, which maketh 181 pounds 2 s 8 pence, and so much money is to be received at London for the said 1430 Livres Turnois, after 6 s 4 pence every △ of 50 soulx.

In London is delivered 200 pound sterling by exchange for Paris, at 5 s—9 d ȳ △ of 50 soulx Tournois. The question is, at what price the said △ is to bee returned to gaine six pounds upon the C. pounds sterling at London.

Answere. First say (by the rule of three direct) if $5\frac{3}{4}$ s sterling give 50 soulx Tournois, what shall 200 pounds sterling give? Work, & you shal finde 1739 Francks or Livres, two soulx $\frac{14}{23}$. Then, the which to returne and gain 6 pounds upon the 100 l in London, say by the rule of 3 direct, if 1739 Francks, two soulx $\frac{14}{23}$
yeeld

yeeld 212 pounds, what the △ of 50 Soult? worke, and finde 6 s—1 d $\frac{7}{50}$ the effect required in the question.

A Merchant delivered in London 160 l' sterling, to receive in Biskay for every 3 s 6 d, 1 Ducket of 374 Marveids. The question is, how many Marveids ought I to receive at Biskay?

Answere. Say, if 5 $\frac{1}{2}$ s sterling give 374 Marveids, what shall 160 l. sterling give? Multiplie and divide, & you shall find 217600 Marveides, and so many I ought to receive at Biskay for my 160 pounds sterling.

A Merchant delivered in Bayon 40000 marveides to receive in London 5 s -- 8 d sterling for every Ducket of 274 Marveids. The question is now, how much sterling money payeth the Bils of Exchange for the said 20000 Marveids?

Answere. Say, if 374 Marveids make 1 Ducket, what 40000 Marveids? Multiply and divide, and find 106 Duckets $\frac{178}{176}$

Then say againe, if 1 Ducket give 5 $\frac{1}{3}$ s, what giveth 106 $\frac{178}{187}$ Duckets? Worke, and find 30 pounds———6 s, and $\frac{34}{561}$ s: which is worth $\frac{408}{561}$ parts of a peny.

Otherwise it is wrought more briefe at one working, as in the last question before, in

considering that 5 s,8 d containeth one Duckate, or 374 Maruedoes. Therefore say by the rule of three, if 374 Marueides giue 5 $\frac{2}{3}$ s, what 40000 Marueides? Worke, and you shall also finde in your quotients 30 pounds 6 s $\frac{34}{561}$. And so many pounds sterling is to be receiued for the 40000 Marueids.

In London 200 l deliuered by Exchange for Vigo, 374 Marueids the Duckate of 5 s, 10 d sterling, maketh 25647 $\frac{1}{7}$ Marueids: the which to returne and gaine 10 l vpon the 100 pounds in London, say by the rule of 3 direct, if 220 pounds require 25457 $\frac{1}{7}$ Marueids, what 5 s 10 d? worke, & find 340 Marueids, prices of euery Duckate in returne: which is the effect in the question required.

These may seeme sufficient for instructions.

NOtwithstanding for the further aid and benefit, hereafter followe 6 speciall & most briefe rules of Practise for English, French, & Flemmish money.

1	teacheth	how to turne Flem. to Eng. sterling.
2		how to turne Eng. sterling to Flem.
3		how to turne Flemmish to French.
4		how to turne French into Flemish.
5		how to turne sterling into French.
6		how to turne French into sterling.

The fifteenth Chapter intreateth of the said six Rules of Breuitie, and of valuation of English, Flemish, and French money, *and how each of them may easily be brought to other* value.

How briefly to reduce pounds, shillings, and pence Flemish, into pounds, shillings, and pence English sterling.

It is to bee noted, that 7 l Flemish maketh but 6 pounds sterling: 7 s flemish, maketh 6 s sterling, and 7 d flemish 6 d sterling, so that 7 yeeldeth but 6. Wherin is euident, that then is lost $\frac{1}{7}$, (if it may be so called) when it is reduced into English money. Wherfore to know how much 233 l, 13 s, 4 d, flemmish maketh English, you must subtract from it $\frac{1}{7}$, beginning with the pounds, &c. & that which resteth after this subtraction is the summe required: so that 233 l, 13 s, 4 d flemish maketh 200 pounds, 5 s, 8 $\frac{4}{7}$ pence sterling.

Example.

l	s	d
233	13	4
$\frac{1}{7}$ 33	7	7 $\frac{3}{7}$
200	5	8 $\frac{4}{7}$ ster.

Another example

l	s	d
311	0	0
$\frac{1}{7}$ 44	8	6 $\frac{6}{7}$
264	11	5 $\frac{1}{7}$

To reduce pounds, shillings, and pence Sterling, into pounds shillings and pence Flemmish.

Note, that a pound sterling maketh 1 l, 3 s, 4 d flemish that is, 1 l $\frac{1}{6}$: 1 s sterling maketh 1 s $\frac{1}{6}$ flem. & 1 ster. maketh 1 $\frac{1}{6}$ flem. So that there is gained (if it may be so called) $\frac{1}{6}$ of the sum beeing thus reduced to flem: for of $\frac{6}{6}$ is made $\frac{7}{6}$, which is one whole and $\frac{1}{6}$. Then to know how much 237 l, 7 s, 6 d ster. maketh flem. subtract from your ster. the $\frac{1}{6}$ of the whole sum, and adde it to the same sum, & it maketh 276 l, 18 s, 4 d, which is the sum required.

Example.

l	s	d
237	7	6
$\frac{1}{6}$ 39	11	3
276	18	9 ster.

Another Example.

l	s	d
337		
$\frac{1}{6}$ 56	3	4
393	3	4

Ye shall note that the equality of flemmish and french money is this, that is to say, the pound flemmish maketh 7 pound $\frac{1}{7}$ french, or Tournois, 1 s flemmish maketh 7 s $\frac{1}{7}$ french, and a groat flemish, maketh 7 d $\frac{1}{7}$ french.

Wherefore to know how much 143 l 4 shillings 9 d flemmish maketh french, yee must multiply the whole number twice by 6, begin-

ning at 8 d, and so forward, and the product of your second multiplication diuide by 5, so the worke is finished. Or multiply the said sum by 7, and take out of it $\frac{1}{5}$, adding it to the product of your multiplication by 7, & that is your number required. So that as well by the one as by the other, 143 l, 4 s, 9 d Flemmish, maketh 1031 l—6 s—2 d $\frac{2}{5}$ French or Turnois.

Example.

l	s	d
143—	4 —	9 flem
		6
859 —	8 —	6
		6
5156 —	11 —	0
$\frac{1}{5}$ 1031 —	6 —	2 $\frac{2}{5}$ fren

The same otherwise.

	l	s	d
	143 —	4 —	9
			7
	1002 —	13 —	3
$\frac{1}{5}$	28 —	12 —	11 $\frac{2}{5}$
	1031 —	6 —	2 $\frac{2}{5}$ frē

Another example.

143 l Flem
6
858
6
5148
$\frac{1}{5}$ 1029 l. or 12 s Fren.

Or thus.

143
7
1001
$\frac{1}{5}$ 28 — 12
1029 l — 12 — french.

A briefe Reduction of l, s and d, French, into l, s, and d Flemmish.

Multiply 233 l— 8 s 4 d French by 5, & divide the product twice by 6, that is, the said number by 6, & the product again by 6, and the quotient of this second division is the thing required. So that 233 l—8 s—4 d French, maketh 32 l, 8 s, 4 d $\frac{5}{9}$ Flemmish.

Example. Another.

l	s	d	l	s	d
233	—8—	4 fren.	753		Fren.
		5	5		
1167	—1	—8	2756		
$\frac{1}{6}$ 194	10—	3 $\frac{2}{6}$	$\frac{1}{6}$ 627—	10	
$\frac{1}{6}$ 52—	8—	4 $\frac{5}{9}$ Fle	$\frac{1}{6}$ 104—	11—	8 flem

To reduce l. s, and d Sterling into l, s, and d French, or Tournois.

The l sterling maketh 8 l, 8 s French, that is to say, 8 l $\frac{2}{5}$: the s maketh 8 s $\frac{2}{5}$, and the peny 8 d $\frac{2}{5}$ French. Wherefore to know what 231 l 13 s, 4 d sterling maketh French, yee must multiply your whole summe by 42, that is, by 7, and the product of it by 6, and divide this

ſecond product by 5, and that is the ſumme required.

Otherwiſe, multiply the ſumme ſterling by 8, and adde twice to the product $\frac{1}{5}$, and it ſhall produce the ſum required. So that both waies 231 l, 13 s 4 d ſterling, maketh 1946 pound french, as hereunder followeth.

Example. | The ſame otherwiſe.

l	s	d	l	s	d
231	13	4 ſter.	231	13	4 ſter
		6			8
1390	0	0	1853	6	8
		7	46	6	8
$\frac{1}{5}$ 9730	0	0	46	6	8
$\frac{1}{5}$ 1946	0	0 fren.	1946	0	0 fren

Another example. | The ſame.

Another example	The ſame
753 ſterl.	753 ſter.
6	8
4518	6024
7	$\frac{1}{5}$ 150 — 12
31626	150 — 12
$\frac{1}{5}$ 6325 — 4 fren.	6325 — 4 fren.

To

To reduce pounds, ſhillings, and pence French, into pounds, ſhillings and pence Sterl.

To know how much 1256 l, 12 s, 6 d fren. maketh in ſterling money, multiply the ſum by 5, and diuide the product by 7 & 6 at twice; and the laſt quotient ſhall bee the thing required, that is to ſay, 1256 l, 12 s, 6 d, maketh 149 pounds, 11 s, 11 d $\frac{4}{7}$ ſterling.

Example. Another.

l	s	d	l	s	d
1259	12	6 fren.	2531		Fren.
		5	5		
6283	2	6	12755		
$\frac{1}{6}$ 1047	3	9	$\frac{1}{6}$ 2109	3	4
$\frac{1}{7}$ 149	11	$\frac{4}{7}$ ſter.	$\frac{1}{7}$ 301	6	2 $\frac{2}{7}$ ſter.

Note, that when any money is giuen by exchange at London for Roan at 7 d $\frac{1}{3}$, or rather 71 $\frac{1}{7}$ for the crowne of 50 s french, there is neither gaine nor loſſe: for it is one money for another, accounting 818 s french for 2 pound ſterling. So the giuer loſeth the time of payment, which is about 5 daies, and hee that taketh it, hath the gaine of the ſame.

They of Roan that put forth or take money by exchange for London, ought to haue like conſideration.

Item when any man giueth at London 64 pence $\frac{1}{7}$, or rather 64 d $\frac{2}{7}$, to haue at one of the Faires of Lyons a crowne de Marc, he that so giueth the money looseth the time, and he that taketh it, gaineth the same: for 62 pence $\frac{2}{7}$ is equal in value to 45 s French. He that putteth or taketh money at Lyons for London, ought to consider the same.

Item, when any deliuer in Antwerpe 75 d, to receiue at Lyons a crowne of Marc, hee that putteth it forth, loseth the time, and hee that taketh it, gaineth the same. For 75 groates Flemmish is equal in value to 45 s French.

Thus for this time I make an end of the practise of exchange and the instructions thereunto belonging: & according to my promise, yet further to gratifie such as are desirous to know the common coines vsed for trafficke among Merchants in these Cities following, here followeth a briefe declaration of their moneys, and the reckonings, & accounts of them.

The

The sixteenth Chapter containeth a declaration of the valuation and diuersity of Coynes of most places of Christendome for trafficke; And the maner of exchange in those places from one Citie or Towne to another: which knowne is right necessary for Merchants, by meanes whereof they do find the gaine or losse vpon the exchange.

ITem for as much as the greatest diuersity of money of exchange is at Lyons, therefore I will begin duly of the money of that place.

At Lyons they vse Franckes, Soulx, and Deniers Tournoies. A Francke maketh 20 Soulx, and one Soulx 12 Deniers. But the Merchants to keepe their bookes of accounts doe vse French Crownes of the marke at 45 soulx the péece, and doe diuide it into 20 s, 1 s and 12 d.

Item, a Marke of Gold maketh 65 △ of the Marke, which serueth for exchange, and diuide it into 8 ounces. The ounce into 24 pence or deniers, the denier into 24 graines, and so the summe or whole by imagination or ghesse.

Also at Lyons there are 4 faires in a yeere, at the which they doe commonly exchange, which are from thrée moneths to 3 moneths.

At

At Geanes they vse the soulx: one Ducket maketh l 3.

At Naples they vse Duckets, Taries, and Graines, the Ducket maketh 5 Tares, and one Tarie 20 graines: but they take 6 Duckets (which maketh thirtie Taries) for the ounce.

A Ducket maketh ten Carlins, and a Carlin ten graines, so that 2 Carlins make a Tarie, and 100 graines make a ducket.

At Rome they vse the duckets of the Chamber: one ducket is worth 12 Guylis, and one Guylis 10 soulx.

At Venice they vse duckets Currant at 124 soulx a péece or 24 deniers, and one denier maketh 32 picolis.

At Palerme and Messine they write, after ounce, tarie, and graines, & 1 ounce is worth 6 duckets or 30 taries, and 1 tarie is 20 grains, and 1 graine 6 picolis, 1 ducket is also worth 24 Carlins.

At Millan they vse l, s, d, of Ducket Imperials, and △ of exchange is worth 4 l.

At Lucques, Florence and Ancone, they vse the △ of Gold: in Gold the french crowne is worth l 7, but at Boloigne l 3, 10 s.

At Barselone they vse the soulx; the ducket of exchange is worth 22 soulx.

At Valence and Saragosse they vse the Liuer, Soulx and denier; the French crowne of exchange is worth 20 soulx, and 1 soulx is

12

12 deniers.

At the Faires of Castill they vse the Marueides, the ducket is worth 375 Marueides.

At Lisbone they vse the Raies, one ducket of exchange is worth 400 rayes.

At Noremburge, Franckford, and Augusta in Germany, they vse the Krentzars, whereof 60 make a flozen.

At Antwerpe they vse s, & d de Gros, and they exchange into the denier de Gros, to wit, our English peny.

At London they vse the 1 l sterling, and 1 d sterling, and they exchange in 1 d sterling.

The exchange of Lyons at sundry places.

Item at Lyons there is exchange in thrée sorts, at the cities and townes following.

First they deliuer at Lyons one Marke to haue or receiue at Naples almost 41 ½ duckets: at Venice 70 duckets currant, at Rome 63 duckets of the Chamber, at Lucques and Florence 65 △ of Gold, at Millan 82 △.

And contrariwise, at the said Cities aforesaid, they doe giue so much of money to haue a marke at Lyons.

Secondly, they giue at Lisbone one △ of Marke of 45 soulx Tournois a péece, to haue at Geanes almost 68 soulx, at Palerme and Messine almost 24 Carlins, at Barselone 22 soulx, at Valence or Saragosse 20 soulx, at the faire at Castill 350 Marueides, at Lisbone

bone 360 Raies, in Antwerpe 57 deniers de Gros, and at London 70 d sterling.

And contrariwise they giue in the said Cities almost as much of their money to haue a French crowne of the make of Lyons.

Thirdly, they doe giue at Lyons ▵ of the sunne to haue almost 93 Kreutzers at Franckford, Ausburg, Noremberge, or other Cities in Almaine.

Also at Lyons onely they do pay the change, the ⅔ in gold, and ½ in money, or else all in money, in giuing 1 ½ for the hundreth.

Changes at Naples and other townes.

Item at Naples they giue or deliuer almost 112 duckets to receiue at Rome 100 duckets of the Chambes at the old value.

Through Luques and Florence they deliuer 100 duckets Carlins, to receiue there almost 86 ▵ of gold.

Through Palerme and Messine one ducket of 5 Tarie, to receiue there almost 164 grains.

Through Millan one ducket to receiue there almost 90 soulx.

Through Geanes one ducket, to receiue there almost 65 soulx. The whole summe to be paid within 10 daies after the sight of the bill of Exchange.

Also at Naples they deliuer one ducket to receiue in Antwerpe almost 67 d or deniers de gros, within 2 moneths. At London almost

most 60 d sterling in thrée moneths. At Barselone almost 20 soulx within two moneths. At Valence almost 18 soulx within 2 moneths, at Lisbone 333 Raies, within thrée moneths, and at the faire at Castill, almost 340 Marueids at the same faire.

Change of Venice to other places.

At Venice they deliuer 100 Duckets currant to receiue in Almaine almost 140 Florenes at 60 Krentzers the péece.

At Lucques and Florence almost 108 △ of gold in 10 daies.

Likewise at Venice they deliuer a Ducket currant to receiue at Palerme and Messine almost 21 Carlins, at Millan almost 93 soulx: at Geanes almost 62 soulx, the whole at tenne daies end.

Of the Pair or Pari.

As touching the exchange, it is necessary to vnderstand or know the Pair, which the Italians call Pari; which is no other thing then to make the money of the change of one City or Towne, to or with the money of another, by meanes whereof they doe finde the gaines or losse vpon the exchange.

Example.

Item hauing receiued letters of credite of one of Antwerpe, that the △ of the Sunne is there worth 7 soulx: The question is, what the same is worth at London, when the Pair

or

or exchange goeth for 23 shillings.

Answere. Say, if 23 giue but 20, what giueth 7? Worke and finde 6 s, 1 $\frac{1}{23}$ d, & so much is the ▵ of the Sunne worth at London.

The seuenteenth Chapter containeth also a declaration of the diuersitie of the waights and measures of most places of Christendome for trafficke. At the end of which discourse are two Tables, the one for waight, and the other for measure, proportionate and reduced to an equalitie of our English measure and waight, by the side whereof the ingenions may easily by the rule of three, conuert the one into the other at their pleasure, &c.

At London, and so all England through, are vsed two kinds of waight and measures, as the Troy waight, & the Haberdepoise: from the Troy waight is deriued the proportion and quantitie of all kind of dry and liquid measures, as Pecks, Bushels, Quarters, &c. wherwith is bought and sold all kind of grain and other commodities mette by the Bushell. And in liquid, Ale, Béere, Wine, Oyle, Butter Hony, &c. vpon these grounds and statutes is bread made, and sold by the Troy waight. And so is gold, siluer, pearle, precious stones, and Jewels. The least quantity of this Troy

waight

waight is a graine: 24 of these graines make a peny waight, 20 penny waights an ounce, and 12 ounces a pound: 2 pounds or 2 pintes of this waight maketh a quart. And so ascending into bigger quantities is produced the measures whereby are sold our other naturall sustenance: viz. Ale or Beare, with all other necessarie commodities, as Butter, Hony, Herings, Eeles, Sope, &c. All which last before rehearsed, though their measures (wherein they are contained) bee framed and derived from the Troy waight, yet are they in trafficke with divers commodities, as Lead, Tinne, Flax, Waxe, with all other commodities, both of this Realme, and of other forraine Countries whatsoever, bought and sold by the Haberdepoise waight, after 16 ounces to ẏ poũd, and 112 pound to the C. waight. And unto every C is allowed 12 l waight at the common beame. From hence is also derived the waight of Suffolk Chéese, which containeth 32 cloues, 8 l to a cloue, and wayeth in all 256 pounds. And also the barrel of Suffolk Butter is or should be of like waight with the waight of chéese, viz. 256 pounds. More, 14 of these pounds make a stone, and 26 stone containeth a sacke of English wool. Forrain wools, to wit, French, Spanish, and Estrich, is also sold by the pound or C. waight, but most commonly by the Roue, 25 pound to a Roue: other commodities of Tale are bought and sold by the C.

fiue-

finescore to the C. Except headed ware, to wit cattell, nailes, and fish, which are sold after sixe score to the C. There is also two other sorts of measures, to wit, the Elle, and the Yeard. By the Elle is usually mette Linnen Cloth, as Canuas, &c. And by the yeard, silkes, woollen clothes, &c.

Antwerpe.

At Antwerpe are also 2 sorts of weightes, their gold and siluer waight, and their common waight. Gold and siluer is weighed by ẏ Marke, the Marke is 8 ounces, the ounce 20 Esterlings, and the Esterling 32, as our graines. The Goldsmiths diuide that into smaller, but not the Merchants: the proofe of gold is made by caracts, whereof 24 maketh a Marke of fine gold: the carack is 24 graines, the proofe of the money is made by Deniers, 12 deniers is 1 s fine, that is, a Marke of fine siluer; the denier also is diuided into 24 graines, and the graine into foure quarters.

Rule.

Item, 100 Markes in Antwerpe, Troy waight, maketh at Lyons 103 Markes, two $\frac{1}{2}$ ounces, and 20 graines 23 l. At Noremburge 103 Markes, 2 $\frac{1}{2}$ ounces, 2 quints, 3 deniers, at Frankford 105 Markes. at Ausburge 104 Markes, three ounces, 1 quint. At Venice 103 Markes, 1 ounce, 7 deniers, 18 graines. At London 66 pounds.

The Marke of Gold or siluer at Antwerpe,

Troy waight, which is 8 ounces, maketh 7 $\frac{5}{3}$ ounces common waight, with which all other merchandise is weighed. So that the Troy waight is greater then the common waight by 6 $\frac{1}{4}$ in the C. By this waight of Troy, they also weigh Muske, Amber, Pearle, &c.

All silkes are bought at Antwerpe, by the Burges elle, which is greater than the common measure, by which they retaile by two in the hundred. Their common elle is $\frac{3}{4}$ of our yard, and $\frac{3}{5}$ of our elle.

Lyons.

At Lyons is used 3 sorts of waight, whereof the first is the common towne waight, with which they weigh all kinde of Spicery, and divers other merchandise. The second is called Geneua waight, which is 8 in the C greater then the common weight, with which they weigh silkes, &c. The third is French weight, called commonly the Marke weight, and 100 pounds thereof maketh 106 $\frac{1}{4}$ pound Geneua, & 114 $\frac{1}{4}$ of their common weight: with which Frẽch weight is weighed all things that paied custome or toll.

At Lyons is also used two sorts of elles or Aulnes. The one wherewith they measure grosse Clothes, as Canuas, and such like. The other is called the French Elle or Aulne, with which they measure all other kinde of Merchandise, whereof seuen common towne elles

maketh 11 ordinary French Elles.

Roan.

At Roan 6 $\frac{1}{2}$ Muydes of salt being the measure of the place, make an hundred at Armuiden in Zeland, & the C. of Bronage measure of Armuiden maketh at Roan 11 Muydes, 30 Mines make a last of Corne, and 16 a Last of Dates, 100 pound weight there, maketh at London 114 $\frac{1}{4}$, and 109 $\frac{2}{8}$ at Antwerpe. And 100 Elles make at London 115 $\frac{2}{8}$.

Noremburge.

A 100 pound weight at Noremburge maketh at London 111 $\frac{3}{4}$: at Antwerpe 107 $\frac{1}{2}$, And 100 Elles at Noremburge make at London 74 $\frac{2}{5}$, at Antwerpe 95 $\frac{3}{5}$, &c.

Lisbone.

The 100 weight at Lisbone maketh foure Roues, every Roue 32 pounds, so that their C. weight is 128 pounds, and their pound containeth 14 ounces, and a 100 pounds of their weight maketh at London 113 $\frac{1}{8}$.

Their silke, cloth of Gold, and Woollen is measured with a measure which they call a Cubite, containing about $\frac{3}{4}$ of a Warre of Castile. Howbeit, their common measure is called a Warre, which maketh 5 Palmes, and cōtaineth 1 $\frac{1}{4}$ of Warre of Castile, our Elle of London is equall with the Warre of Lisbone.

All

All kind of merchandise brought from Flã-ders, Roan, or Britaine payeth at Lisbone, as a dutie or custome to the king, 20 in the 100, which they call the tenth in merchandise, and the other tenth in money.

Note also that all kind of merchandise comming to Lisbone by land, payeth lesse in custome then that that commeth by water.

Ciuill.

The Roue of Ciuill is 30 pound, 4 Roues make their C. waight, which is 120 pounds. The 100 pounds of Ciuill maketh at London 102 pounds. Their other common measure is a Varre, whereof 100 maketh at London 74 Elles and at Rome 40 Canes, &c.

Venice.

At Venice be two sorts of waights, the one called La Grosse, the other La Suttle, with the grosse is weyed all kind of great wares, and with the small all kinde of spicerie, and such like: 96 poũds of grosse waight there maketh at London 100 pounds, and 100 pounds of spicerie there without any tare or allowance, make at London 64, and with tare 65.

Their other common measure are Braces, whereof 100 make at London 55½ Elles, at Antwerpe 92½, &c.

Florence.

At Florence the 100 l waight maketh at

Aquila, for Saffron 110, & 145 pounds of Florence make at Roane but 100 pounds, the weight of Florence and that of Luke is al one.

Their other measures are braces, whereof 100 maketh at Antwerpe Burges measure, 81 $\frac{2}{3}$ elles. 100 Braces there make at London 49 elles, &c.

Lucque.

The Lucque Sattens are commonly sold at Lyons by weight, & 133 $\frac{1}{2}$ pounds, maketh at Lions 100 pound, so that 1 pound $\frac{1}{3}$ maketh at Lyons but one pound.

Their other measures are Braces, whereof 100 of them make at London 50 Elles, at Antwerpe 83 $\frac{1}{3}$ Elles, &c.

Aquila.

At Aquila their 100 pounds maketh at London 71 $\frac{1}{4}$, their 136 $\frac{2}{3}$ pounds of Saffron maketh at Geneua but 100, and 11 pound of Geneua maketh 15 pound at Aquila.

Valentia.

At Valentia be 2 sorts of weights, a great and a small. The 100 weight of great weight containeth 4 Roues, the Roue 36 l, so the 100 great weight is 144 l, and the C weight smal containeth but 120 pounds, and is also parted into 4 Roues, which is 30 pounds to a Roue. By the small is sold the Scarlet grain, with al other kind of spicery, and by the great is sold wooll, with all such like grosse wares: The $\frac{5}{7}$ pounds

pounds of silke at Valentia, maketh at Lyons 1 pound Geneua weight. The charge of great merchandise at Valentia containeth 432 l, and in small wares 360 pounds. The weight here and at Barsellone is all one. Their 100 pound weight maketh at London 78 l, and at Antwerpe 75.

Dansicke.

At Dansick or Spruce-land the rule is, that whosoeuer buyeth any Merchandise there, buyeth it by the ship-pound, which is 320 l, 20 Lispounds make a Ship-pound, and ye Lispound containeth 16 l, which Ship-pound of Dansicke maketh at Antwerpe 266 $\frac{2}{6}$ l. Their 100 l weight maketh at London 86 $\frac{5}{8}$, &c.

Their other common measures are Elles, whereof 100 make at London 72 $\frac{1}{4}$, & at Antwerpe 120 $\frac{1}{2}$ Elles.

Toulouse.

At Toulouse 6 Cabes of Woad maketh a Charge, two Cesternes of Corne, and all kind of graine maketh a Charge, the Cesterne weigheth 160 l weight of that place. Their C l in weight, maketh at London but 91 $\frac{1}{4}$ l.

Geanes.

At Genua or Geanes, C. l of their weight maketh at London 71 $\frac{1}{4}$, and at Antwerpe 67 $\frac{3}{8}$, a 100 l weight at Genua maketh at Venice, to wit, Suttle 106 l.

Their other common measures are Palms, whereof 100 make at London 20 $\frac{3}{5}$ Elles, and at Antwerpe 34 $\frac{3}{5}$.

The eighteenth Chapter treateth of Sports, and Pastimes done by Number.

If you would know the number that any man doth thinke or imagine in his minde, as though you could diuine, bid them triple it, or put twice so much more to it as it is; which done, aske him whether it be euen or odde: if he say od, bid him take one to it, to make it euen, and for that one keep one in your minde. Now after he hath taken one to it to make it euen, bid him giue away halfe, and keepe the other halfe for himselfe, which when hee hath done, bid him triple that halfe, and again after he hath tripled it, aske him whether it be euen or od; if hee say od: then bid him take one to make it euen againe, and for that last one keep two in your minde, now after hee hath made his number euen, bid him cast away the one halfe, and keep the other stil, from which halfe that he keepeth, cause him subtilly to put away or giue you 9 out of his number, and for each 9 that he giueth you, keep 4 in your minde, and thereunto ioine the 3 which I bad you keepe, and you shall haue your desire.

Ex-

Example.

Imagine he thought 7, the triple whereof is 21, and because it is odde, he is to take 1 to make it euen, which first, 1 giuen is one for you to keepe in minde. Then the halfe of his 22 beeing cast away, hee reserueth still 11, which after you haue bid him triple, it maketh 33, then in giuing of him one againe to make it euen: vpon that last 1 reserueth 2 in your minde, then his halfe of 34 maketh 17, from whence he can giue you 9 but once. Therefore that yeelding to you 4, and the 3 that you keep, make 7, your desire.

Another kinde of Diuination, to tell your friend how many pence or single peeces, reckoning them one with another, hee hath in his purse, or should thinke in his minde.

Which to do, first bid him double the peeces he hath in his purse, or the number hee thinketh, if he participate his number or secrecie vnto some one friend that sitteth by him, that can but multiply, and adde neuer so little: if their number be great, then shall they worke as you bid them so much the surer.

Now after hee hath doubled his number, bid him adde thereunto 5 more, which done, bid him multiply that his number by 5 also: which done, bid him tell you the iust summe of

of his laſt Multiplication, which ſumme the giuer thinking it nothing auaileable, becauſe it is ſo great aboue his pretended imagination: yet thereby ſhall you preſently with the helpe of Subtraction tell his propoſed number.

The Rule is this.

Imagine hee thought 17: double 17, and it maketh 34, whereunto if you adde 5, it maketh 39: which multiplied by 5, as here is practiſed in the margent, it yeeldeth 195, which 195 is the ſumme deliuered you in the worke: then for a generall rule, you ſhall euermore cut off the laſt figure towards your right hand, with a daſh of your pen, as here is performed, as a figure nothing auaileable vnto your worke, and then rebate 2 from your firſt figure, after 5 is cut off, and the reſt ſhall euermore be your deſire, as by this example doth appeare.

17
2
———
34
5
———
39
5
———
19|5
2|
———
17

If in any company you are diſpoſed to make them mery by maner of Diuining, in deliuering a King vnto any one of them, which after you haue deliuered it vnto them, that you will abſent your ſelfe frō them & they to deuiſe after you are gone, which of them ſhall haue the keeping thereof, and that you at your returne will tell them what perſon hath it, vpon what hand, vpon what finger, and what ioint.

Which

Which to doe, cause the persons to sit downe all on a rowe, and to keepe likewise an order of their fingers: now, after ye are gone out from them to some other place, say vnto one of the lookers on, that he double the numbers of him that hath the King, and vnto the double, bid him adde 5, and then cause him to multiply ẏ Addition by 5, and vnto the product, bid him adde the number of the finger, of the person ẏ hath the King. And lastly, to end the worke, beyond that number towards his right hand, let him set downe a figure signifying vpon which of the ioints he hath the King, as if it be vpon the second ioint, let him put downe 2. Then demaund of him what number he keepeth, from the which you shall abate 250. And you shall haue three figures remaining at the least. The first towards your left hand shall signifie the number of the person which hath the King, the second or middle number shall declare the number of the finger, and the last figure towards your right hand shall betoken the number of the ioint.

Example.

Imagine the seuenth person is determined to keep the King vpon the fift finger, and the third ioint: first double 7, it maketh 14, thereto adde 5, it maketh 19, which multiplied by 5, yeeldeth 95: vnto which 95, adde the number of the finger, and it maketh 100: and beyond

yond 100 toward the right hand, I set downe 3 the number of the ioynt, all maketh 1003, which is the number that is to bee delivered you, from which abating 250, there resteth 753, which prefigureth unto you the seventh person, the fifth finger, and the third ioint.

But note, that when you have made your subtraction, if there do remaine 0: in the place of tennes, that is to say, in the second place, you must then abate 1, from that figure which is in the place of hundreds, that is to wit, from the figure which is next your left hand, and that shall bee worth 10 tenths, signifying the tenth figure, as if there should remaine 803, you must say, that the seventh person upon his tenth finger, and upon his third ioint, hath the King.

And after the same manner, if a man doe cast 3 Dice, you may know the points of every one of them. For if you cause him to double the points of one Dye, and to the double to adde 5, and the same sum to multiply by 5, & unto the product adde the points of one of the other Dice. And behind the number towards the right hand, to put the figure which signifyeth the points of the last Die, and then to aske what number hee keepeth, from which abate 250, and there will remaine 3 figures, which doe note unto you the points of every Die.

An-

Another.

If thrée diuers things are to bee hidden of thrée diuers persons, and you to diuine, which of the thrée persons hath the thrée diuers things, doe thus: imagine the thrée things to be represented by A B C. Then secondly kéepe well in your minde which of the persons you meane to be the first, second and third. Then take 24 counters or stones, and your thrée things, and giue A to the party whom you imagine to be your first man, and therwithall giue him one of your 24 counters in his hand. And B vnto your second man, and therewithall 2 counters. And C vnto your third man, and therewithall 3 counters: and leaue the rest which are 18 still among them, which done, separate your selfe from them, and afterwards bid them change the things among them as they shall thinke good: which done, after they are agréed, bid him that hath such a thing, as before you haue represented by A, for euery counter that he hath in his hand, to take vp as many more. And for him that hath B, for euery one in his hand to take vp two. And for him that hath C, for euery one in his hand to take vp 4, and the rest of them to leaue still vpon the boord. These thrée things and the thrée persons béeing fully printed in your mind, come to the Table, and you shall euermore finde one of these six numbers, 1, 2, 3, 5, 6, or 7. If therefore one remaine still vp-

on

on the boord, thē haue they made no exchange, but keep them still as they were deliuered vnto them. So that the first man hath A, the second B, and the third C. But if 2 remaine, then the first man hath B, your second man A, and your third man C. The rest of the worke and the order thereof are here apparant by the Table following.

	1	A		1	B
1	2	B	5	2	C
	3	C		3	A
	1	B		1	C
2	2	A	6	2	A
	3	C		3	B
	1	A		1	C
3	2	C	7	2	B
	2	B		3	A

Another

Another Diuination of a number vpon the caſting of two Dice.

Firſt let the caſter caſt both the Dice, and marke well the number: then let him take vp one of them, it maketh no matter which, and looke what number it hath in the bottome, & adde all together: then caſt the dye againe, kéep in his mind what al together maketh: then let the Dice ſtand: bring ſeuen with you, and thereunto adde the reſt of the pits that you ſée vpon the vpper ſide of the ſide of the dice, and ſo many did the caſter caſt in all.

FINIS.

I.D. To the earnest Arithmetician.

MY louing friend to Science bent,
Some thing thou hast by this booke won:
But if thou wilt be excellent,
Another race thou must yet run.

Supplies thereto but fevv do neede,
And none but such as in our phrase,
(By Records *pen) thou maist vvell reade,*
Proceede therefore. Be not stunt dvvase.

The ground most sure, whereon this race
With speedfull courage must be past,
Of late hath turnd his Greekish face,
By English tilth, which ay will last.

The famous Greeke of Platoes *lore,*
Euclide *I meane Geometer:*
So true, so plaine, so fraught with store,
(As in our speech) is yet no where.

A treasure strange, that booke will proue,
With numbers skill matcht in due sort.
This I thee warne of sincere loue,
And to proceed do thee exhort.

Plus oultre

Finally the Author giueth intelligence: That if any bee minded to haue their children or seruants instructed or taught in this noble Arte of Arithmeticke, or any briefe practise thereof. Whose method is such by long custome of teaching, that (God to friend) he will bring them (if their capacitie be any thing) to their desire therein in a short time. As also to learne them to write anie maner of hand vsuall within this realme of England.

Item also after reasonable vnderstanding of Arithmetike, if anie be minded to haue them taught the famous account of Debitor and Creditor, they shall find him ready to accomplish their desire. More also, to further such as are desirous that way, in the principles of Algebar or Cossicke numbers. Lastly, to learne to draw any maner of demonstration, Deuice, or proportion. Or to learne them to draw either white or blacke capitall Letters. Or to draw or reduce any Mappe or Card in true proportion from a great quantitie to a small, or to bring a smaller to a greater. Of all or any these things rehearsed, you shall finde the Author (according to his small talent) ready to accomplish the same for a resonable reward. Whose dwelling is and hath beene these sixteene yeeres, within the Mayes-gate in short Southwarke nigh Battle bridge.

FINIS.

1

1 1

John Greenall

John Greenall

John Greenall

Leg. Lg 1-3